S2 S10

1. Explain what you understand by

(a) a population,

(1)

(b) a statistic.

(1)

A researcher took a sample of 100 voters from a certain town and asked them who they would vote for in an election. The proportion who said they would vote for Dr Smith was 35%.

(c) State the population and the statistic in this case.

(2)

(d) Explain what you understand by the sampling distribution of this statistic.

(1)

a) Population - all possible items from which a sample could be chosen

Statistic - A function from a random sample containing no unknown parameters

b) Population - All the people in the town who can vote

Statistic - The percentage voting for Dr Smith

d) Sampling distribution - The probability distribution of those voting for Dr Smith from all possible samples of 100.

2. Bhim and Joe play each other at badminton and for each game, independently of all others, the probability that Bhim loses is 0.2

Find the probability that, in 9 games, Bhim loses

(a) exactly 3 of the games,

(3)

(b) fewer than half of the games.

(2)

Bhim attends coaching sessions for 2 months. After completing the coaching, the probability that he loses each game, independently of all others, is 0.05

Bhim and Joe agree to play a further 60 games.

(c) Calculate the mean and variance for the number of these 60 games that Bhim loses.

(2)

(d) Using a suitable approximation calculate the probability that Bhim loses more than 4 games.

(3)

a) x = Bhim loses $\quad X \sim B(9, 0.2)$

$$P(x=3) = \binom{9}{3} 0.2^3 0.8^6 = 0.1762$$

b) $P(x \leq 4) = 0.9804$

c) $x \sim B(60, 0.05) \qquad \mu = np = 60 \times 0.05 = 3$
$\sigma^2 = np(1-p) = 3 \times 0.95 = 2.85$

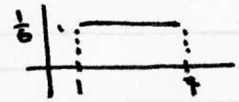
$\approx x \sim Po(3) \qquad P(x > 4) = 1 - P(x \leq 4) = 0.1847$

3. A rectangle has a perimeter of 20 cm. The length, X cm, of one side of this rectangle is uniformly distributed between 1 cm and 7 cm.

Find the probability that the length of the longer side of the rectangle is more than 6 cm long.

(5)

$x \sim U[1, 7]$



$$P(x > 6) \cup P(x < 4) = \frac{1}{6} + \frac{3}{6} = \frac{2}{3}$$

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4. The lifetime, X, in tens of hours, of a battery has a cumulative distribution function $F(x)$ given by

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{4}{9}(x^2 + 2x - 3) & 1 \leqslant x \leqslant 1.5 \\ 1 & x > 1.5 \end{cases}$$

(a) Find the median of X, giving your answer to 3 significant figures.
(3)

(b) Find, in full, the probability density function of the random variable X.
(3)

(c) Find $P(X \geqslant 1.2)$
(2)

A camping lantern runs on 4 batteries, all of which must be working. Four new batteries are put into the lantern.

(d) Find the probability that the lantern will still be working after 12 hours.
(2)

a) $F(Q_2) = 0.5 \Rightarrow \frac{4}{9}(x^2 + 2x - 3) = \frac{1}{2}$

$\Rightarrow 8x^2 + 16x - 24 = 9 \Rightarrow 8x^2 + 16x - 33 = 0 \quad \therefore Q_2 = 1.26$

c) $P(x \geqslant 1.2) = 1 - F(1.2) = 1 - \frac{28}{75} = 0.6267$

b) $f(x) = \frac{d}{dx} F(x) = \frac{4}{9}(2x + 2)$

$\therefore f(x) = \begin{cases} \frac{8}{9}(x+1) & 1 \leq x \leq 1.5 \\ 0 & \text{otherwise} \end{cases}$

d) $0.6267^4 = 0.1542$

5. A company has a large number of regular users logging onto its website. On average 4 users every hour fail to connect to the company's website at their first attempt.

(a) Explain why the Poisson distribution may be a suitable model in this case.
(1)

Find the probability that, in a randomly chosen **2 hour** period,

(b) (i) all users connect at their first attempt,

(ii) at least 4 users fail to connect at their first attempt.
(5)

The company suffered from a virus infecting its computer system. During this infection it was found that the number of users failing to connect at their first attempt, over a 12 hour period, was 60.

(c) Using a suitable approximation, test whether or not the mean number of users per hour who failed to connect at their first attempt had increased. Use a 5% level of significance and state your hypotheses clearly.
(9)

a) Connections are independent and occur at a constant rate

b) x = failed connection $\quad x \sim Po(8)$

i) $P(x=0) = e^{-8} = 0.000335$

ii) $P(x \geqslant 4)$
$P(x > 3)$ $\Rightarrow 1 - P(x \leqslant 3) = 0.9576$

c) $x \sim Po(48) \approx N(48, 48)$

$H_0 : \lambda = 48 \quad P(x \geqslant 60)$
$H_1 : \lambda > 48 \quad P(x > 59) \Rightarrow cc \; P(x > 59.5)$

$\approx P(Z > \frac{59.5 - 48}{\sqrt{48}}) \approx P(Z > 1.66) = 1 - \Phi(1.66)$
$= 0.0485 \; (< 0.05)$

$\therefore$ There is enough evidence to reject null hypothesis as result is significant

$\therefore$ enough evidence to suggest failed connections increased.

6. A company claims that a quarter of the bolts sent to them are faulty. To test this claim the number of faulty bolts in a random sample of 50 is recorded.

(a) Give two reasons why a binomial distribution may be a suitable model for the number of faulty bolts in the sample. **(2)**

(b) Using a 5% significance level, find the critical region for a two-tailed test of the hypothesis that the probability of a bolt being faulty is $\frac{1}{4}$. The probability of rejection in either tail should be as close as possible to 0.025 **(3)**

(c) Find the actual significance level of this test. **(2)**

In the sample of 50 the actual number of faulty bolts was 8.

(d) Comment on the company's claim in the light of this value. Justify your answer. **(2)**

The machine making the bolts was reset and another sample of 50 bolts was taken. Only 5 were found to be faulty.

(e) Test at the 1% level of significance whether or not the probability of a faulty bolt has decreased. State your hypotheses clearly. **(6)**

a) Bolt can be faulty or not faulty. Prob of faulty components are independent.
Probability of being faulty is constant.

b) x = faulty bolt $\quad x \sim B(50, 0.25)$

$P(x \leq L) \approx 0.025$ $\quad P(x \geq U) \approx 0.025$

$P(x > U-1) \approx 0.025$

$P(x \leq 6) = 0.0194$ $\quad 1 - P(x \leq U-1) \approx 0.025$

$P(x \leq 7) = 0.0453$ $\quad P(x \leq U-1) \approx 0.975$

$\therefore L = 6$

$P(x \leq 18) = 0.9713$ $\quad \therefore U-1 = 18$

CR $(x \leq 6) \cup (x \geq 19)$ $\quad P(x \leq 19) = 0.9861$ $\quad \therefore U = 19$

c) ASL $= 0.0194 + 0.0287 = 0.0481$ (4.81%)

b) 8 is not in the critical region $\therefore$ not significant
$\therefore$ enough evidence to support claim

Question 6 continued

e) $H_0: p = \frac{1}{4}$ $\quad P(x \leq 5) = 0.007$ (< 0.01)
$H_1: p < \frac{1}{4}$

$\therefore$ the result is significant, so there is enough evidence to reject null hypothesis
$\therefore$ enough evidence to suggest probability of getting a faulty bolt is reduced.

7. The random variable Y has probability density function f(y) given by

$$\mathrm{f}(y) = \begin{cases} ky(a-y) & 0 \leqslant y \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

where k and a are positive constants.

(a) (i) Explain why $a \geqslant 3$

(ii) Show that $k = \dfrac{2}{9(a-2)}$

(6)

Given that E(Y) = 1.75

(b) show that $a = 4$ and write down the value of k.

(6)

For these values of a and k,

(c) sketch the probability density function,

(2)

(d) write down the mode of Y.

(1)

a) i) a must be ≥ 3, otherwise Probability would be negative when y=3 which is impossible.

ii) $\int f(y)dy = 1 \Rightarrow k\int_0^3 ay - y^2 dy = 1$

$\Rightarrow k\left[\frac{1}{2}ay^2 - \frac{1}{3}y^3\right]_0^3 = 1 \Rightarrow k\left(\frac{9}{2}a - 9\right) = 1$

$\Rightarrow 9k(a-2) = 2 \quad \therefore k = \frac{2}{9(a-2)}$ #

b) $E(y) = \int yf(y)dy = k\int_0^3 ay^2 - y^3 dy$

$= k\left[\frac{1}{3}ay^3 - \frac{1}{4}y^4\right]_0^3 \Rightarrow k\left(9a - \frac{81}{4}\right) = \frac{7}{4}$

$\Rightarrow k(36a - 81) = 7 \Rightarrow k = \frac{7}{9(4a-9)}$

blank

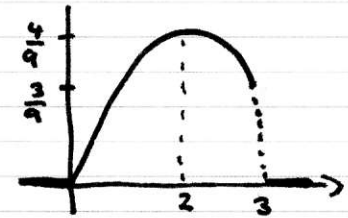
Question 7 continued

$\therefore \frac{2}{\cancel{9}(a-2)} = \frac{7}{\cancel{9}(4a-9)} \Rightarrow 8a - 18 = 7a - 14$

$\Rightarrow \underline{a = 4}$

$k = \frac{2}{9(4-2)} = \underline{\frac{1}{9}}$

$f(y) = \frac{1}{9}y(4-y)$



d) mode = 2.