January 2009 MA - S2

1) $X \sim Po(3)$

a) $P(X>2) = 1 - P(X \leq 2)$

$= 1 - 0.4232$

$= 0.5768$ (4dp)

b) $P(X=5,6) = P(X \leq 6) - P(X<5)$

$= 0.9665 - 0.8153$

$= 0.1512$ (4dp)

c) $\mu = \frac{\sum x}{f}$ $\quad$ $Var(X) = \frac{\sum x^2}{f} - \mu^2$

$= \frac{295}{80}$ $\quad$ $= \frac{1386}{80} - \left(\frac{295}{80}\right)^2$

$= 3.69$ (3sf) $\quad$ $= 3.72$ (3sf)

d) The Poisson distribution's mean and variance are equal
3.69 ≈ 3.72 to (2sf) therefor Poisson is a good model.

1) $Y \sim Po(\mu)$

e) $P(Y=4) = e^{-\mu}\dfrac{\mu^4}{4!}$

$= \dfrac{0.0250 \times 184.9}{24}$

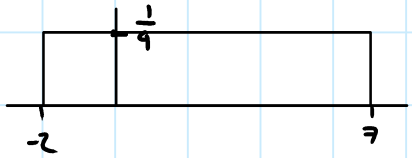
$= 0.1929$ 4sf

2) $X \sim U[-2, 7]$

a) $\dfrac{1}{7 - -2} = \dfrac{1}{9}$

$$f(x) = \begin{cases} \frac{1}{9} & -2 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$$

b)



c) $\displaystyle\int_{-2}^{7} x^2 f(x)\,dx = \int_{-2}^{7} \frac{x^2}{9}\,dx$

$= \left[\dfrac{x^3}{27}\right]_{-2}^{7}$

$= \left[\dfrac{343}{27} - \dfrac{-8}{27}\right]$

$= 13$

OR

$Var(X) = \frac{1}{12}(7 - -2)^2$

$= \dfrac{27}{4}$

$E(X) = \frac{1}{2}(-2 + 7)$

$= \dfrac{5}{2}$

$Var(X) = E(X^2) - E(X)^2$

$\dfrac{27}{4} = E(X^2) - \dfrac{25}{4}$

$\therefore E(X^2) = 13$

d) $P(-0.2 < X < 0.6) = \displaystyle\int_{-0.2}^{0.6} f(x)\,dx$

$= \left[\frac{1}{9}x\right]_{-0.2}^{0.6}$

$= \left[\dfrac{0.6}{9} - \dfrac{-0.2}{9}\right]$

$= \dfrac{4}{45}$

3) $X \sim B(20, 0.3)$

a) $P(x \leq 1) = 0.0076$ $P(x \leq 9) = 0.9520$

* $P(x \leq 2) = 0.0355$ $P(x \leq 10) = 0.9829$ *

$\Downarrow$

* closest to 2.5% $P(X \geq 11) = 1 - P(x \leq 10)$

$= 0.0171$

Critical Region $(X \leq 2) \cup (X \geq 11)$

b) $0.0355 + 0.0171 = 0.0526$

c) $H_0: p = 0.3$

$H_1: p \neq 0.3$ 2tail test

Accept H_0, Reject H_1

3 is not in the critical region so there is insufficient evidence reject H_0

4) $f(t) = \begin{cases} kt & 0 \le t \le 10 \\ 0 & \text{otherwise} \end{cases}$

a) $\int_{-\infty}^{\infty} f(t)\,dt = 1$

$= \left[\frac{k}{2}t^2\right]_0^{10} + 0 + 0$

$= \frac{k}{2}\left[100 - 0\right]$

$1 = 50k \quad \therefore k = \frac{1}{50}$

b) $P(T > 6) = \int_6^{10} f(t)\,dt$

$= \frac{1}{100}\left[100 - 36\right]$

$= \frac{16}{25}$

c) $E(T) = \int_{-\infty}^{\infty} t f(t)\,dt$ $\qquad E(T^2) = \int_{-\infty}^{\infty} t^2 f(t)\,dt$

$= \frac{1}{150}\left[t^3\right]_0^{10}$ $\qquad = \frac{1}{200}\left[t^4\right]_0^{10}$

$= \frac{20}{3}$ $\qquad = 50$

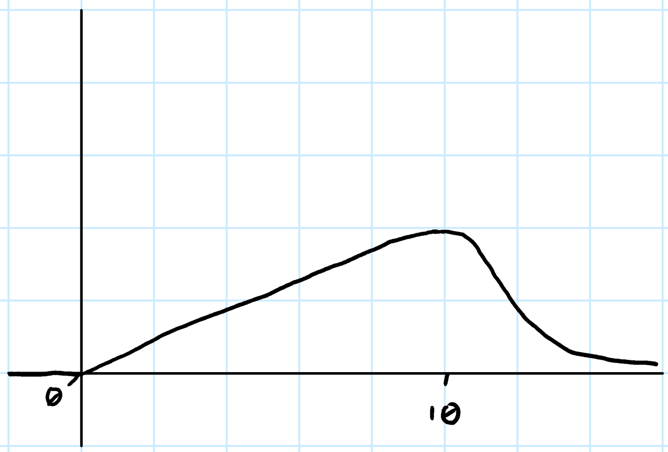
$Var(T) = 50 - \left(\frac{20}{3}\right)^2$

$= \frac{50}{9}$

4)

d) 10 The mode is the highest point on the graph of f(t)

e)



A phone call cannot be negative therefore the function should be 0 where $t < 0$
A phone call can however be longer than 10 min (f(t) assumes no call is ever longer than 10 min)
Since f(t) has previously been used, assume true for $0 \leq t \leq 10$ and add a tail to right of $t = 10$

5) $X \sim B(10, 0.01)$

a) $P(X=1) = \binom{10}{1} 0.01^1 \times 0.99^9$

$= 0.0914$ (4dp)

b) $P(X=0) = \binom{10}{0} 0.01^0 \times 0.99^{10}$

$= 0.9044$ 4dp

$P(X \geq 2) = 1 - P(X=1) - P(X=0)$

$= 1 - 0.0914 - 0.9044$

$= 0.0043$ (4dp)

c) $Y \sim B(250, 0.01)$

Small p, large n : therefore use Poisson approximation

$250 \times 0.01 = 2.5$

$Z \sim Po(2.5)$

$P(1 \leq Z \leq 4) = P(Z \leq 4) - P(Z=0)$

$= 0.8912 - 0.0821$

$= 0.8091$ (4dp)

6) $W \sim Po(7)$

i) $H_0: \lambda = 7$ 1tail test

$H_1: \lambda > 7$ at 10%

$P(W \leq 10) = 0.9015$

$P(W \leq 9) = 0.8305$

$\therefore P(W \geq 11) < 0.1$

critical value $= 11$

$10 < 11$

10 is not in critical region
Accept H_0, reject H_1
Not a significant result so conclude that the rate of visits on Saturday is unchanged.

ii) $W = 11$

Finding critical values in part (i) is a faster method as it solves part (ii) as well.
The critical region is the minimum number of visits for a for a significant result.

b) (For Poisson distribution) We assume that visits are independent, occur randomly and at a constant rate.

b) $X \sim Po(14)$

$Y \sim N(14, \sqrt{14}^2)$

$Z \sim N(0, 1)$

c) $H_0: \lambda = 14$

$H_1: \lambda > 14$

$P(X \geq 20) \approx P(Y \geq 19.5)$

$$z = \frac{14.5 - 14}{\sqrt{14}}$$

$$= 1.47$$

$P(Y \geq 19.5) = P(Z \geq 1.47)$

$= 1 - P(Z \leq 1.47)$

$= 0.0708$

$0.0708 < 0.1$

Reject H_0, accept H_1

There is sufficient evidence to conclude that rate of visits increases on a Saturday

7)

a)

$$f(x) = \begin{cases} -\frac{2}{9}x + \frac{8}{9} & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \int f(x)\,dx$$

$$= -\frac{2}{9}\frac{x^2}{2} + \frac{8}{9}x + c$$

$$= -\frac{1}{9}x^2 + \frac{8}{9}x + c$$

$F(4) = 1$, $-\frac{16}{9} + \frac{32}{9} + c = 1$

$c = -\frac{7}{9}$

$$-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9}$$

b)

$$F(x) = \begin{cases} 0 & x < 1 \\ -\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} & 1 \leq x \leq 4 \\ 1 & 4 < x \end{cases}$$

7) c)

UQ : $F(x) = 0.75$

$$-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} - \frac{3}{4} = 0$$

$$\frac{-\frac{8}{9} \pm \sqrt{\frac{64}{81} - \frac{55}{81}}}{-\frac{2}{9}} = \frac{-8 \pm 3}{-2}$$

$$x = 2.5,\ 5.5$$

$x \leq 4 \quad \therefore \quad x = 2.5$

LQ : $F(x) = 0.25$

$$-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} - \frac{1}{4} = 0$$

$$\frac{-\frac{8}{9} \pm \sqrt{\frac{64}{81} - \frac{37}{81}}}{-\frac{2}{9}} = \frac{-8 \pm 3\sqrt{3}}{-2}$$

$$= 1.40,\ 6.60$$

$x \leq 4 \quad \therefore \quad x = 1.40$ (3sf)

d)

$$E(X) = \int_1^4 x f(x)\,dx$$

$$= \frac{1}{9}\left[-\frac{2x^3}{3} + 4x^2\right]_1^4$$

$$= \frac{64}{27} - \frac{10}{27}$$

$$= 2$$

Mode is 1 (highest point on negative linear graph is lowest constraint)
Median is 1.88
Mean is E(x) is 2

2 > 1.88 > 1
Mean > Median > Mode
therefore positive skew