JUNE 2009 S2 MA

1a)

$$X \sim B(30, 0.15)$$

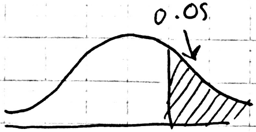
$$P(X \leq 6) = \underline{0.8474}$$

b) $60 \times 0.15 = 9$

$$Y \sim Po(9)$$

$$P(X < 13) = P(X \leq 12) = 0.8758$$

2) $2.5 \times 2 = 5$



$\therefore X \sim Po(5)$

$H_0 : \lambda = 5$

$H_1 : \lambda < 5$

$P(X \leq 1) = 0.0404$

$0.0404 < 0.0500$

$\therefore$ reject H_0, there has been evedence of decrease

3a) • A STATISTIC IS A FUNCTION OF $X_1, X_2, \cdots X$

• IT IS A QUANTITY CALCULATED ONLY FROM THE DATA

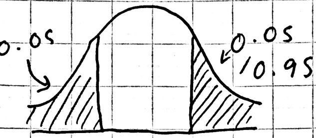
• IT HAS <u>NO UNKNOWN PARAMETERS</u>

b) SAMPLING DISTRIBUTION = PROBABILITY DISTRIBUTION OF Y.

c) (ii) CANNOT BE A STATISTIC AS 'μ' & '
ARE UNKNOWN $\therefore$ HAS UNKNOWN PARAME

4 a) $p = 0.3$

$n = 20 \therefore X \sim B(20, 0.3)$



USING TABLES FIND CLOSEST x TO 0.05 AND 0.95 IN $X \sim B(20, 0.3)$ RANGE

$P(X \leq 2) = 0.0355 = \boxed{3.55\%}$

$P(X \leq 9) = 0.9520 = 95.2\%$

$1 - 0.9520 = 0.048$

$1 - P(X \leq 9) = P(X \geq 10) = 0.048 = \boxed{4.8\%}$

critical region: $\boxed{10 \leq X \leq 2}$

b) significance level $= 0.0355 + 0.048$

$= \underline{0.0835} = 8.35\%$

c) $11 > 10$ $\therefore$ IT IS IN THE CRITICAL REGION

$\therefore$ THERE IS A CHANGE IN PROPORTION OF COSTUMERS WHO HAD BOUGHT BAKED BEANS IN A SINGLE TIN.

5 a) $\frac{3}{1000} = 0.003$

$0.003 \times 2000 = 6$

$X \sim Po(6)$

$P(X \geq 4) = 1 - P(X \leq 3)$

$= 1 - 0.1512$

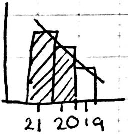
$= 0.8488$

b) $0.003 \times 8000 = 24$

$X \sim Po(24) \quad P(X \leq 20)$

NORMAL APPROXIMATION

$Y \sim N(24, (\sqrt{24})^2)$



$P(Y < 20.5)$

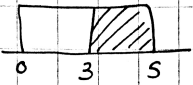
$z = \frac{x - \mu}{\sigma} = \frac{\overset{20.5-24}{\cancel{24-20.5}}}{\sqrt{24}} = -0.71$

$P(Z < -0.71) = 1 - \phi(z)$

$= 1 - 0.7611$

$= 0.2389$

6

a) 

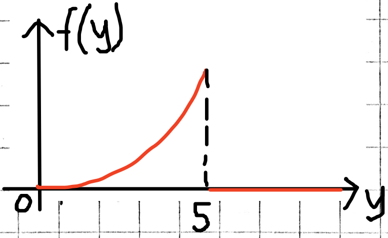
$P(A>3) = \frac{2}{5} = \underline{0.4}$

b) $0.4 \times 0.4 \times 0.4 = \underline{0.064}$

c) $f(y) = \frac{d}{dy} F(y)$

$$\frac{d}{dy}\left(\frac{y^3}{125}\right) = \frac{3y^2}{125}$$

PDF $= f(y) = \begin{cases} \frac{3y^2}{125} & 0 \leq y \leq 5 \\ 0 & \text{otherwise} \end{cases}$

) 

) mode = highest value = 5

f) $E(Y) = \int y f(y)\,dy$

$= \int_0^5 y \frac{3y^2}{125}\,dy$

$\int_0^5 \frac{3y^3}{125}\,dy = \left[\frac{3y^4}{500}\right]_0^5$

$= \frac{15}{4} - 0 = \frac{15}{4}$

g) $1 - F(3)$

$= 1 - \frac{3^3}{125} = 1 - \frac{27}{125} = \underline{\underline{0.784}}$

7a) $E(x)$ = highest point = $\underline{2}$

b) gradient from 0-2 $= \frac{\frac{1}{2}}{2} = \frac{1}{4}x$

$\therefore\ a = \frac{1}{4}x$

$b - \frac{1}{4}x = 1 - \frac{1}{4}x$

$\therefore\ b = 1$

(y intercept of gradient)

c) $\sigma^2 = \int x^2 f(x)\,dx - \mu^2$

$$= \int_0^2 x^2\left(\frac{1}{4}x\right)dx + \int_2^4 x^2\left(1-\frac{1}{4}x\right)dx - 2^2$$

$$= \int_0^2 \frac{1}{4}x^3\,dx + \int_2^4 x^2 - \frac{1}{4}x^3\,dx - 4$$

$$= \left[\frac{1}{16}x^4\right]_0^2 + \left[\frac{1}{3}x^3 - \frac{1}{16}x^4\right]_2^4 - 4$$

$$= 1 + \left(\frac{16}{3} - \frac{5}{3}\right) - 4$$

$$= \frac{2}{3}$$

$\sigma^2 = \frac{2}{3}$ $\quad\therefore\quad$ $\sigma = \sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3} = \underline{0.816}$

d) $\int_0^q \frac{1}{4}x\,dx = \frac{1}{4}$

$\left[\frac{1}{8}x^2\right]_0^q = \frac{1}{4}$

$\frac{1}{8}q^2 = \frac{1}{4}$

$q^2 = 2$

$q = \sqrt{2} = 1.414.$

e) $2 - \sigma = 2 - 0.816 = 1.184$

$2 + \sigma = 2.816$

THIS IS WIDER THAN THE IQR

$\therefore$ GREATER THAN O.S

$1.184 < 1.414$

8a) $p = \frac{2}{15}$ $\quad p$ in $15m = \frac{2}{15} \times 15 = 2$

$X \sim Po(2)$

$P(X=4) = P(X \leq 4) - P(X \leq 3)$

$= 0.9473 - 0.8571$

$= 0.0902$

b) in ~~15m~~ $60m = \frac{2}{15} \times 60 = 8$

$Y \sim Po(8)$

$P(Y > 10) = 1 - P(Y \leq 10)$

$= 1 - 0.8159$

$= \underline{0.1841}$

c) $\lambda = x \times \frac{2}{15}$

$e^{-\frac{2x}{15}} = 0.8$ $\quad$ to 2 s.f $\therefore 1.65, 1.75$

when $x = 1.65$ $\quad e^{-\frac{2x}{15}} = 0.8025$

when $x = 1.75$ $\quad e^{-\frac{2x}{15}} = 0.7918$

8c continued) When $x = 1.65$
value > 0.8
When $x = 1.75$
value < 0.8
$\therefore$ value lies between
≈ 1.7 to 2 sf.

d) no faults $= 0.8$
faults $= 0.2$

$0.8 \times 1200 = 960$
$0.2 \times 1200 = 240$

$(960 \times 0.6) - (240 - 1.5)$
$= 576 - 360$
$= \underline{\underline{£216}}$ profit