

Mark Scheme (Results) January 2007

GCE

GCE Mathematics

Statistics S2 (6684)



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6684 Statistics S2
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Question Number	Scheme	Marks
<p>1. (a)</p> <p>(b) (i)</p> <p>(ii)</p>	<p>A random variable; function of known observations (from a population). data OK</p> <p>Yes</p> <p>No</p>	<p>B1 B1 (2)</p> <p>B1 (1)</p> <p>B1 (1)</p> <p>Total 4</p>
<p>2. (a)</p> <p>(b)</p>	<p>$P(J \geq 10) = 1 - P(J \leq 9)$ or $= 1 - P(J < 10)$</p> <p>$= 1 - 0.9919$ implies method</p> <p>$= 0.0081$ awrt 0.0081</p> <p>$P(K \leq 1) = P(K = 0) + P(K = 1)$ both, implied below even with '25' missing</p> <p>$= (0.73)^{25} + 25(0.73)^{24}(0.27)$ clear attempt at '25' required</p> <p>$= 0.00392$ awrt 0.0039 implies M</p>	<p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>M1</p> <p>A1 (3)</p> <p>Total 5</p>

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3. (a)	Let W represent the number of white plants. $W \sim B(12, 0.45)$ $P(W = 5) = P(W \leq 5) - P(W \leq 4)$ $= 0.5269 - 0.3044$ $= 0.2225$	use of ${}^{12}C_5 0.45^5 0.55^7$ or equivalent award B1M1 values from correct table implies B awrt 0.222(5) B1 M1 A1 (3)
(b)	$P(W \geq 7) = 1 - P(W \leq 6)$ $= 1 - 0.7393$ $= 0.2607$	or $= 1 - P(W < 7)$ implies method awrt 0.261 M1 A1 (2)
(c)	$P(3 \text{ contain more white than coloured}) = \frac{10!}{3!7!} (0.2607)^3 (1 - 0.2607)^7$ $= 0.256654\dots$	use of $B, n=10$ awrt 0.257 M1A1 A1 (3)
(d)	mean = $np = 22.5$; var = $npq = 12.375$ $P(W > 25) \approx P\left(Z > \frac{25.5 - 22.5}{\sqrt{12.375}}\right)$ $\approx P(Z > 0.8528\dots)$ $\approx 1 - 0.8023$ ≈ 0.1977	\pm standardise with σ and μ ; ± 0.5 c.c. awrt 0.85 ‘one minus’ awrt 0.197 or 0.198 B1B1 M1;M1 A1 M1 A1 (7)
		Total 15

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4.		
(a)	$\lambda > 10$ or large μ ok	B1 (1)
(b)	The Poisson is discrete and the normal is continuous.	B1 (1)
(c)	Let Y represent the number of yachts hired in winter $P(Y < 3) = P(Y \leq 2)$ $P(Y \leq 2)$ & Po(5) $= 0.1247$ awrt 0.125	M1 A1 (2)
(d)	Let X represent the number of yachts hired in summer $X \sim \text{Po}(25)$. N(25,25) all correct, can be implied by standardisation below $P(X > 30) \approx P\left(Z > \frac{30.5 - 25}{5}\right)$ \pm standardise with 25 & 5; ± 0.5 c.c. $\approx P(Z > 1.1)$ 1.1 $\approx 1 - 0.8643$ ‘one minus’ ≈ 0.1357 awrt 0.136	B1 M1;M1 A1 M1 A1 (6)
(e)	no. of weeks $= 0.1357 \times 16$ ANS (d)x16 $= 2.17$ or 2 or 3 ans>16 M0A0	M1 A1 (2) Total 12

Question Number	Scheme	Marks
5. (a)	$f(x) = \begin{cases} \frac{1}{\beta - \alpha}, & \alpha < x < \beta, \\ 0, & \text{otherwise.} \end{cases}$ <p>function including inequality, 0 otherwise</p>	B1,B1 (2)
(b)	$\frac{\alpha + \beta}{2} = 2, \quad \frac{3 - \alpha}{\beta - \alpha} = \frac{5}{8}$ <p>or equivalent</p> $\alpha + \beta = 4$ $3\alpha + 5\beta = 24$ $3(4 - \beta) + 5\beta = 24$ $2\beta = 12$ $\beta = 6$ $\alpha = -2$ <p>both</p>	B1,B1 M1 A1 (4)
(c)	$E(X) = \frac{150 + 0}{2} = 75 \text{ cm}$	75 B1 (1)
(d)	<p>Standard deviation = $\sqrt{\frac{1}{12}(150 - 0)^2}$</p> $= 43.30127\dots \text{cm}$ <p>$25\sqrt{3}$ or awrt 43.3</p>	M1 A1 (2)
(e)	$P(X < 30) + P(X > 120) = \frac{30}{150} + \frac{30}{150}$ <p>1st or at least one fraction, + or double</p> $= \frac{60}{150} \text{ or } \frac{2}{5} \text{ or } 0.4 \text{ or equivalent fraction}$	M1,M1 A1 (3) Total 12

Question Number	Scheme	Marks
6. (a)	$H_0 : p = 0.20, H_1 : p < 0.20$ Let X represent the number of people buying family size bar. $X \sim B(30, 0.20)$ $P(X \leq 2) = 0.0442$ or $P(X \leq 2) = 0.0442$ awrt 0.044 $P(X \leq 3) = 0.1227$ $CR X \leq 2$ $0.0442 < 5\%$, so significant. Significant There is evidence that the no. of family size bars sold is lower than usual.	B1,B1 M1A1 M1 A1 (6)
(b)	$H_0 : p = 0.02, H_1 : p \neq 0.02$ $\lambda = 4$ etc ok both Let Y represent the number of gigantic bars sold. $Y \sim B(200, 0.02) \Rightarrow Y \sim Po(4)$ can be implied below $P(Y = 0) = \mathbf{0.0183}$ and $P(Y \leq 8) = \mathbf{0.9786} \Rightarrow P(Y \geq 9) = \mathbf{0.0214}$ first, either Critical region $Y = 0 \cup Y \geq 9$ $Y \leq 0$ ok N.B. Accept exact Bin: 0.0176 and 0.0202	B1 M1 B1,B1 B1,B1
(c)	Significance level = $0.0183 + 0.0214 = 0.0397$ awrt 0.04	B1 (1) Total 13

Question Number	Scheme	Marks
7. (a)	$1 - F(0.3) = 1 - (2 \times 0.3^2 - 0.3^3)$ $= 0.847$	'one minus' required M1 A1 (2)
(b)	$F(0.60) = 0.5040$ $F(0.59) = 0.4908$ <p>0.5 lies between therefore median value lies between 0.59 and 0.60.</p>	both required awrt 0.5, 0.49 M1A1 B1 (3)
(c)	$f(x) = \begin{cases} -3x^2 + 4x, & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$	attempt to differentiate, all correct M1A1 (2)
(d)	$\int_0^1 xf(x)dx = \int_0^1 -3x^3 + 4x^2 dx$ $= \left[\frac{-3x^4}{4} + \frac{4x^3}{3} \right]_0^1$ $= \frac{7}{12} \text{ or } 0.58\dot{3} \text{ or } 0.583 \text{ or equivalent fraction}$	attempt to integrate $xf(x)$ sub in limits M1 A1 (3)
(e)	$\frac{df(x)}{dx} = -6x + 4 = 0$ $x = \frac{2}{3} \text{ or } 0.\dot{6} \text{ or } 0.667$	attempt to differentiate $f(x)$ and equate to 0 M1 A1 (2)
(f)	mean < median < mode, therefore negative skew.	Any pair, cao B1,B1 (2) Total 14