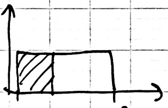
JUNE 2008 S2 MA

1) $E(x) = \underline{\underline{5}}$ $\qquad [0, 10]$

a variance$(x) = \frac{1}{12}(10-0)^2$

$= \frac{100}{12} = \underline{8.33\ldots}$

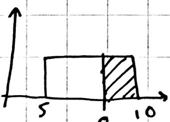
b) $P(x \leq 2)$



$= \frac{2}{10} = \underline{0.2}$

c) $(0.2)^5 = \underline{0.00032}$

d) $P(x > 8)$ $\quad [5, 10]$



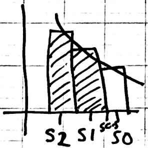
$= \frac{2}{5} = \underline{0.4}$

2) $X \sim B(100, 0.58)$

$P(X > 50) = P(X \geq 51)$

$Y \sim N(np, npq)$

$Y \sim N(58, 24.36)$



$P(Y > 50.5)$

$z = \dfrac{y - \mu}{\sigma} = \dfrac{50.5 - 58}{\sqrt{24.36}}$

$P(Z > -1.52)$

$1 - \Phi(-1.52)$

$1 - (1 - \Phi(1.52)) = \Phi(1.52)$

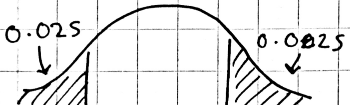
$= \underline{0.9357}$

3)

$H_0 : \lambda = 9$

$H_1 : \lambda \neq 9$

from tables:



$P(X \leq 3) = 0.0212$

$P(X \leq 15) = 0.978$

$1 - P(X \leq 15) = 0.022$

$1 - P(X \leq 15) = P(X \geq 16)$

CRITICAL REGION : $\boxed{16 \leq X \leq 3}$

b) P of rejecting $H_0 = 0.0212 + 0.022$

$= \underline{0.0432}$

4 a) $X \sim B(11000, 0.0005)$

b) mean $= np = 11000 \times 0.0005 = \underline{5.5}$

c) Variance $= np(1-p) = 11000 \times 0.9995 \times 0.0$

$= \underline{\underline{5.49725}}$

c) $Y \sim Po(5.5)$

$P(X \leq 2) = \underline{0.0884}$

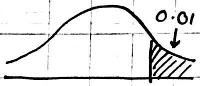
5) $X \sim B(15, 0.5)$

b) $P(x = 8) = P(X \leq 8) - P(X \leq 7)$
$= 0.6964 - 0.5000$
$= \underline{0.1964}$

c) $P(X \geq 4) = 1 - P(X \leq 3)$
$= 1 - 0.0176$
$= \underline{0.9824}$

d) p is the probability of heads
$H_0 : p = 0.5$
$H_1 : p > 0.5$

$X \sim B(15, 0.5)$



find using tables value closest to $1 - 0.01 \approx 0.99$

$P(X \leq 12) = 0.9963$

$1 - P(X \leq 12) = P(X \geq 13) = 0.0037$

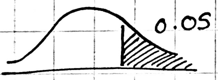
Critical region ≥ 13
$13 \geq 13$ ∴ 13 lies within the CR
∴ reject H_0 ∴ sufficient evidence tha the coin is biased in favour of heads

6a) calls occur INDEPENDENTLY and RANDOMLY.
calls occur SINGLY

b) 60 mins = 18
$\therefore$ 15 mins = 4.5
$X \sim Po(4.5)$
$P(X=5) = P(X \leq 5) - P(X \leq 4)$
$= 0.7029 - 0.5321$
$= \underline{0.1708}$

ii) $P(X>8) = 1 - P(X \leq 8)$
$= 1 - 0.9597$
$= \underline{0.0403}$

c) 30 mins = 9 calls
$H_0 : \lambda = 9$
$H_1 : \lambda > 9$



$X \sim Po(9)$
CR :
$1 - P(X \leq 14) = 1 - 0.9585$
$= 0.0415$
$1 - P(X \leq 14) = P(X \geqslant 15)$

CR $X \geqslant 15$ $\therefore$ 14 is not in the critical region $\therefore$ accept H_0. There is no evidence to suggest the rate has increased.

7)

$$f(x) = \begin{cases} \frac{1}{2}x & 0 \leq x < 1 \\ kx^3 & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

a) $\int_0^1 \frac{1}{2}x \, dx = \left[\frac{1}{4}x^2\right]_0^1 = \frac{1}{4}$

$\int_1^2 kx^3 \, dx = \left[\frac{1}{4}kx^4\right]_1^2 = 4k - \frac{1}{4}k$

equate: $\frac{1}{4} + 4k - \frac{1}{4}k = 1$

$k\left(4 - \frac{1}{4}\right) = \frac{3}{4}$

$\frac{15}{4}k = \frac{3}{4} \quad \therefore \quad k = \frac{1}{5}$

b) $E(X) = \int_0^1 \left(\frac{1}{2}x\right)x \, dx + \int_1^2 x\left(\frac{1}{5}x^3\right) dx$

$= \int_0^1 \left(\frac{1}{2}x^2\right) dx + \int_1^2 \frac{1}{5}x^4 \, dx$

$= \left[\frac{1}{6}x^3\right]_0^1 + \left[\frac{1}{25}x^5\right]_1^2$

$= \frac{1}{6} + \frac{32}{25} - \frac{1}{25} = \frac{211}{150} = \underline{1.40\dot{6}}$

c)

$$F(x)\begin{cases} 0 & x<0 \\ \frac{1}{4}x^2+c_1 & 0\le x\le 1 \\ \frac{1}{20}x^4+c_2 & 1<x\le 2 \\ 1 & x>2 \end{cases}$$

$\frac{1}{4}x^2+c_1=0$ when $x=0$

$\therefore\ c_1=0$

$\frac{1}{20}x^4+c_2=1$ when $x=2$

$\therefore\ c_2=1-\frac{1}{20}2^4 = \frac{1}{5}$

$\therefore$ CDF =

$$F(x)\begin{cases} 0 & x<0 \\ \frac{1}{4}x^2 & 0\le x\le 1 \\ \frac{1}{20}x^4+\frac{1}{5} & 1<x\le 2 \\ 1 & x>2 \end{cases}$$

d) $F(0.5)$

$$\frac{1}{20}m^4 + \frac{1}{5} = 0.5$$

$$m^4 = 6$$

$$m = \pm\sqrt[4]{6} = \underline{1.57\ldots}$$

$\underline{\text{or}}$ $\frac{1}{4}m^2 = 0.5$

$$m = \pm\sqrt{2} = \underline{1.41\ldots}$$

reject $\sqrt{2}$ as smaller,

accept $m = \pm\sqrt[4]{6} = \underline{\underline{1.57}}$

e) if

mean < median < mode

$\therefore$ negative skew

$1.40\dot{6} < 1.57$

mean < median $\therefore$ negative skew