S2 January 2002 (MA)

Q1a) A set of individuals / items.

b) A random variable that is a function of known observations from a population.

c) College students are the population.
The statistic is the mean approval rating of 75%

d) The probability distribution of all possible mean approval ratings of sample size 50.

Q2) $H_0: \lambda = 2.5$ for 1 week, $X \sim Po(2.5)$
$H_1: \lambda > 2.5$ for 4 weeks, $X \sim Po(10)$

$$P(X \geq 14) = 1 - P(X \leq 13) = 1 - 0.8645$$

$$= \boxed{0.1355}$$

$0.1355 > 0.05$

$\therefore$ Result is insignificant.
Accept H_0.
Evidence suggests that the new salesman has not increased house sales.

Q3a) $X \sim B[200, 0.3]$ where X = no. of passengers that don't turn up for their flight.

b) $P(\text{more than } 196) = P(X \leq 3)$

$X \simeq \sim Po(6)$ (n high, p low).

$P(X \leq 3) = \boxed{0.1512} = P(\text{required})$.

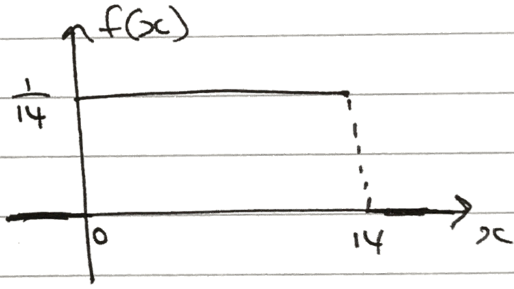
c) $P(\text{at least 1 empty seat}) = P(X \geq 5) = 1 - P(X \leq 4)$

$= 1 - 0.2851 = \boxed{0.7149}$

Q4a) $X \sim U[0, 14]$ (Continuous Uniform Distribution)

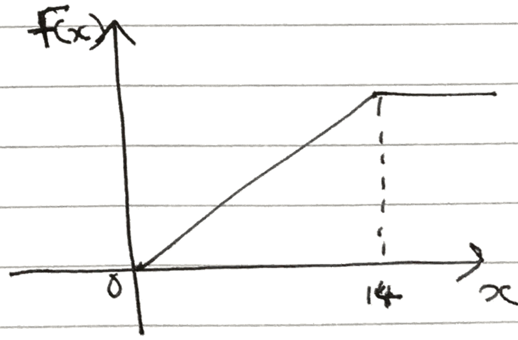
b) $E(X) = \frac{a+b}{2} = \frac{0+14}{2} = \boxed{7}$

c) $f(x) = \frac{1}{14}$



$\therefore F(x) = \int_0^x \left(\frac{1}{14}\right) dx$

$= \frac{x}{14}$ //



so $F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{14}, & 0 \leq x \leq 14 \\ 1, & x > 14 \end{cases}$

d) $P(X>10) = \dfrac{14-10}{14} = \boxed{\dfrac{2}{7}}$

Q5a) - Each failed connection occurs independently of others.

- Failed connections occur at a constant rate of 3 per hour.

bi) $X \sim Po(3)$ where X = no. of failed connections hourly.

$P(X=0) = e^{-3} = \boxed{0.0498}$

ii) $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.8153$

$= \boxed{0.1847}$

c) $X \sim Po(24)$ $(3 \times 8 = 24)$

d) λ is large so $X \approx\sim N(24, 24)$

(applying c.c): $P(X \geq 12) = P(X > 11) = P(X > 11.5)$

$P(X > 11.5) = P\left(Z > \dfrac{11.5 - 24}{\sqrt{24}}\right)$

$= P(Z e -2.55)$

$= \boxed{0.9946}$

Q6a) $X \sim B[20, 0.4]$

b) $P(5 < X < 15) = P(6 \leq X \leq 14)$

$= P(X \leq 14) - P(X \leq 5)$

$= 0.9984 - 0.1256$

$= \boxed{0.8728}$

c) $E(X) = np = 20 \times 0.4 = \boxed{8}$

$Var(X) = np(1-p) = 20(0.4)(1-0.4) = 4.8$

$\therefore \sigma = \sqrt{4.8} = \boxed{2.19}$

d) $H_0 : p = 0.4$ $\quad Y \sim B[10, 0.4]$

$H_1 : p > 0.4$ $\quad P(X \geq 8) = 1 - P(X \leq 7)$

$= 1 - 0.9877$

$= \boxed{0.0123}$

$0.0123 < 0.050$

$\therefore$ Result is significant.
Reject H_0.
Evidence suggests that proportion is higher than claim made.

$(P(X=2)=1)$

Q7a) $8k = 1$ $\therefore \boxed{k = \frac{1}{8}}$

b) $F(x) = 0.5000$

$k(x^2 + 2x) = 0.5$

$x^2 + 2x = 4$

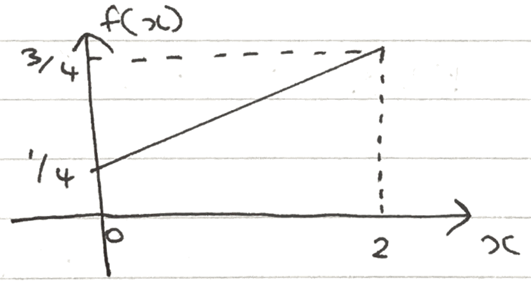
$x^2 + 2x - 4 = 0$

By Quadratic formula, $x = \boxed{\sqrt{5} - 1}$ $(x < 2)$

c) $\frac{d}{dx}\left(\frac{1}{8}x^2 + \frac{1}{4}x\right) = \frac{1}{4}x + \frac{1}{4}$ //

$\therefore$ $f(x) = \begin{cases} \frac{1}{4}(x+1), & 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$

d)



e) mode $= \boxed{2}$

f) $E(X) = \int_0^2 x f(x)\, dx = \frac{1}{4}\int_0^2 (x^2 + x)\, dx = \frac{1}{4}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^2$

$= \frac{1}{4}\left[\frac{8}{3} + 2\right] - [0] = \boxed{\frac{7}{6}}$

$\frac{7}{6} < \sqrt{5} - 1 < 2$

g) Mean < median < mode

$\therefore$ Negative skew