S2 January 2007 (MA)

Q1a) A statistic is a random variable that is a function of unown observations from a population.

bi) Yes (contains no population parameters)

ii) No (contains a population parameter (μ)).

Q2a) $J \sim Po(4)$

$$P(J \geq 10) = 1 - P(J \leq 9) = 1 - 0.9919$$

$$= \boxed{0.0081}$$

b) $K \sim B[25, 0.27]$

$$P(K \leq 1) = P(K=1) + P(K=0)$$

$$= \binom{25}{1}(0.27)^1(0.73)^{24}$$

$$+ \binom{25}{0}(0.27)^0(0.73)^{25}$$

$$= \boxed{0.00392}$$

Q3a) $W \sim B[12, 0.45]$ where W = no. of white flowers.

$$P(W=5) = \binom{12}{5}(0.45)^5(0.55)^7 = 0.2225$$
$$= \boxed{0.223}$$

b) $P(W>6) = P(W \geq 7) = 1 - P(W \leq 6)$

$$= 1 - 0.7393 = \boxed{0.2607}$$

c) for 10 batches....

$X \sim B[10, 0.2607]$

$$P(X=3) = \binom{10}{3}(0.2607)^3(1-0.2607)^7$$
$$= \boxed{0.257}$$

d) for 50 plant batch... $Y \sim B[50, 0.45]$

$np = 50 \times 0.45 = 22.5$

$np(1-p) = 50 \times 0.45(1-0.45) = 12.375$

n is large, $p \simeq 0.5$

so $Y \simeq \sim N(22.5, 12.375)$

(applying c.c) $P(Y>25) = P(Y>25.5) = P\left(Z > \frac{25.5-22.5}{\sqrt{12.375}}\right)$

$= P(Z > 0.85) = 1 - P(Z < 0.85)$

$= 1 - 0.8023 = \boxed{0.1977}$

Q4a) $\lambda > 10$

b) Because poisson is 'discrete' and normal uses 'continuous' data.

c) $X \sim Po(5)$ for a week in winter.

$P(X \leq 2) = \boxed{0.1247}$

d) for the summer, $X_2 \sim Po(25)$

$\lambda > 10 \therefore X_2 \simeq \sim (25, 25)$ //

(c.c) $P(\text{required}) = P(X_2 > 30) = P(X_2 > 30.5)$

$= P\left(Z > \frac{30.5 - 25}{\sqrt{25}}\right) = P(Z > 1.10)$

$= 1 - P(Z < 1.10) = 1 - 0.8643$

$= \boxed{0.1357}$

e) let X_3 = no. of saturdays in summer where demand can't be met,

$X_3 \sim B[16, 0.1357]$

$np = \text{mean} = E(X_3) = 16 \times 0.1357$
$= 2.1712$

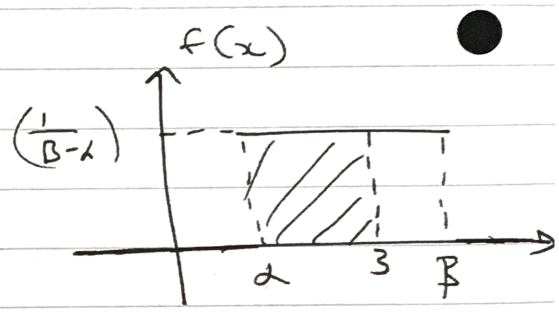
So $\boxed{2}$ Saturdays

(Q5a)

$$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha}, & \alpha \le x \le \beta \\ 0, & \text{otherwise.} \end{cases}$$

b) $E(X) = \dfrac{\alpha + \beta}{2} = 2$

$\therefore \alpha + \beta = 4 \sim ①$



$P(X < 3) = \dfrac{5}{8}$

$(3 - \alpha) \times \dfrac{1}{(\beta - \alpha)} = \dfrac{5}{8}$

$\dfrac{3 - \alpha}{\beta - \alpha} = \dfrac{5}{8}$

$8(3 - \alpha) = 5(\beta - \alpha)$

$24 - 8\alpha = 5\beta - 5\alpha$

$3\alpha + 5\beta = 24 \sim ②$

$① \times 3 : \quad 3\alpha + 3\beta = 12$

$$\begin{array}{r} (3\alpha + 3\beta = 12) \\ -(3\alpha + 5\beta = 24) \\ \hline 0 \quad -2\beta = -12 \end{array}$$

$\therefore \boxed{\beta = 6}$

$\alpha = 4 - \beta = \boxed{-2}$

c) $X \sim U[0, 150]$

$E(X) = \dfrac{150 + 0}{2} = \boxed{75\text{cm}}$

d) $\text{Var}(X) = \dfrac{(b-a)^2}{12}$

$= \dfrac{(150-0)^2}{12}$ //

$\therefore \sigma = \sqrt{\dfrac{(150)^2}{12}} = \boxed{43 \cdot 3}$ cm.

e) $P(X < 30) + P(X > 120)$

if $X > 120$ then the other piece will be shorter than 30cm.

$\dfrac{30}{150} + \dfrac{30}{150} = P(\text{required}) = \boxed{0 \cdot 4}$

Q6a) $H_0: p = 0.20$; $H_1: p < 0.20$ $\Big\}$ $X \sim B[30, 0.20]$

$P(X \leq 2) = 0.0442$

$0.0442 < 0.05$

$\therefore$ Result is significant.
Reject H_0.
Evidence suggests that the no. of family size bars sold is lower than usual.

b) $H_0: p = 0.02$; $H_1: p \neq 0.02$ $\Big\}$ $Y \sim B[200, 0.02]$

n large, p low

$\therefore \quad Y \simeq \sim Po(4)$

$P(Y \leq 0) = 0.0183$
$P(Y \leq 1) = 0.0916$ $\Big\}$ 0.0183 is closer to 0.025.

$P(Y \geqslant 9) = 0.0214$
$P(Y \geqslant 8) = 0.0511$ $\Big\}$ 0.0214 is closer to 0.025.

so critical region is $Y \geqslant 9$ and $Y \leq 0$
$(Y = 0)$

c) $0.0183 + 0.0214 = \boxed{0.0397}$.

(3.97 %)

Q7a)

$$F(x) = \begin{cases} 0, & x < 0 \\ 2x^2 - x^3, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

$$P(X > 0.3) = 1 - F(0.3)$$

$$= 1 - \left[2(0.3)^2 - (0.3)^3\right]$$

$$= \boxed{0.847}$$

b) $F(0.59) = 0.4908$, $F(0.60) = 0.5040$ $\}$ $F(0.59) < 0.50$, $F(0.60) > 0.50$

hence median lies between 0.59 and 0.60.

c) $\frac{d}{dx}(2x^2 - x^3) = 4x - 3x^2$ //

$$\therefore f(x) = \begin{cases} 4x - 3x^2, & 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

d) $E(X) = \int_0^1 (4x^2 - 3x^3)\,dx = \left[\frac{4x^3}{3} - \frac{3x^4}{4}\right]_0^1$

$$= \left[\frac{4}{3} - \frac{3}{4}\right] = \boxed{\frac{7}{12}}$$

e) $f'(x) = 4 - 6x = 0 \quad \therefore x = \frac{4}{6} = \boxed{\frac{2}{3}}$

f) mean < median < mode

$\frac{7}{12}$ < 0.59 – 0.60 < $\frac{2}{3}$

$\therefore$ $\boxed{\text{Negative skew}}$