

4. The lifetime, X , in tens of hours, of a battery has a cumulative distribution function $F(x)$ given by

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{4}{9}(x^2 + 2x - 3) & 1 \leq x \leq 1.5 \\ 1 & x > 1.5 \end{cases}$$

- (a) Find the median of X , giving your answer to 3 significant figures. (3)
- (b) Find, in full, the probability density function of the random variable X . (3)
- (c) Find $P(X \geq 1.2)$ (2)

A camping lantern runs on 4 batteries, all of which must be working. Four new batteries are put into the lantern.

- (d) Find the probability that the lantern will still be working after 12 hours. (2)



Leave
blank

Question 4 continued

A series of horizontal lines for writing the answer to Question 4.



Leave blank

5. A company has a large number of regular users logging onto its website. On average 4 users every hour fail to connect to the company's website at their first attempt.

(a) Explain why the Poisson distribution may be a suitable model in this case. (1)

Find the probability that, in a randomly chosen **2 hour** period,

(b) (i) all users connect at their first attempt,
(ii) at least 4 users fail to connect at their first attempt. (5)

The company suffered from a virus infecting its computer system. During this infection it was found that the number of users failing to connect at their first attempt, over a 12 hour period, was 60.

(c) Using a suitable approximation, test whether or not the mean number of users per hour who failed to connect at their first attempt had increased. Use a 5% level of significance and state your hypotheses clearly. (9)

Horizontal lines for writing answers.



Leave
blank

6. A company claims that a quarter of the bolts sent to them are faulty. To test this claim the number of faulty bolts in a random sample of 50 is recorded.

(a) Give two reasons why a binomial distribution may be a suitable model for the number of faulty bolts in the sample.

(2)

(b) Using a 5% significance level, find the critical region for a two-tailed test of the hypothesis that the probability of a bolt being faulty is $\frac{1}{4}$. The probability of rejection in either tail should be as close as possible to 0.025

(3)

(c) Find the actual significance level of this test.

(2)

In the sample of 50 the actual number of faulty bolts was 8.

(d) Comment on the company's claim in the light of this value. Justify your answer.

(2)

The machine making the bolts was reset and another sample of 50 bolts was taken. Only 5 were found to be faulty.

(e) Test at the 1% level of significance whether or not the probability of a faulty bolt has decreased. State your hypotheses clearly.

(6)



Leave blank

7. The random variable Y has probability density function $f(y)$ given by

$$f(y) = \begin{cases} ky(a-y) & 0 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

where k and a are positive constants.

(a) (i) Explain why $a \geq 3$

(ii) Show that $k = \frac{2}{9(a-2)}$

(6)

Given that $E(Y) = 1.75$

(b) show that $a = 4$ and write down the value of k .

(6)

For these values of a and k ,

(c) sketch the probability density function,

(2)

(d) write down the mode of Y .

(1)



