S2 W12

1. The time in minutes that Elaine takes to checkout at her local supermarket follows a continuous uniform distribution defined over the interval [3, 9].

Find

(a) Elaine's expected checkout time, **(1)**

(b) the variance of the time taken to checkout at the supermarket, **(2)**

(c) the probability that Elaine will take more than 7 minutes to checkout. **(2)**

Given that Elaine has already spent 4 minutes at the checkout,

(d) find the probability that she will take a total of less than 6 minutes to checkout. **(3)**

a) $E(X) = \frac{a+b}{2} = 6$ b) $V(x) = \frac{(b-a)^2}{12} = 3$

c) $P(x > 7) = \frac{2}{6} = \frac{1}{3}$

d) $P(x < 6 \mid x > 4) = \frac{P(x<6) \cap P(x>4)}{P(x>4)}$

$= \frac{\frac{2}{6}}{5/6} = \frac{2}{5}$

2

2. David claims that the weather forecasts produced by local radio are no better than those achieved by tossing a fair coin and predicting rain if a head is obtained or no rain if a tail is obtained. He records the weather for 30 randomly selected days. The local radio forecast is correct on 21 of these days.

Test David's claim at the 5% level of significance.

State your hypotheses clearly. **(7)**

x = forecast is correct $x \sim B(30, 0.5)$

$H_0: P = 0.5$ $P(x \geq 21)$ $P(x > 20)$
$H_1: P > 0.5$

$= 1 - P(x \leq 20) = 1 - 0.9786$

$= 0.0214 \; (< 0.05)$

2

∴ enough evidence to reject null hypothesis as result is significant
∴ enough evidence to reject David's claim.

3. The probability of a telesales representative making a sale on a customer call is 0.15

Find the probability that

(a) no sales are made in 10 calls, **(2)**

(b) more than 3 sales are made in 20 calls. **(2)**

Representatives are required to achieve a mean of at least 5 sales each day.

(c) Find the least number of calls each day a representative should make to achieve this requirement. **(2)**

(d) Calculate the least number of calls that need to be made by a representative for the probability of at least 1 sale to exceed 0.95 **(3)**

$x \sim B(10, 0.15)$ x = Sale is made

a) $P(x = 0) = 0.85^{10} = 0.197$

2

b) $P(x' > 3) = 1 - P(x \leq 3) = 0.352$ $x' \sim B(20, 0.15)$

2

c) $y \sim B(n, 0.15)$

$\mu = np = 5$ ∴ $n = \frac{5}{0.15} = 33.3$ ∴ $n = 34$

2

d) $P(y \geq 1) = 1 - P(y = 0) > 0.95$

∴ $P(y = 0) < 0.05$

$\Rightarrow 0.85^n < 0.05 \Rightarrow n \log 0.85 < \log 0.05$

$\Rightarrow n > \frac{\log 0.05}{\log 0.85} \Rightarrow n > 18.4$

∴ $n = 19$

Leave blank

4. A website receives hits at a rate of 300 per hour.

(a) State a distribution that is suitable to model the number of hits obtained during a 1 minute interval. **(1)**

(b) State two reasons for your answer to part (a). **(2)**

Find the probability of

(c) 10 hits in a given minute, **(3)**

(d) at least 15 hits in 2 minutes. **(3)**

The website will go down if there are more than 70 hits in 10 minutes.

(e) Using a suitable approximation, find the probability that the website will go down in a particular 10 minute interval. **(7)**

a) 300 per hour = 5 per min

x = hit $x \sim Po(5)$ Poisson

b) Constant rate / Hits are Independent

c) $P(x=10) = \frac{e^{-5} \times 5^{10}}{10!} = 0.0181$

d) $P(y \geq 15) = 1 - P(y \leq 14)$ $y \sim Po(10)$

$P(y > 14)$ $= 0.0835$

e) $t \sim Po(50) \approx t \sim N(50, 50)$

$P(t > 70)$ cc $P(t > 70.5) \approx P(Z > \frac{70.5-50}{\sqrt{50}})$

$P(t \geq 71)$

$\approx P(Z > 2.90) = 1 - \Phi(2.9)$

$= 0.0019$

5. The probability of an electrical component being defective is 0.075
The component is supplied in boxes of 120

(a) Using a suitable approximation, estimate the probability that there are more than 3 defective components in a box. **(5)**

A retailer buys 2 boxes of components.

(b) Estimate the probability that there are at least 4 defective components in each box. **(2)**

a) x = defective component

$x \sim B(120, 0.075)$ Binomial.

$P(x > 3) = 1 - P(x \leq 3)$

$\approx x \sim Po(9)$ Poisson

$= 0.9788$

b) $P(x \geq 4) = 0.9788$

$\therefore 0.9788^2 = 0.958$

6. A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{2} & 0 \leqslant x < 1 \\ x - \frac{1}{2} & 1 \leqslant x \leqslant k \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

(a) Sketch the graph of $f(x)$. **(2)**

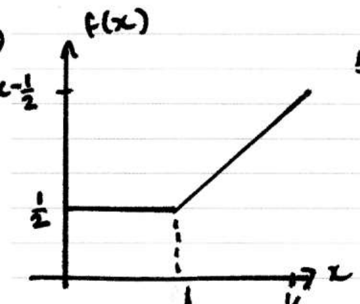
(b) Show that $k = \frac{1}{2}(1+\sqrt{5})$. **(4)**

(c) Define fully the cumulative distribution function $F(x)$. **(6)**

(d) Find $P(0.5 < X < 1.5)$. **(2)**

(e) Write down the median of X and the mode of X. **(2)**

(f) Describe the skewness of the distribution of X. Give a reason for your answer. **(2)**



b) Area = 1

$\frac{1}{2} + \frac{(\frac{1}{2} + k - \frac{1}{2}) \times k - 1}{2}$

$= \frac{1}{2} + \frac{1}{2}k(k-1) = 1$

$\Rightarrow k(k-1) = 1$

$\Rightarrow k^2 - k = 1 \Rightarrow (k - \frac{1}{2})^2 = \frac{5}{4} \Rightarrow k - \frac{1}{2} = \frac{\sqrt{5}}{2}$

$k = \frac{1}{2} + \frac{\sqrt{5}}{2} \quad \therefore \quad k = \frac{1}{2}(1+\sqrt{5})$

blank

Question 6 continued

c) $0 \leq x < 1 \quad F(x) = \int_0^x \frac{1}{2}\,dt = [\frac{1}{2}t]_0^x = \frac{1}{2}x$

$1 \leq x \leq k \quad F(x) = F(1) + \int_1^x t - \frac{1}{2}\,dt = \frac{1}{2} + [\frac{1}{2}t^2 - \frac{1}{2}t]_1^x$

$= \frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{2}$

$$\therefore F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}x & 0 \leq x < 1 \\ \frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{2} & 1 \leq x \leq \frac{1+\sqrt{5}}{2} \\ 1 & x > \frac{1+\sqrt{5}}{2} \end{cases}$$

d) $P(\frac{1}{2} < X < \frac{3}{2}) = F(\frac{3}{2}) - F(\frac{1}{2}) = \frac{7}{8} - \frac{1}{4} = \underline{\underline{\frac{5}{8}}}$

e) mode $= \underline{\underline{\frac{1+\sqrt{5}}{2}}}$ $\quad F(Q_2) = 0.5$

$F(1) = 0.5 \quad \therefore \underline{\underline{Q_2 = 1}}$

f) Median (1) < mode (1.62) $\therefore$ negative skew

alt negative skew from shape of graph of f(x) more values to the right.

7. (a) Explain briefly what you understand by

(i) a critical region of a test statistic,

(ii) the level of significance of a hypothesis test.

(2)

(b) An estate agent has been selling houses at a rate of 8 per month. She believes that the rate of sales will decrease in the next month.

(i) Using a 5% level of significance, find the critical region for a one tailed test of the hypothesis that the rate of sales will decrease from 8 per month.

(ii) Write down the actual significance level of the test in part (b)(i).

(3)

The estate agent is surprised to find that she actually sold 13 houses in the next month. She now claims that this is evidence of an increase in the rate of sales per month.

(c) Test the estate agent's claim at the 5% level of significance. State your hypotheses clearly.

(5)

a) range of values of a test statistic which would provide enough evidence to reject null hypothesis.

ii) the probability of incorrectly rejecting the null hypothesis

b) x = houses sold p/m $\quad x \sim Po(8)$

$H_0: \lambda = 8 \quad P(x \leq L) < 0.05 \quad \therefore L = 3$

$H_1: \lambda < 8 \quad P(x \leq 3) = 0.0424$

$P(x \leq 4) = 0.0996$

CR $x \leq 3 \quad$ ii) ASL $= 0.0424 \quad 4.24\%$

c) $H_0: \lambda = 8 \quad P(X \geq 13) = 1 - P(X \leq 12) = 0.0638$

$H_1: \lambda > 8 \quad P(X > 12)$

> 0.05

not enough evidence to reject null hypothesis as result is not significant

$\therefore$ not enough evidence to support her claim.