STATISTICS (C) UNIT 2 TEST PAPER 7

1. (i) Briefly explain the difference between a one-tailed test and a two-tailed test. [2]

   (ii) State, with a reason, which type of test would be more appropriate to test the claim that this decade's average temperature is greater than the last decade's. [2]

2. A company that makes string wants to assess the breaking strain of its product.

   (i) Explain why a sample, and not the whole population, should be used. [2]

   A child cuts a 30 cm piece of string into two parts, cutting at a random point.

   (ii) Find the probability that one part of the string is more than twice as long as the other. [2]

   (iii) Sketch the probability density function of L, the length of the longer part of string. [2]

3. When a park is redeveloped, it is claimed that 70% of the local population approve of the new design. A conservation group, however, carries out a survey of 20 people, and finds that only 9 approve.

   (i) Use this information to carry out a hypothesis test on the original claim, working at the 5% significance level. State your conclusion clearly. [5]

   If the conservationists are right, and only 45% approve of the new park,

   (ii) use a suitable approximation to the binomial distribution to estimate the probability that in a larger survey, of 500 people, less than half will approve. [6]

4. A certain type of steel is produced in a foundry. It has flaws (small bubbles) randomly distributed, and these can be detected by X-ray analysis. On average, there are 0·1 bubbles per cm^3, and the number of bubbles per cm^3 has a Poisson distribution.

   In an ingot of 40 cm^3, find

   (i) the probability that there are less than two bubbles, [3]

   (ii) the probability that there are between 3 and 10 bubbles (inclusive). [3]

   A new machine is being considered. Its manufacturer claims that it produces fewer bubbles per cm^3. In a sample ingot of 60 cm^3, there are just two bubbles.

   (iii) Carry out a hypothesis test at the 5% significance level to decide whether the new machine is better. State your hypotheses and conclusion carefully. [5]

   (iv) Explain what a Type I error is in this context. [2]

5. The fraction of sky covered by cloud is modelled by the random variable X with probability density function

$$f(x) = kx(1 - x) \qquad 0 \le x \le 1,$$

$$f(x) = 0 \qquad \text{otherwise.}$$

(i) Find k and sketch the graph of f(x). [4]

(ii) Find the mean and the standard deviation of X. [6]

(iii) Given that flying is prohibited when 81% of the sky is covered by cloud, show that cloud conditions allow flying nearly 90% of the time. [3]

6. In a particular parliamentary constituency, the percentage of Conservative voters at the last election was 35%, and the percentage who voted for the Monster Raving Loony party was 2%. Use suitable approximations to find

(i) the probability that a random sample of 500 electors will include at least 200 who voted either Conservative or Monster Raving Loony, [6]

(ii) the probability that a random sample of 200 electors will have at least 5 Monster Raving Loony voters in it. [5]

One of (i) or (ii) requires an adjustment to be made before a calculation is done. Explain what this adjustment is, and why it is necessary. [2]

STATISTICS 2 (C) TEST PAPER 7 : ANSWERS AND MARK SCHEME

1.	(i) One-tailed : is a parameter greater (or less) than a given value?	B1	
	Two-tailed : is a parameter different from a given value?	B1	
	(ii) One-tailed, as testing for ‘warmer’ rather than ‘different’	B1 B1	4
2.	(i) If every rope were tested to breaking point, none would be left	B2	
	(ii) Needs to be cut in either of the 10 cm ends, so prob. = $^2/_3$	M1 A1	
	(iii) Graph drawn : 1/15 for 15 Š L Š 30, 0 elsewhere	B2	6
3.	(i) Taking H0 : $p = 0{\cdot}7$, no. approving is $X \sim B(20, p)$	B1 B1	
	Under H0, $P(X < 10) = P(X \le 9) = 0{\cdot}0171 < 5\%$	M1 A1	
	so at 5% level, reject H_0 and conclude that less than 70% approve	A1	
	(ii) No. of approvals is $B(500, 0{\cdot}45) \approx N(225, 123{\cdot}75)$, so	M1 A1	
	$P(X < 250) = P(X < 249{\cdot}5) = P(Z < 24{\cdot}5/11{\cdot}12)$	M1 A1	
	$= P(Z < 2{\cdot}20) = 0{\cdot}986$	M1 A1	11

4.	(i) $X \sim \text{Po}(4)$, so $P(X < 2) = 0{\cdot}0916$	B1 M1 A1
	(ii) $P(3 \le X \le 10) = 0{\cdot}9972 - 0{\cdot}2381 = 0{\cdot}759$	M1 M1 A1
	(iii) H_0 : mean number of bubbles is still 0·1 per cm^3;	
	H_1 : mean < 0·1	B1
	Under H_0, no. of bubbles in 60 cm^3 is Po(6)	B1
	Then $P(X \le 2) = 0{\cdot}062$, so do not reject H_0 at 5% level	M1 A1 A1
	(iv) Type I error is to reject the old machine in favour of the new, when in fact it is no better	B2 13
5.	(i) Need $k \int x - x^2 \, dx = 1$ $\quad k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1 \quad k = 6$	M1 A1 A1
	Graph sketched : parabola, vertex upwards, through (0, 0), (1, 0)	B1
	(ii) Mean = 0·5, by symmetry	M1 A1 A1
	$\text{Var}(X) = 6\int x^3 - x^4 \, dx - 0.5^2 = 6(0{\cdot}25 - 0{\cdot}2) - 0{\cdot}25 = 0{\cdot}05$	M1 A1
	so standard deviation = $\sqrt{0{\cdot}05} = 0{\cdot}224$	A1
	(iii) $P(x \le 81\%) = 6\int_0^{0.81} x^3 - x^4 \, dx = 0{\cdot}9054$, so cloud	M1 A1
	cover is ≤ 81% for about 90% of the time	A1 13
6.	(i) No. of Cons or MRL ~ B(500, 0·37) ≈ N (185, 116·55), so	M1 A1
	$P(X \ge 200) = P(X > 199{\cdot}5) = P(Z > 14{\cdot}5/10{\cdot}79) = P(Z > 1{\cdot}34)$	M1 A1 M1
	$= 1 - 0{\cdot}9099 = 0{\cdot}0901$	A1
	(ii) No. of MRL ~ B(200, 0·02) ≈ Po(4)	M1 A1
	so $P(X \ge 5) = 1 - 0{\cdot}6288 = 0{\cdot}371$	M1 A1 A1
	Binomial to Normal needs continuity correction, going from a discrete	B1
	to a continuous distribution	B1 13