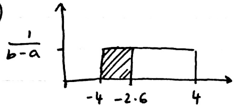
JUNE 2006 S2

1a) it would be : quicker or easier or cheaper

b) a list or register of all the members

c) club members

2a)



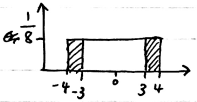
$P(L < -2.6)$

$\frac{1}{b-a} \times (-2.6 - -4)$

$= \frac{1}{4 - -4} \times 1.4$

$= \frac{1}{8} \times 1.4 = \underline{0.175}$

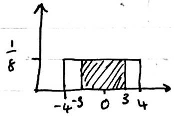
b)



$= 2 \times \frac{1}{8} \times (4-3)$

$= 2 \times \frac{1}{8} \times 1 = \frac{1}{4}$

c)



success = within 3mm

$1 - \frac{1}{4} = \frac{3}{4} = \underline{0.75}$

$X \sim B(20, 0.75) \leftarrow$ not in table ∴ redefine

Redifine success = outside 3mm

$p = \frac{1}{4} = 0.25$

probabilitity X is <u>less</u> than half $P(X<10)$

$= P(X \leq 9)$

$= \underline{0.9861}$

3a) $X \sim Po(7)$

Poisson events occur:

- singly in space or time
- independently of each other (the sale of one house does not influence the sale of the next house)
- at a constant rate

they occur randomly.

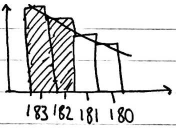
b) $P(X=5)$

$$e^{-7} \times \frac{7^5}{5!} = 0.1277$$

c) $Po(7 \times 24) = Po(168)$ ← as λ is large use normal approx

$Y \sim N(168, 168)$

$P(X > 181) \rightarrow$ NORMAL



continuity correction

$P(Y \geq 181.5)$

$$z = \frac{x-\mu}{\sigma} = \frac{181.5 - 168}{\sqrt{168}}$$

$z = 1.04$

$P(z \geq 1.04)$

$= 1 - \Phi(1.04) = 1 - 0.8508$

$= 0.1492$

4) $X \sim Po(1.25)$

$P(X<3) = P(0) + P(1) + P(2)$

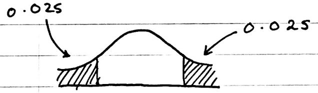
$= e^{-1.25} \times \frac{1.25^0}{0!} + e^{-1.25} \times \frac{1.25^1}{1!} + e^{-1.25} \times \frac{1.25^2}{2!}$

$= 0.868467\ldots$

$= \underline{0.8685}$

b) $Y \sim Po(1.25 \times 4) = Y \sim Po(5)$

$\left.\begin{array}{l} H_0: \lambda = 5 \\ H_1: \lambda \neq 5 \end{array}\right\} \therefore$ This is a two tailed test



$P(Y \geq 11) = 1 - P(Y \leq 10)$

$= 1 - 0.9863 = \underline{0.0137}$

$0.0137 < 0.025 \therefore$ it is in the critical region

$\therefore$ reject H_0 and accept H_1
therefor there is evidence that the rate of break downs has changed

5a) $X \sim B(10, 0.06)$

b) $P(X=3) = \binom{10}{3}(0.06)^3(0.94)^7 = 0.0168085\ldots$

$= \underline{0.0168}$

c) $X \sim B(125, 0.06)$

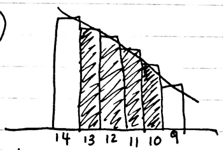
$Y \sim Po(125 \times 0.06) = Y \sim Po(7.5)$

$P(10 \leq X \leq 13)$

$= P(X \leq 13) - P(X \leq 9)$

$= 0.9784 - 0.7764 = \underline{0.2020}$

d)



$P(9.5 \leq X \leq 13.5)$

$\sigma^2 = npq = 7.5 \times 0.94$

$= 7.05$

CONTINUITY CORRECTION

$X \sim N(7.5, 7.05)$

$$z = \frac{x - \mu}{\sigma}$$

$$P\left(\frac{9.5 - 7.5}{\sqrt{7.05}} \leq X \leq \frac{13.5 - 7.5}{\sqrt{7.05}}\right)$$

$P(0.75 \leq X \leq 2.26)$

$\Phi(2.26) - \Phi(0.75)$

$0.9881 - 0.7734 = \underline{\underline{0.2147}}$

6 a) ~~ka~~ area = 1

$$\therefore \quad \frac{1}{k}\int_1^4 1+x \, dx = 1$$

$$\frac{1}{k}\left[x+\frac{1}{2}x^2\right]_1^4 = 1$$

$$\frac{1}{k}\left(12-\frac{3}{2}\right) = 1$$

$$\frac{21}{2k} = 1$$

$$21 = 2k \quad \Rightarrow \quad k = \frac{21}{2}$$

b)

$$f(x) = \begin{cases} \dfrac{1+x}{10.5} & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} 0 & x<1 \\ \dfrac{2}{21}x+\dfrac{1}{21}x^2+c & 1 \leq x \leq 4 \\ 1 & x \geq 4 \end{cases}$$

when $x = 4$

$$\frac{2}{21}x+\frac{1}{21}x^2+c = 1$$

$$\frac{2}{21}(4)+\frac{1}{21}(4)^2-1 = -c$$

$$-c = \frac{1}{7} \qquad c = -\frac{1}{7}$$

$$\therefore \quad F(x) = \begin{cases} 0 & x<1 \\ \dfrac{2x+x^2-3}{21} & 1 \leq x \leq 4 \\ 1 & x \geq 4 \end{cases}$$

c) $E(X) = \int x f(x)\, dx$

$= \int_1^4 \frac{2}{21} x(x+1)\, dx$

$= \frac{2}{21} \int_1^4 x^2 + x\, dx$

$= \frac{2}{21} \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 \right]_1^4$

$= \frac{2}{21} \left(\frac{88}{3} - \frac{5}{6} \right)$

$= \frac{19}{7} \approx 2.71$

d) median $f(m) = 0.5$

$\frac{x^2 + 2x - 3}{21} = 0.5$

$x^2 + 2x - \frac{27}{2} = 0$ $\quad \frac{-2 \pm \sqrt{2^2 - 4(-\frac{27}{2})}}{2}$

$= \frac{-2 \pm \sqrt{58}}{2}$

$x = 2.81\ldots$ or $x = -4.81\ldots$

$\underline{\underline{x = 2.81}}$

e) pdf reaches highest point at 4

$\therefore$ mode $= 4$

f) negative skew if:

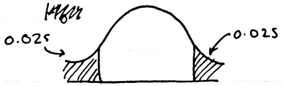
mean < median < mode

$2.71 < 2.81 < 4$

$\therefore$ distribution is negatively skewed.

7a)

$X \sim B(25, 0.2)$



from table

$P(X \leq 1) = 0.0274$

$P(X \leq 9) = 0.9287$

$\Rightarrow$ critical when $P(X \geq 10) = 0.0173$

critical region $(X \leq 1 \cup X \geq 10)$

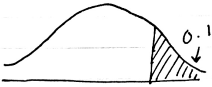
b) α / significance level $= 0.0173 + 0.0274$

$= 0.0447$

$= 4.47\%$

c) $Y \sim B(20, 0.2)$

→ one tailed



$H_0: p = 0.2$

$H_1: p < 0.\cancel{0}2$

$P(Y \leq 2) = \cancel{0.0261}\ 0.2061$

$P(Y \geq 2) = 1 - P(Y \leq 1)$

$= 1 - 0.0692$

$= 0.9308$

$0.2061 > 0.1$

numbers are very close $\therefore$ there is insignificant evidence to reject H_0 or H_1 or to suggest proportion of defective bowls has decreased