S2 W14R

1. The probability of a leaf cutting successfully taking root is 0.05

Find the probability that, in a batch of 10 randomly selected leaf cuttings, the number taking root will be

(a) (i) exactly 1
(ii) more than 2
(5)

A second random sample of 160 leaf cuttings is selected.

(b) Using a suitable approximation, estimate the probability of at least 10 leaf cuttings taking root.
(3)

X = number of cuttings that take root

$X \sim B(10, 0.05)$

a) $P(X=1) = \binom{10}{1} 0.05^1 \times 0.95^9 = \underline{0.3152}$

ii) $P(X>2) = 1 - P(X \leq 2) = 1 - 0.9885 = \underline{0.0115}$

b) $np = 160 \times 0.05 = 8$

y = cuttings taking root from sample of 160

$y \sim Po(8)$

$P(y \geq 10) = 1 - P(y \leq 9) = 1 - 0.7166 = \underline{0.2834}$

2. Bill owns a restaurant. Over the next four weeks Bill decides to carry out a sample survey to obtain the customers' opinions.

(a) Suggest a suitable sampling frame for the sample survey.
(1)

(b) Identify the sampling units.
(1)

(c) Give one advantage and one disadvantage of taking a census rather than a sample survey.
(2)

Bill believes that only 30% of customers would like a greater choice on the menu. He takes a random sample of 50 customers and finds that 20 of them would like a greater choice on the menu.

(d) Test, at the 5% significance level, whether or not the percentage of customers who would like a greater choice on the menu is more than Bill believes. State your hypotheses clearly.
(6)

a) list of all customers who eat at the restaurant

b) Customers chosen in the sample

c) advantage - everyone included
more accurate

disadvantage - take too long, expensive.

d) x = customers wanting more choice

$x \sim B(50, 0.3)$ $H_0: p = 0.3$
$H_1: p > 0.3$

$P(x \geq 20) = 1 - P(x \leq 19)$
$= 1 - 0.9152 = 0.0848$

8.48% > 5% ∴ result is not statistically significant
∴ not enough evidence to reject null hypothesis
∴ accept Bill's claim.

3. The continuous random variable X has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{6}x(x+1) & 0 \leqslant x \leqslant 2 \\ 1 & x > 2 \end{cases}$$

(a) Find the value of a such that $P(X > a) = 0.4$

Give your answer to 3 significant figures. **(3)**

(b) Use calculus to find (i) $E(X)$

(ii) $Var(X)$. **(8)**

a) $P(X \leq a) = 0.6 \Rightarrow F(a) = 0.6$

$\frac{1}{6}a(a+1) = 0.6 \Rightarrow a^2 + a = 3.6$

$\Rightarrow (a+\frac{1}{2})^2 = 3.6 + 0.25 = 3.85$

$\Rightarrow a = -\frac{1}{2} \pm \sqrt{3.85} \quad \therefore a = \underline{1.46}$

b) $f(x) = \frac{d}{dx}F(x) = \frac{d}{dx}(\frac{1}{6}x^2 + \frac{1}{6}x) \quad 0 \leq x \leq 2$

$f(x) = \frac{1}{3}x + \frac{1}{6} \quad 0 \leq x \leq 2 \quad$ 0, otherwise.

$E(X) = \int x f(x)\,dx = \int_0^2 \frac{1}{3}x^2 + \frac{1}{6}x \, dx$

$= [\frac{1}{9}x^3 + \frac{1}{12}x^2]_0^2 = \frac{11}{9} - 0 = \frac{11}{9}$

$V(X) = E(X^2) - E(X)^2$

$E(X^2) = \int x^2 f(x)\,dx = \int_0^2 \frac{1}{3}x^3 + \frac{1}{6}x^2\,dx$

$= [\frac{1}{12}x^4 + \frac{1}{18}x^3]_0^2 = \frac{16}{9} - 0 = \frac{16}{9}$

$\therefore V(X) = \frac{16}{9} - (\frac{11}{9})^2 = \frac{23}{81}$

4. The number of telephone calls per hour received by a business is a random variable with distribution $Po(\lambda)$.

Charlotte records the number of calls, C, received in 4 hours.

A test of the null hypothesis $H_0: \lambda = 1.5$ is carried out.

H_0 is rejected if $C > 10$

(a) Write down the alternative hypothesis. **(1)**

(b) Find the significance level of the test. **(3)**

Given that $P(C > 10) < 0.1$

(c) find the largest possible value of λ that can be found by using the tables. **(3)**

a) $H_1: \lambda > 1.5$

b) $\lambda = 1.5$ in 1hr $\Rightarrow \lambda = 6$ in 4hrs

C = Calls received in 4hrs $\quad C \sim Po(6)$

$P(C > 10) = 1 - P(C \leq 10) = 1 - 0.9574 = 0.0426$

$\therefore$ ASL = 4.26%.

c) $P(C > 10) < 0.1 \Rightarrow 1 - P(C \leq 10) < 0.1$

$\Rightarrow P(C \leq 10) > 0.9 \quad$ If $\lambda = 7$ $P(C \leq 10) = 0.9015$
$\lambda = 7.5$ $P(C \leq 10) = 0.8622$

$\therefore \lambda = 7$

5. A school photocopier breaks down randomly at a rate of 15 times per year.

(a) Find the probability that there will be exactly 3 breakdowns in the next month. **(3)**

(b) Show that the probability that there will be at least 2 breakdowns in the next month is 0.355 to 3 decimal places. **(2)**

(c) Find the probability of at least 2 breakdowns in each of the next 4 months. **(2)**

The teachers would like a new photocopier. The head teacher agrees to monitor the situation for the next 12 months. The head teacher decides he will buy a new photocopier if there is more than 1 month when the photocopier has at least 2 breakdowns.

(d) Find the probability that the head teacher will buy a new photocopier. **(5)**

x = number of breakdowns per month.

15 per year $\Rightarrow \lambda = 1.25 \quad x \sim Po(1.25)$

$P(x=3) = \frac{e^{-1.25} \times 1.25^3}{3!} = 0.0933$

b) $P(x \geq 2) = 1 - P(x=0) - P(x=1)$

$= 1 - e^{-1.25} - e^{-1.25} \times 1.25 = 0.335$ (3dp)

c) $0.33536..^4 = 0.0159$

d) y = number of months there are more than 2 breakdowns

$y \sim B(12, 0.355..)$

$P(y>1) = 1 - P(y \leq 1) = 1 - P(y=0) - P(y=1)$

$= 1 - 0.355..^{12} - \binom{12}{1} 0.355^{11} \times 0.645^1 = 0.961$

6. The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} k(x+1)^2 & -1 \leqslant x \leqslant 1 \\ k(6-2x) & 1 < x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

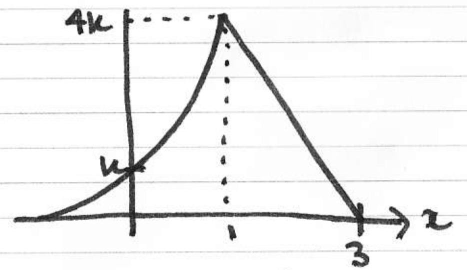
(a) Sketch the graph of $f(x)$. **(2)**

(b) Show that the value of k is $\frac{3}{20}$ **(5)**

(c) Define fully the cumulative distribution function $F(x)$. **(5)**

(d) Find the median of X, giving your answer to 3 significant figures. **(3)**

a)



b) $\int f(x)\,dx = 1 \Rightarrow \int_{-1}^{1} k(x^2+2x+1)\,dx + \int_{1}^{3} k(6-2x)\,dx$

$= k\left[\left[\frac{x^3}{3} + x^2 + x\right]_{-1}^{1} + \left[6x - x^2\right]_{1}^{3}\right] = 1$

$= k\left[\left(\frac{7}{3}\right) - \left(-\frac{1}{3}\right) + (9) - (5)\right] = 1 \quad \frac{20}{3}k = 1 \therefore k = \frac{3}{20}$

c) $F(x) = \int f(x)\,dx$

$-1 \leq x \leq 1 \quad F(x) = \frac{3}{20}\int_{-1}^{x} t^2 + 2t + 1\,dt = \frac{3}{20}\left[\frac{t^3}{3} + t^2 + t\right]_{-1}^{x}$

$$= \left(\frac{x^3}{20} + \frac{3}{20}x^2 + \frac{3}{20}x\right) - \left(\frac{-1}{20} + \frac{3}{20} - \frac{3}{20}\right) = \frac{x^3}{20} + \frac{3x^2}{20} + \frac{3x}{20} + \frac{1}{20}$$

$F(1) = \frac{8}{20}$ $\therefore\ 1 < x \le 3$ $F(x) = \frac{8}{20} + \frac{3}{20}\int_1^x 6-2t\,dt$

$$= \frac{8}{20} + \frac{3}{20}\left[6t - t^2\right]_1^x = \frac{18}{20}x - \frac{3}{20}x^2 - \frac{15}{20} + \frac{8}{20}$$

$$\therefore\ F(x) = \begin{cases} 0 & \text{if } x < -1 \\ \frac{1}{20}(x^3 + 3x^2 + 3x + 1) & \text{if } -1 \le x \le 1 \\ \frac{1}{20}(-3x^2 + 18x - 7) & \text{if } 1 < x \le 3 \\ 1 & \text{if } x > 3 \end{cases}$$

d) $F(Q_2) = 0.5$ Since $F(1) = 0.4$ $1 < Q_2 \le 3$

$\frac{1}{20}(-3x^2 + 18x - 7) = \frac{1}{2} \Rightarrow -3x^2 + 18x - 7 = 10$

$\therefore\ 3x^2 - 18x + 17 = 0$ $x = \frac{18 \pm \sqrt{18^2 - 4(3)(17)}}{6}$

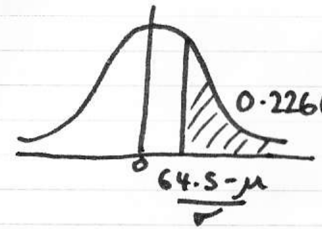
$\therefore\ Q_2 = \underline{1.17}$

7. The random variable $Y \sim B(n, p)$.

Using a normal approximation the probability that Y is at least 65 is 0.2266 and the probability that Y is more than 52 is 0.8944

Find the value of n and the value of p.

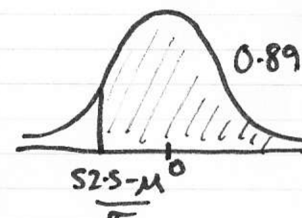
(12)



$P(y \geq 65)$ $\Rightarrow$ cc $P(y > 64.5) = 0.2266$
$P(y > 64)$

$\phi\left(\frac{64.5-\mu}{\sigma}\right) = 0.7734$

$\therefore\ \frac{64.5-\mu}{\sigma} = 0.75$ $\Rightarrow\ 64.5 - \mu = 0.75\sigma$



$P(y > 52)$ cc $P(y > 52.5) = 0.8944$
$P(y \geq 53)$

$\phi(1.25) = 0.8944$

$\therefore\ \frac{52.5-\mu}{\sigma} = -1.25$

$\Rightarrow\ 52.5 - \mu = -1.25\sigma$

$$\therefore\ \begin{array}{r} 64.5 - \mu = 0.75\sigma \\ 52.5 - \mu = -1.25\sigma \\ \hline 12 = 2\sigma \end{array} \quad - \qquad \therefore\ \sigma = 6 \quad \mu = 60$$

$\mu = np \Rightarrow 60 = np$ $\sigma^2 = np(1-p) \Rightarrow 36 = np(1-p)$

$\therefore\ 36 = 60(1-p) \Rightarrow 1 - p = 0.6$

$\underline{p = 0.4}$

$\underline{n = 150}$