S2 June 2002 (MA)

Q1a) Survey is quicker. so easier to process / analyse results.

b) List of club members.

c) Members.

Q2a) Y is a random variable that includes any function of 'X_i' and no other quantities.

b) $Y_2 = \sum X$

c) When all possible samples are taken, values of Y found and then the values form a probability distribution.

Q3a) $R \sim U[\alpha, \beta]$

$E(R) = \frac{\alpha+\beta}{2} = 3 \quad \therefore \quad \alpha+\beta = 6 \sim (1)$

$Var(R) = \frac{(\beta-\alpha)^2}{12} = \frac{25}{3}$

$(2) \sim \quad (\beta-\alpha)^2 = 100$

from (1), $\beta = 6 - \alpha$. $\quad (6-2\alpha)^2 = 100$

$6 - 2\alpha = \pm 10$.

$2\alpha = 6 \mp 10$

$\alpha = 3 \mp 5$

$\therefore \alpha = -2$ or $\alpha = 8$

$\downarrow$ $\downarrow$

$\beta = 6 - -2 = 8$ // $\beta = 6 - 8 = -2$ //

accept // we are told $\alpha \leq R \leq \beta$.

$\therefore \beta > \alpha$.

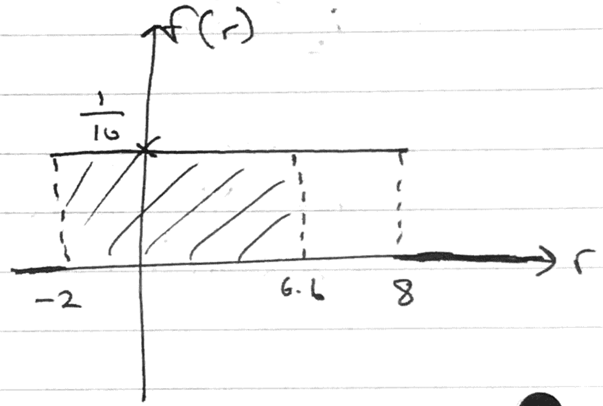
hence reject the above solution

so $\boxed{\alpha = -2}$, $\boxed{\beta = 8}$

b) $P(R < 6.6) =$ shaded area

$= (6.6 + 2) \times \frac{1}{10}$

$= \boxed{0.86}$



Q4a) $H_0: p = 0.2$

$H_1: p < 0.2$

$X \sim B[25, 0.2]$

$P(X \leq 2) = 0.0982$ //

$0.0982 > 0.05$

$\therefore$ Result is insignificant.

Accept H_0.

Evidence suggests that the % was not lower that day.

4b) $Y \sim B[300, 0.03]$ where Y = no. of customer's buying bumper packs.

$H_0: p = 0.03$
$H_1: p \neq 0.03$

n large, p small

$\therefore Y \simeq \sim Po(9)$

$P(Y \leq 3) = 0.0212$
$P(Y \leq 4) = 0.0550$
0.0212 is closer to 0.025.

0.0220 is closer to 0.025
$P(Y \geq 15) = 0.0415$
$P(Y \geq 16) = 0.0220$

So C.R is $(Y \geq 16)$ and $(Y \leq 3)$

c) $0.0220 + 0.0212 = 0.0432$

(4.32%)

Q5a) $L \sim N(\mu, 0.3^2)$

$P(L < 150) = 0.05$

$P\left(L < \frac{150 - \mu}{0.3}\right) = 0.05$

$P\left(L > \frac{\mu - 150}{0.3}\right) = 0.05$

from tables, $P(Z > 1.6449) = 0.05$

$$\therefore \quad 1.6449 = \frac{\mu - 150}{0.3}.$$

$$\mu = 150 + 0.3(1.6449) = \boxed{150.5}$$

b) let X = no. of canes less then 150cm,

$$X \sim B[10, 0.05]$$

$$P(\text{required}) = P(X \leq 2) = \boxed{0.9885}$$

c) for new customer, $Y \sim B[500, 0.05]$ //

n is large, p is small and $np > 10$.
so use normal approximation,

$$np = 500 \times 0.05 = 25$$
$$np(1-p) = 25(0.95) = 23.75$$

$$Y \simeq\sim N[25, 23.75]$$

$$P(Y < 35) = P(Y < 34.5) = P\left(Z < \frac{34.5 - 25}{\sqrt{23.75}}\right)$$

$$= P(Z < 1.95) = \boxed{0.9744}$$

Q6a) $T \sim Po(1.5)$

$P(T=4) = \dfrac{(e^{-1.5})(1.5^4)}{4!} = \boxed{0.0471}$

b) For $100m^2$, $T_2 \sim Po(6)$

$P(Y<6) = P(Y \leq 5) = 0.4457$

now let X = no. of 100m balls containing less than 6 faults,

$X \sim B[3, 0.4457]$

$P(X=1) = \binom{3}{1}(0.4457)^1(1-0.4457)^2$

$= \boxed{0.411}$

c) For $500m^2$, $T_3 \sim Po(30)$

$P(23 \leq T_3 \leq 33) = P(22 < T_3 < 34)$

λ is large $(np > 10)$, $T_3 \simeq \sim N[30, 30]$

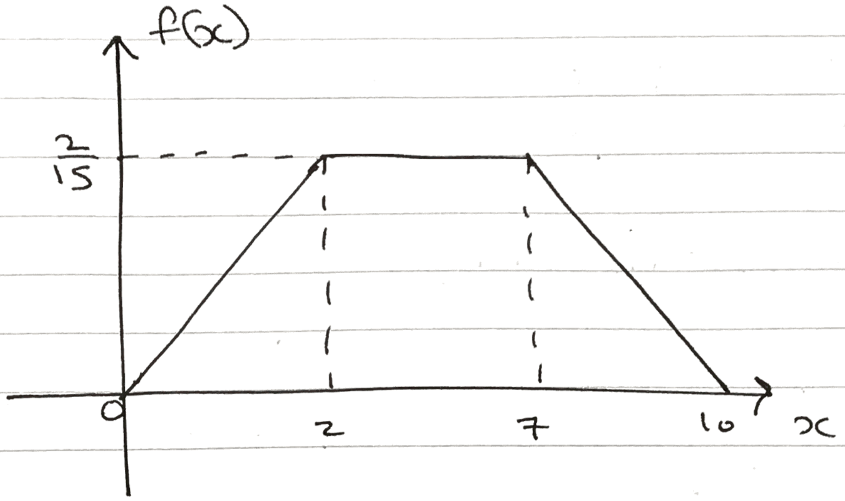
(applying c.c) : $P(22 < T_3 < 34) = P(22.5 < T_3 < 33.5)$

$= P\left(\frac{22.5-30}{\sqrt{30}} < Z < \frac{33.5-30}{\sqrt{30}}\right)$

$= P(-1.37 < Z < 0.64)$

$= P(Z < 0.64) - P(Z < -1.37) = \boxed{0.6536}$

(Q7a)



For $0 \le x \le 2$

$$\int_0^x \left(\frac{x}{15}\right) dx = \left[\frac{x^2}{30}\right]$$

For $2 < x < 7$

$$F(2) + \int_2^x \left[\frac{2}{15}\right] dx = \frac{2}{15} + \left[\frac{2}{15}x\right]_2^x$$

$$= \frac{2}{15} + \frac{2}{15}x - \frac{4}{15}$$

$$= \frac{2}{15}(x-1)$$

For $7 \le x \le 10$

$$F(7) + \int_7^x \left[\frac{4}{9} - \frac{2x}{45}\right] dx$$

$$\frac{2}{15}(7-1) + \left[\frac{4}{9}x - \frac{2x^2}{90}\right]_7^x$$

$$= \frac{12}{15} + \left[\frac{4}{9}x - \frac{2}{90}x^2\right] - \left[\frac{91}{45}\right]$$

$$= -\frac{1}{45}x^2 + \frac{4}{9}x - \frac{11}{9}$$

$$= -\frac{1}{9}(5x^2 - 4x + 11)$$

$$\text{so} \ldots \quad F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{30}, & 0 \leq x \leq 2 \\ \frac{2}{15}(x-1), & 2 < x < 7 \\ -\frac{1}{9}(5x^2 - 4x + 11), & 7 \leq x \leq 10 \\ 1, & x > 10 \end{cases}$$

c) $F(8.2) = P(X \leq 8.2) = -\frac{1}{9}(5(8.2)^2 - 4(8.2) + 11)$

$$= \boxed{0.928}$$

d) $E(X) = \int_0^2 \left(\frac{2x^3}{30}\right) dx + \int_2^7 \left[\frac{2x}{15}\right] dx + \int_7^{10} \left[\frac{4x}{9} - \frac{2x^2}{45}\right] dx$

$$= \left[\frac{x^3}{45}\right]_0^2 + \left[\frac{2x^2}{30}\right]_2^7 + \left[\frac{4x^2}{18} - \frac{2x^3}{135}\right]_7^{10}$$

$$= \frac{8}{45} + 3 + \frac{200}{27} - \frac{784}{135}$$

$$= \frac{43}{9} = \boxed{4.78}$$