# STATISTICS (A) UNIT 2

# TEST PAPER 7

1. A company that makes ropes for mountaineering wants to assess the breaking strain of its ropes.

   (a) Explain why a sample survey, and not a census, should be used. **(2 marks)**

   (b) Suggest an appropriate sampling frame. **(1 mark)**

2. It is thought that a random variable $X$ has a Poisson distribution whose mean, $\lambda$, is equal to 8. Find the critical region to test the hypothesis $H_0 : \lambda = 8$ against the hypothesis $H_1 : \lambda < 8$, working at the 1% significance level. **(5 marks)**

3. A child cuts a 30 cm piece of string into two parts, cutting at a random point.

   (a) Name the distribution of $L$, the length of the longer part of string, and sketch the probability density function for $L$. **(4 marks)**

   (b) Find the probability that one part of the string is more than twice as long as the other. **(3 marks)**

4. A supplier of widgets claims that only 10% of his widgets have faults.

   (a) In a consignment of 50 widgets, 9 are faulty. Test, at the 5% significance level, whether this suggests that the supplier's claim is false. **(6 marks)**

   (b) Find how many faulty widgets would be needed to provide evidence against the claim at the 1% significance level. **(3 marks)**

5. In a survey of 22 families, the number of children, $X$, in each family was given by the following table, where $f$ denotes the frequency:

| $X$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f$ | 3 | 8 | 5 | 3 | 2 | 1 |

   (a) Find the mean and variance of $X$. **(4 marks)**

   (b) Explain why these results suggest that $X$ may follow a Poisson distribution. **(1 mark)**

   (c) State another feature of the data that suggests a Poisson distribution. **(1 mark)**

   It is sometimes suggested that the number of children in a family follows a Poisson distribution with mean 2·4. Assuming that this is correct,

   (d) find the probability that a family has less than two children. **(3 marks)**

   (e) Use this result to find the probability that, in a random sample of 22 families, exactly 11 of the families have less than two children. **(3 marks)**

6. When a park is redeveloped, it is claimed that 70% of the local population approve of the new design. Assuming this to be true, find the probability that, in a group of 10 residents selected at random,

(a) 6 or more approve, **(3 marks)**

(b) exactly 7 approve. **(3 marks)**

A conservation group, however, carries out a survey of 20 people, and finds that only 9 approve.

(c) Use this information to carry out a hypothesis test on the original claim, working at the 5% significance level. State your conclusion clearly. **(5 marks)**

If the conservationists are right, and only 45% approve of the new park,

(d) use a suitable approximation to the binomial distribution to estimate the probability that in a larger survey, of 500 people, less than half will approve. **(7 marks)**

7. A continuous random variable $X$ has probability density function $f(x)$ given by

$$f(x) = \frac{2x}{3} \qquad 0 \le x < 1,$$

$$f(x) = 1 - \frac{x}{3} \qquad 1 \le x \le 3,$$

$$f(x) = 0 \qquad \text{otherwise.}$$

(a) Sketch the graph of $f(x)$ for all $x$. **(3 marks)**

(b) Find the mean of $X$. **(5 marks)**

(c) Find the standard deviation of $X$. **(7 marks)**

(d) Show that the cumulative distribution function of $X$ is given by

$$F(x) = \frac{x^2}{3} \qquad 0 \le x < 1,$$

and find $F(x)$ for $1 \le x \le 3$. **(6 marks)**

 