S2 January 2005 (MA)

Q1a) $P(R=5) = \binom{15}{5}(0.3^5)(0.7)^{10}$

$= \boxed{0.2061}$

b) $P(S=5) = \dfrac{(e^{-7.5})(7.5^5)}{5!} = \boxed{0.1093}$

c) $P(T=5) = 0$. (Continuous data)

Q2ai) A collection of individuals / items

ii) A list of all sampling units in the population

b) Can't always keep the list 100% up-to-date

ci) Employees in a small company – small, easily found population.

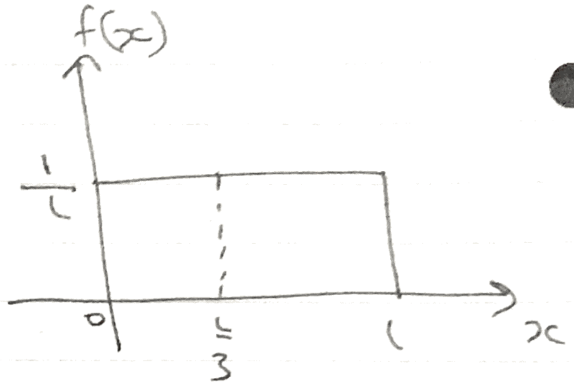
ii) Employees in a large company – population is large, may be hard to track every single employee.

Q3a) $X \sim U[0, l]$ (continuous uniform)

$$f(x) = \begin{cases} \frac{1}{l}, & 0 \leq x \leq l \\ 0, & \text{otherwise.} \end{cases}$$

b) $P(X < \frac{1}{3}L) = \frac{L}{3} \times \frac{1}{L}$

$= \boxed{\frac{1}{3}}$



c) $E(X) = \frac{a+b}{2} = \frac{0+L}{2} = \boxed{\frac{L}{2}}$

d) $\left(\frac{1}{3}\right)^2 = \boxed{\frac{1}{9}}$

Q4a) - Trials must be independent of each other.
- Probability of success / failure should be constant.

b) $X \sim B[250, 0.10]$ where X = no. of students able to distinguish...

$H_0: p = 0.1$
$H_1: p > 0.1$

$\begin{bmatrix} np = 25 \\ np(1-p) = 22.5 \end{bmatrix}$

$\therefore X \approx \sim N(25, 22.5)$

$P(X \geq 40) = P(X > 39)$

(applying c.c) $= P(X > 39.5) = P\left(Z > \frac{39.5 - 25}{\sqrt{22.5}}\right)$

$= P(Z > 3.06) \approx 1 - P(Z < 3.05)$

$\approx 0.0011 < 0.01$

$\therefore$ Result is significant
Reject H_0.
Evidence suggests claim is not justified. p is actually higher.

For Normal Approximation

c) - n is large - this is true
- p close to 0.5 - this is not true. so assumption isn't valid.

For Binomial Model

- success/failure isn't always clear. Ability to distinguish a difference may not be a simple yes or no.

Q5a) $X \sim B[10, 0.032]$ where X = no. of defective articles

$$P(X=2) = \binom{10}{2}(0.032^2)(1-0.032)^8$$

$$= \boxed{0.0355}$$

b) $X_2 \sim B[100, 0.032]$

n is large, p is small

$\therefore X_2 \simeq \sim Po(3.2)$

$$P(X_2 \leq 3) = P(X_2=0) + P(X_2=1) + P(X_2=2) + P(X_2=3)$$

$$= e^{-3.2} + (e^{-3.2})(3.2) + \frac{(e^{-3.2})(3.2)^2}{2!} + \frac{(e^{-3.2})(3.2)^3}{3!}$$

$$= \boxed{0.603}$$

c) $X_3 \sim B[1000, 0.032]$

$np = 1000 \times 0.032 = 32 > 10$.
This is too large for a poisson approximation, so use normal.

$np(1-p) = 32(1-0.032) = 30.976$

$\therefore X_3 \simeq \sim N(32, 30.976)$

$P(\text{required}) = P(X_3 > 42)$

(applying c.c) $= P(X_3 > 42.5)$

$= P\left(Z > \frac{42.5-32}{\sqrt{30.976}}\right)$

$= P(Z > 1.89) = 1 - P(Z < 1.89)$

$= \boxed{0.0294}$

Q6a) $X \sim Po(3)$

$P(X > 4) = 1 - P(X \leq 4) = 1 - 0.8513$

$= \boxed{0.1487}$

b) for 3 months, $X_2 \sim Po(9)$

$P(X_2 > 4) = 1 - P(X_2 \leq 4)$

$= 1 - 0.0550$

$= \boxed{0.9450}$

c) $\left.\begin{array}{l} H_0 : \lambda = 3 \\ H_1 : \lambda < 3 \end{array}\right\}$ $P(X \leq 1) = 0.1991$ //

$0.1991 > 0.05$

$\therefore$ Result is insignificant.
Accept H_0.
Evidence suggests that the mean no. of accidents has not reduced.

d) For 2-years, $X_3 \sim Po(72)$ $(3 \times 24 = 72)$

λ is large, $\therefore X_3 \simeq \sim N(72, 72)$

$\left.\begin{array}{l} H_0 : \lambda = 3 \\ H_1 : \lambda < 3 \end{array}\right\}$ $P(X_3 \leq 55) = P(X_3 < 56)$

$= P(X_3 < 55.5)$

$= P\left(Z < \frac{55.5 - 72}{\sqrt{72}}\right) = P(Z < -1.94)$

$= 1 - P(Z < 1.94)$

$= 1 - 0.9738 = \boxed{0.0262}$

$0.0262 < 0.05$

$\therefore$ Result is significant.
Reject H_0.
Evidence suggests mean no. of road accidents per month has indeed reduced.

Q7a) $\int_1^4 f(x)\,dx = 1$

$\Rightarrow k\int_1^4 (-x^2 + 5x - 4)\,dx = 1$

$\Rightarrow k\left[\frac{-x^3}{3} + \frac{5x^2}{2} - 4x\right]_1^4 = 1$

$\Rightarrow k\left[-\frac{64}{3} + \frac{80}{2} - 16\right] - k\left[-\frac{1}{3} + \frac{5}{2} - 4\right] = 1$

$\Rightarrow \frac{9k}{2} = 1$

$\Rightarrow k = \boxed{\frac{2}{9}}$

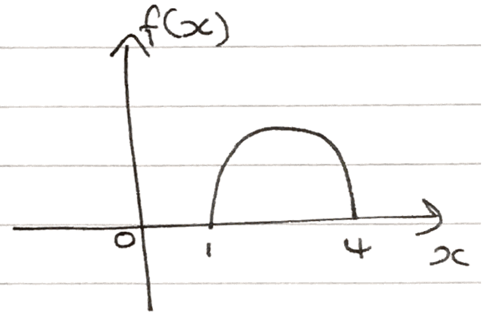
b) $E(X) = \int_1^4 x f(x)\,dx = \frac{2}{9}\int_1^4 (-x^3 + 5x^2 - 4x)\,dx$

$$= \frac{2}{9}\left[\frac{-x^4}{4} + \frac{5x^3}{3} - 2x^2\right]_1^4$$

$$= \frac{2}{9}\left[\frac{-256}{4} + \frac{320}{3} - 32\right] - \frac{2}{9}\left[\frac{-1}{4} + \frac{5}{3} - 2\right]$$

$$= \frac{2}{9}\left(\frac{32}{3} + \frac{7}{12}\right) = \boxed{2.5}$$

c) mode = highest point.



$f(x) = -\frac{2}{9}x^2 + \frac{10}{9}x - \frac{8}{9}$

$f'(x) = -\frac{4}{9}x + \frac{10}{9} = 0$

$\Rightarrow \quad 4x = 10$

$\Rightarrow \quad x = \frac{10}{4} = \boxed{2.5}$

d) $F(x) = \frac{2}{9}\int_1^x (-x^2 + 5x - 4)\,dx = \frac{2}{9}\left[\frac{-x^3}{3} + \frac{5x^2}{2} - 4x\right]_1^x$

$$= \frac{2}{9}\left[-\frac{x^3}{3} + \frac{5x^2}{2} - 4x\right] - \frac{2}{9}\left[-\frac{1}{3} + \frac{5}{2} - 4\right]$$

$$= \frac{2}{9}\left[\frac{-x^3}{3} + \frac{5x^2}{2} - 4x + \frac{11}{6}\right]$$

$$F(x) = \begin{cases} 0, & x < 1 \\ \frac{2}{9}\left(-\frac{x^3}{3} + \frac{5}{2}x^2 - 4x + \frac{11}{6}\right), & 1 \le x \le 4 \\ 1, & x > 4 \end{cases}$$

e) $P(X \le 2.5) = F(2.5) = \frac{2}{9}\left[-\frac{1}{3}\left(\frac{5}{2}\right)^3 + \frac{5}{2}\left(\frac{5}{2}\right)^2 - 4\left(\frac{5}{2}\right) + \frac{11}{6}\right]$

$= \boxed{0.5}$

f) Median = 2.5 since the distribution is symmetrical.

(see graph on previous page)