

STATISTICS (A) UNIT 2**TEST PAPER 8**

1. (a) Briefly explain the difference between a one-tailed test and a two-tailed test. (2 marks)
- (b) State, with a reason, which type of test would be more appropriate to test the claim that this decade's average temperature is greater than the last decade's. (2 marks)
2. (a) Give one advantage and one disadvantage of
- (i) a sample survey, (2 marks)
- (ii) a census. (2 marks)
- (b) Suggest a situation in which each could be used. (2 marks)
3. A pharmaceutical company produces an ointment for earache that, in 80% of cases, relieves pain within 6 hours. A new drug is tried out on a sample of 25 people with earache, and 24 of them get better within 6 hours.
- (a) Test, at the 5% significance level, the claim that the new treatment is better than the old one. State your hypotheses carefully. (6 marks)
- A rival company suggests that the sample does not give a conclusive result;
- (b) Might they be right, and how could a more conclusive statement be achieved? (3 marks)
4. A centre for receiving calls for the emergency services gets an average of 3.5 emergency calls every minute. Assuming that the number of calls per minute follows a Poisson distribution,
- (a) find the probability that more than 6 calls arrive in any particular minute. (3 marks)
- Each operator takes a mean time of 2 minutes to deal with each call, and therefore seven operators are necessary to cope with the average demand.
- (b) Find how many operators are required for there to be a 99% probability that a call can be dealt with immediately. (3 marks)
- It is found from experience that a major disaster creates a surge of emergency calls. Taking the null hypothesis H_0 that there is no disaster,
- (c) find the number of calls that need to be received in one minute to disprove H_0 at the 0.1 % significance level. (3 marks)
5. The random variable X has a continuous uniform distribution on the interval $a \leq X \leq 3a$.
- (a) Without assuming any standard results, prove that μ , the mean value of X , is equal to $2a$ and derive an expression for σ^2 , the variance of X , in terms of a . (7 marks)
- (b) Find the probability that $|X - \mu| < \sigma$ and compare this with the same probability when x is modelled by a Normal distribution with the same mean and variance. (6 marks)

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6. Two people are playing darts. Peg hits points randomly on the circular board, whose radius is a . If the distance from the centre O of the point that she hits is modelled by the variable R ,

(a) explain why the cumulative distribution function $F(r)$ is given by

$$\begin{aligned} F(r) &= 0 & r < 0, \\ F(r) &= \frac{r^2}{a^2} & 0 \leq r \leq a, \\ F(r) &= 1 & r > a. \end{aligned} \quad (4 \text{ marks})$$

(b) By first finding the probability density function of R , show that the mean distance from O of the points that Peg hits is $\frac{2a}{3}$. (7 marks)

Bob, a more experienced player, aims for O , and his points have a distance X from O whose cumulative distribution function is

$$F(x) = 0, \quad x < 0; \quad F(x) = \frac{x}{a} \left(2 - \frac{x}{a} \right), \quad 0 \leq x \leq a; \quad F(x) = 1, \quad x > a.$$

(c) Find the probability density function of X , and explain why it shows that Bob is aiming for O . (5 marks)

7. In an orchard, all the trees are either apple or pear trees. There are four times as many apple trees as pear trees. Find the probability that, in a random sample of 10 trees, there are

- (a) equal numbers of apple and pear trees, (3 marks)
 (b) more than 7 apple trees. (3 marks)

In a sample of 60 trees in the orchard,

- (c) find the expected number of pear trees. (1 marks)
 (d) Calculate the standard deviation of the number of pear trees and compare this result with the standard deviation of the number of apple trees. (2 marks)
 (e) Find the probability that exactly 35 in the sample of 60 trees are pear trees. (4 marks)
 (f) Find an approximate value for the probability that more than 15 of the 60 trees are pear trees. (5 marks)