STATISTICS (C) UNIT 2 TEST PAPER 6

1. An agricultural inspector wishes to select a random sample of farms in an area. To do so, he randomly chooses a point on a map, and visits the farm whose land that point is in.
   (i) State whether this method produces a random sample of farms. [2]
   (ii) Suggest a better method of selecting the sample. [2]

2. The lengths of a certain type of fish are thought to be normally distributed. It is found that 12% of those caught are longer than 37 cm and 9% are shorter than 31 cm.
   Find the mean and standard deviation of the lengths of these fish. [6]

3. Bags of sugar are filled at a refinery. Their weights are normally distributed, with mean 504 grams and standard deviation 7 grams.
   (i) Find the probability that a randomly-chosen bag has a weight below 495 grams. [2]
   A sample of 5 bags is taken and their average weight is found to be 498·8 grams.
   (ii) Working at the 1% significance level, and assuming that the standard deviation is 7 grams, determine whether this indicates that the machine is measuring less than the correct mean weight. [5]

4. It is thought that a random variable X has a Poisson distribution with mean 8.
   (i) Find the critical region to test the hypothesis $H0 : \lambda = 8$ against the hypothesis $H1 : \lambda < 8$, working at the 5% significance level. [2]
   (ii) Find the critical region if, instead, the alternative hypothesis is $\lambda \neq 8$. [3]
   (iii) If, in fact, $\lambda = 5{\cdot}5$, find the probability of making a Type II error in case (ii). [3]

5. A secretarial agency carefully assesses the work of a new recruit, with the following results after 150 pages:

No of errors	0	1	2	3	4	5	6
No of pages	16	38	41	29	14	10	2

   (i) Calculate unbiased estimates of the mean and variance of the number of errors per page. [4]
   (ii) Explain how these results support the idea that the number of errors per page follows a Poisson distribution. [1]
   (iii) After two weeks at the agency, the secretary types a fresh piece of work, six pages long, which is found to contain 20 errors.
   The director suspects that the secretary was trying especially hard during the early period and is now less conscientious. Test this hypothesis at the 5% significance level. [6]

6. A random variable X has a probability density function given by

$$f(x) = \frac{4x^2(3-x)}{27} \qquad 0 \le x \le 3,$$

$$f(x) = 0 \qquad \text{otherwise.}$$

(i) Find the mode of X. [3]

(ii) Find the mean of X. [3]

(iii) Show that the median, m, of X satisfies the equation $m^4 - 4m^3 + 13{\cdot}5 = 0$, and hence show that $1{\cdot}84 < m < 1{\cdot}85$. [6]

7. A certain Sixth Former is late for school once a week, on average. In a half term of seven five-day weeks, lateness on more than ten occasions results in loss of privileges the following half term.

(i) Use an appropriate normal distribution to estimate the probability that he loses his privileges. [7]

In a certain half term, he is only late on 4 days altogether.

(ii) Using a 5% significance level, carry out a hypothesis test to decide whether this supports his claim that his attendance has improved. [5]

STATISTICS 2 (C) TEST PAPER 6 : ANSWERS AND MARK SCHEME

1.	(i) No - farms with larger areas are more likely to be chosen	B2	
	(ii) Allocate each farm a number, then select numbers randomly	B2	4
2.	Top 12% needs $z = 1{\cdot}175$ and bottom 9% needs $z = -1{\cdot}34$	B1 B1	
	Get $\mu + 1{\cdot}175\sigma = 37$ and $\mu - 1{\cdot}34\sigma = 31$,	B1 (both)	
	so $2{\cdot}515\sigma = 6$, and $\sigma = 2{\cdot}39$ cm; then $\mu = 34{\cdot}2$ cm	M1 A1 A1	6
3	(i) $P(X < 495) = P(Z < -{}^9/_7) = P(Z < -1{\cdot}286) = 1 - 0{\cdot}9008 = 0{\cdot}0992$	M1 A1	
	(ii) With H_0 : mean = 504, $\bar{X}$ is distributed $N(504, {}^{49}/_5)$, so	B2	
	$P(X < 498{\cdot}8) = P(Z < (498{\cdot}8 - 504) / (7/\sqrt{5})) = P(Z < -1{\cdot}661)$	M1 A1	
	$= 0{\cdot}0484$ Do not reject H_0 at 1% level	A1	7
4.	(i) From tables, reject H_0 if $X = 0, 1, 2$ or 3	B2	
	(ii) Split 5% into two tails of 2·5% each :	M1	
	reject H_0 if $X = 0, 1, 2$ or $X > 13$	A1 A1	
	(iii) If $\lambda = 5{\cdot}5$, $P(2 < X < 14) = 0{\cdot}9983 - 0{\cdot}0884 = 0{\cdot}910$	M1 A1 A1	8

5.	(i) Mean = 325/150 = 2·167	M1 A1
	variance = 1009/149 – (150/149) ×$2{\cdot}1666^2$ = 2·05	M1 A1
	(ii) Mean ≈ Variance, which suggests Poisson	B1
	(iii) Assuming $H_0 : \lambda = 2{\cdot}1666$, no. of errors in 6 pages is Po(13)	B1 B1
	Then $P(X \geq 20) = 1 - 0{\cdot}9573 = 0{\cdot}0427 < 5\%$, so has got worse	M1 M1 A1 A111
6.	(i) $f'(x) = {}^4/_{27}(6x - 3x^2) = 0$ ie T.P when $x = 2$, so mode = 2	M1 A1 A1
	(ii) Mean = ${}^4/_{27} \int 3x^3 - x^4 \, dx = \left[\frac{4}{27}\left(\frac{3x^4}{4} - \frac{x^5}{5}\right)\right]_0^3 = 1{\cdot}8$	M1 A1 A1
	(iii) For median m, need ${}^4/_{27} \int 3x^3 - x^4 \, dx = 0{\cdot}5$	M1
	so $m^3 - (m^4)/_4 = {}^{27}/_8$, i.e. $m^4 - 4m^3 + 13{\cdot}5 = 0$	A1 A1
	Put $m = 1{\cdot}84$, get $0{\cdot}044 > 0$	M1 A1
	Put $m = 1{\cdot}85$, get $-0{\cdot}113 < 0$, so $1{\cdot}84 < m < 1{\cdot}85$	A1 12
7.	(i) No of lates in 7 weeks is distributed B(35, 0·2) ~ N(7, 5·6)	M1 A1
	$P(X > 10) = P(X > 10{\cdot}5) = P(Z > 3{\cdot}5/\sqrt{5{\cdot}6}) = P(Z > 1{\cdot}479)$	M1 A1 A1
	$= 1 - 0{\cdot}9305 = 0{\cdot}0695$	M1 A1
	(ii) H0 : $p = 0{\cdot}2$; then $P(X \leq 4) = P(X < 4{\cdot}5) = P(Z < -2{\cdot}5/\sqrt{5{\cdot}6})$	B1 M1 A1
	$= P(Z < -1{\cdot}056) = 0{\cdot}146 > 5\%$	A1
	Therefore, no significant evidence of a change from $p = 0{\cdot}2$	A1 12