S2 January 2006 (MA)

Q1a) $X \sim B[4, \frac{1}{2}]$ where X = no. of heads.

$P(X=2) = P(\text{equal no. of heads/tails})$

$$= \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \boxed{0.375}$$

b) $P(X=4) + P(X=0)$

$$= \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 = \boxed{0.125}$$

c) $$\left.\begin{aligned} P(HHTT) &= \left(\tfrac{1}{2}\right)^4 \\ P(HHTH) &= \left(\tfrac{1}{2}\right)^4 \end{aligned}\right\} \left(\tfrac{1}{2}\right)^4 + \left(\tfrac{1}{2}\right)^4 = \boxed{0.125}$$

Q2a) $A \sim Po(1.5)$ where A = no. of accidents weekly.

b) $P(A=2) = \dfrac{(e^{-1.5})(1.5^2)}{2!} = \boxed{0.2510}$

c) $P(A \geq 1) = 1 - P(A=0) = [1 - e^{-1.5}]$

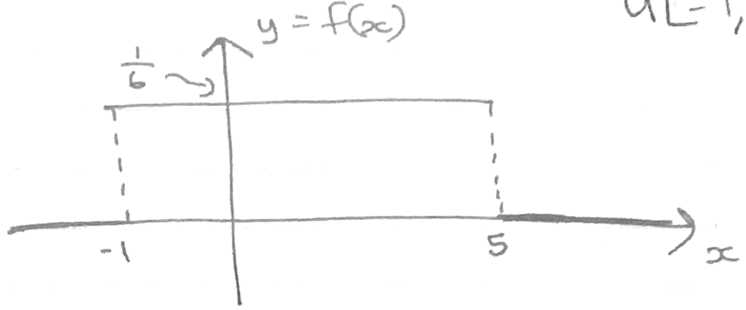
$P(\text{required}) = (1 - e^{-1.5})^3 = \boxed{0.4689}$

d) For 2 weeks, $A_2 \sim Po(3)$

$P(A_2 > 4) = 1 - P(A_2 \leq 4) = \boxed{0.1847}$

$U[-1, 5]$

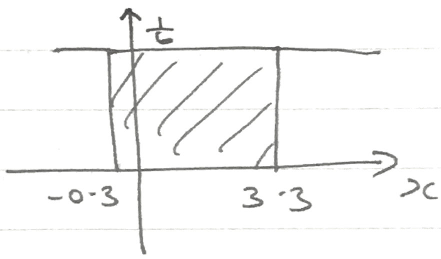
Q3a)



b) $E(X) = \frac{-1+5}{2} = \boxed{2}$

c) $Var(X) = \frac{(b-a)^2}{12} = \frac{(5+1)^2}{12} = \boxed{\frac{36}{12}} = \boxed{3}$ //

d) $P(-0.3 < X < 3.3) = (3.3 + 0.3) \times \frac{1}{6} = \boxed{0.6}$



Q4) $X \sim B[150, 0.02]$

n large, p small so $X \simeq \sim Po(3)$ //

$(150 \times 0.02 = 3)$

$P(X > 7) = 1 - P(X \leq 7) = \boxed{0.0119}$

Q5a) $f(x) = \begin{cases} kx(x-2), & 2 \leq x \leq 3 \\ 0, & \text{otherwise.} \end{cases}$

$$k\int_2^3 (x^2 - 2x)\,dx = 1$$

$$k\left[\frac{x^3}{3} - x^2\right]_2^3 = 1$$

$$k\left[\frac{27}{3} - 9\right] - k\left[\frac{8}{3} - 4\right] = 1$$

$$k\left[\frac{4}{3}\right] = 1 \qquad \therefore \quad k = \boxed{\frac{3}{4}}$$

b) $E(X) = \frac{3}{4}\int_2^3 [x^3 - 2x^2]\,dx = \frac{3}{4}\left[\frac{x^4}{4} - \frac{2x^3}{3}\right]_2^3$

$\left(E(X) = \int_a^b x f(x)\,dx\right) \qquad = \frac{3}{4}\left[\frac{43}{12}\right] = \boxed{\frac{43}{16}}$

$= 2.69$

c) $\frac{3}{4}\int_2^x (x^2 - 2x)\,dx = \frac{3}{4}\left[\frac{x^3}{3} - x^2\right]_2^x$

$$= \frac{3}{4}\left[\frac{x^3}{3} - x^2\right] - \frac{3}{4}\left[\frac{8}{3} - 4\right]$$

$$= \frac{3}{4}\left[\frac{x^3}{3} - x^2 + \frac{4}{3}\right] = \frac{1}{4}\left[x^3 - 3x^2 + 4\right] //$$

$$\therefore F(x) = \begin{cases} 0, & x < 2 \\ \frac{1}{4}(x^3 - 3x^2 + 4), & 2 \le x \le 3 \\ 1, & x > 3 \end{cases}$$

d) $F(2.70) = 0.453$ $\quad F(2.7) < 0.5$

$F(2.75) = 0.527$ $\quad F(2.75) > 0.5$

hence median lies between 2.7 and 2.75 //

Q6a)

0.5	1p
$\frac{1}{3}$	2p
$\frac{1}{6}$	5p

x	1p	2p	5p
$P(X=x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

//

$$E(X) = \sum x P(X=x) = 1\left(\tfrac{1}{2}\right) + 2\left(\tfrac{1}{3}\right) + 5\left(\tfrac{1}{6}\right)$$

$$= \boxed{2}\text{ p.}$$

b) [1, 1] [2, 2] [5, 5]

[1, 2] [2, 5] [1, 5]

[2, 1] [5, 2] [5, 1]

Probability		Mean
(0.5^2)	$[1, 1]$	1
$(\frac{1}{2})(\frac{1}{3})$	$[1, 2]$	1.5
$(\frac{1}{2})(\frac{1}{3})$	$[2, 1]$	1.5
$(\frac{1}{3})^2$	$[2, 2]$	2
$(\frac{1}{3})(\frac{1}{6})$	$[2, 5]$	3.5
$(\frac{1}{3})(\frac{1}{6})$	$[5, 2]$	3.5
$(\frac{1}{6})^2$	$[5, 5]$	5
$(\frac{1}{6})(\frac{1}{2})$	$[5, 1]$	3
$(\frac{1}{6})(\frac{1}{2})$	$[1, 5]$	3

M	1	1.5	2	3	3.5	5
$P(M=m)$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{36}$

(Q7ai) $H_0: p = 0.2$
$H_1: p \neq 0.2$

$X \sim B[20, 0.2]$

$P(X \geq 9) = 1 - P(X \leq 8)$

$= 1 - 0.9900 = 0.01$

$0.01 < 0.025$

$\therefore$ Result is significant
Reject H_0.
Evidence suggests the % is not 20%.

ii) $P(X \geq 8) = 1 - P(X \leq 7) = 0.0321 > 0.025$

$P(X \leq 1) = 0.0692$ and $P(X \leq 0) = 0.0115$

So the number of students would have to be 0 or in the range 9–20.

b) total $n = 100$
total no. of pupils reading Deano $= 18$

$\therefore X \sim B[100, 0.20]$

(np>10
use normal approx)

$np = 20$
$np(1-p) = 16$

$\therefore X \simeq \sim N(20, 16)$

$P(X \leq 18) = P(X < 19) = P(X < 18.5)$

$= P(Z < \frac{18.5 - 20}{\sqrt{16}}) = P(Z < -0.375)$

$= 1 - P(Z < 0.38) = \boxed{0.352}$

$0.352 > 0.025$
$\therefore$ Result is insignificant
Accept H_0.
No reason to doubt 20% of pupils read Deano regularly.

c) The results from the two tests were different. The sample size in (b) was a lot bigger so it is reasonable to use the conclusion there.