S2 June 2001 (MA)

Q1ai) use census since population size will be small.
(use an electoral register as sampling frame).

ii) Sample survey
(use a specific list of times on weekdays where sample can be taken).

b) $X \sim Po(\lambda)$ where X = no. of vehicles passing through a point in a 10 min period from 9am – 9pm.

Q2a) $X \sim Po(0.9)$

$$P(X=0) = (e^{-0.9}) = 0.4065\ldots$$

$$= \boxed{0.407}$$

b) for 6 months, $X_2 \sim Po(5.4)$ $(6 \times 0.9 = 5.4)$

$$P(X_2 = 2) = \frac{(e^{-5.4})(5.4)^2}{2!} = \boxed{0.066}$$

c) Y = no. of months with no accidents,

$$Y \sim B[4, 0.407]$$

$$P(Y=2) = \binom{4}{2}(0.407)^2(1-0.407)^2 = \boxed{0.350}$$

5% each tail

Q3) $H_0: p = \frac{1}{4}$ $\quad X \sim B[20, \frac{1}{4}]$

$H_1: p \neq \frac{1}{4}$

$P(X \leq 2) = 0.0913$

$P(X \leq 1) = 0.0243$

$\left.\begin{array}{l} P(X \geq 8) = 0.1018 \\ P(X \geq 9) = 0.0409 \end{array}\right\}$ $X \geq 9$ is part of C.R.

so C.R is $X \leq 1 \cap X \geq 9$.

$P(X \leq 2) = 0.0913 < 0.05$

$X = 2$ is not in C.R

so accept H_0.

Evidence suggests proportion of gold beads has not changed.

Q4a) $X \sim B[10, 0.20]$ where X = no. of first class letters.

$P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.6778$

$= \boxed{0.3222}$

b) $P(X < 2) = P(X \leq 1) = \boxed{0.3758}$

c) Y = no. of first class letters in 70.

$Y \sim B[70, 0.20]$

only suitable approximation is normal.

$$np = 70 \times 0.2 = 14$$
$$np(1-p) = 14(1-0.2) = 11.2$$

$$\therefore Y \approx \sim N(14, 11.2)$$

(applying c.c) $P(Y \leq 12) = P(Y < 13) = P(Y < 12.5)$

$$= P\left(Z < \frac{12.5 - 14}{\sqrt{11.2}}\right)$$

$$= P(Z > 0.45) = 1 - P(Z < 0.45)$$

$$= 1 - 0.6736 = \boxed{0.3264}$$

d) The 70 letters are independent of each other.

Q5a) $X \sim Po(2)$ where X = no. of light bulb replacement requests.

$$P(X = 4) = \frac{(e^{-2})(2^4)}{4!} = \boxed{0.0902}$$

b) $P(X > 5) = 1 - P(X \leq 5) = 1 - 0.9834$

$$= \boxed{0.0166}$$

c) Y = no. of requests in 3 weeks, $Y \sim Po(6)$

$P(\text{required}) = P(Y \leq 5) = \boxed{0.4457}$

d) $H_0 : \lambda = 2$ (or $\lambda = 8$)

$H_1 : \lambda < 2$ (or $\lambda < 8$)

let A = no. of requests in 4 weeks,

$A \sim Po(8)$

$P(A \leq 3) = 0.0424$

$0.0424 < 0.05$

$\therefore$ Result is significant
Reject H_0
Evidence suggests that the rate of replacement requests has decreased.

(Q6a) $\frac{d}{dx}\left(\frac{1}{27}(-x^3 + 6x^2 - 5)\right) = \frac{1}{27}(-3x^2 + 12x)$

$$\text{so } f(x) = \begin{cases} \frac{1}{27}(-3x^2 + 12x), & 1 \leq x \leq 4 \\ 0, & \text{otherwise.} \end{cases}$$

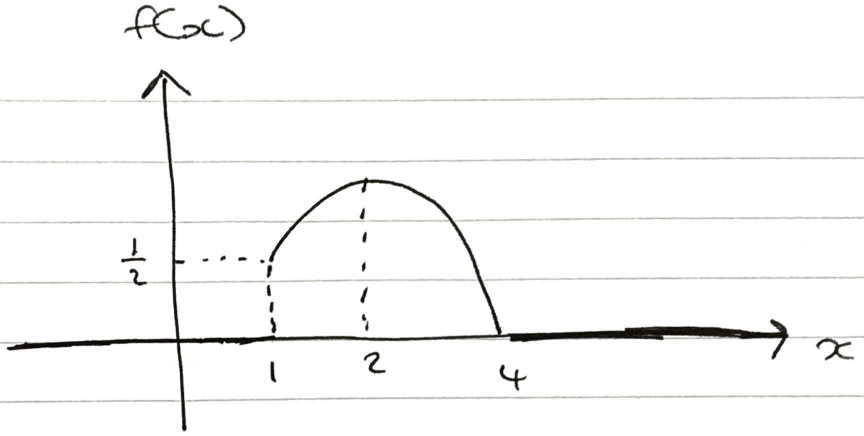
b) $f'(x) = \frac{1}{27}(-6x + 12) = 0$

let $f'(x) = 0$ to find turning point

$6x = 12$

$\boxed{x = 2}$ is the mode.

c)



d) $E(X) = \int_1^4 xf(x)\,dx = \frac{1}{27}\int_1^4 \left[-3x^3 + 12x^2\right] dx$

$$= \frac{1}{27}\left[\frac{-3x^4}{4} + \frac{12x^3}{3}\right]_1^4$$

$$= \frac{1}{27}\left[-192 + 256\right] - \frac{1}{27}\left[\frac{-3}{4} + \frac{12}{3}\right]$$

$$= \frac{1}{27}\left[64\right] - \frac{1}{27}\left[\frac{13}{4}\right] = \boxed{\frac{9}{4}}$$

e) $F(\mu) = F(2.25) = \frac{1}{27}\left(-(2.25)^3 + 6(2.25)^2 - 5\right)$

$$= \boxed{0.517} > 0.5$$

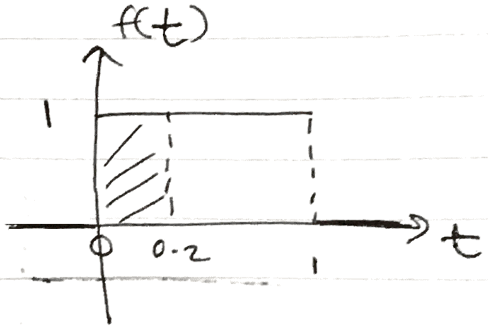
f) $F(\mu) > 0.5$

$F(2) = \frac{1}{27}(-8 + 24 - 5) = \frac{11}{27} = 0.407 < 0.5$

$(F(m) = 0.5)$

hence median (m) is less than μ and greater than mode

Q7a) $T \sim U[0,1]$



$P(T < 0.2) = 1 \times \boxed{0.2}$

b) $E(T) = \dfrac{1+0}{2} = \boxed{0.5}$

c) $f(t) = 1$

$$E(T^2) = \int_0^1 t^2 f(t)\,dt = \int_0^1 (t^2)\,dt$$

$$= \left[\frac{t^3}{3}\right]_0^1 = \frac{1}{3}$$

$$Var(T) = E(T^2) - (E(T))^2$$

$$= \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \boxed{\frac{1}{12}}$$

d) let X = no. of children stopping the star in less than 0.2s.

$$X \sim B[20, 0.2]$$

$$P(\text{required}) = P(X \leq 4) = \boxed{0.6296}$$

e) mean would be close to $\frac{1}{2}$ and the variance would decrease as all children aim to stop it at 0.5s.

f) $g(t) = \begin{cases} 4t, & 0 \leq t \leq 0.5 \\ 4-4t, & 0.5 < t \leq 1 \\ 0, & \text{otherwise.} \end{cases}$

$$\int_0^{0.2} (4t)\,dt = \left[\frac{4t^2}{2}\right]_0^{0.2} = \left[2 \times 0.2^2\right]$$

$$= \boxed{0.08}$$

g) let D denote no. of them stopping the star in less than 0.2s,

$$D \sim B[75, 0.08]$$

n is large, p is small. $(np = 6)$

$$\therefore D \approx \sim Po(6)$$

$$P(D > 7) = 1 - P(D \leq 7) = 1 - 0.7440$$

$$= \boxed{0.2560}$$