STATISTICS 2 (A) TEST PAPER 6 : ANSWERS AND MARK SCHEME

1.	(a) List of all drivers insured with that company	B1	
	(b) Individual drivers (c) Sample is quicker and cheaper	B1; B2	4
2.	(a) Continuous uniform distribution on $[0, 360]$	B1	
	(b) Mean = 180 s.d. = $\sqrt{(360^2 \div 12)} = \sqrt{10800} = 103{\cdot}9$	B1 M1 A1	4
3.	(a) Mean = $322/150 = 2{\cdot}15$, variance = $982/150 - 2{\cdot}147^2 = 1{\cdot}94$	M1 A1 M1 A1	
	(b) Mean ≈ Variance, which suggests Poisson	B1	
	(c) Assuming $\lambda = 2{\cdot}14666$, no. of errors in 6 pages is $\text{Po}(12{\cdot}88)$	B1 B1	
	then $P(X \geq 15) > P(X = 15) = 0{\cdot}0868 > 5\%$, so do not reject H_0	M1 A1 A1	10
4.	(a) $X \sim B(5, 0{\cdot}2)$ $P(X = 0) = 0{\cdot}8^5 = 0{\cdot}3277$	M1 A1	
	(b) $P(X > 2) = 1 - P(X \leq 2) = 1 - 0{\cdot}9421 = 0{\cdot}0579$ (from tables)	M1 A1 A1	
	(c) No of lates in 7 weeks is distributed $B(35, 0{\cdot}2) \approx N(7, 5{\cdot}6)$	M1 A1	
	$P(X > 10) = P(X > 10{\cdot}5) = P(Z > 3{\cdot}5/\sqrt{5.6}) = P(Z > 1{\cdot}48)$	M1 A1 A1	
	$= 1 - 0{\cdot}9306 = 0{\cdot}0694$	M1 A1	12
5.	(a) $X \sim \text{Po}(4)$, so $P(X < 2) = 0{\cdot}0916$	B1 M1 A1	
	(b) $P(3 < X < 10) = 0{\cdot}9919 - 0{\cdot}4335 = 0{\cdot}558$	M1 M1 A1	
	(c) H_0 : mean number of bubbles is still $0{\cdot}1$ per cm^3; H_1 : mean $< 0{\cdot}1$	B1 B1	
	Under H_0, no. of bubbles in 60 cm^3 is $\text{Po}(6)$	B1	
	Then $P(X \leq 1) = 0{\cdot}0174$, so do not reject H_0 at 1% level	M1 A1 A1	12
6.	(a) $f'(x) = \frac{4}{27}(6x - 3x^2) = 0$ when $x = 2$, so mode = 2	M1 A1 A1	
	(b) $E(x) = \frac{4}{27}\int_0^3 3x^3 - x^4 \, dx = \frac{4}{27}\left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^3 = 1{\cdot}8$	M1 A1 A1	
	(c) $F(x) = 0$ $(x < 0)$, $F(x) = \frac{4}{27}\left(x^3 - \frac{x^4}{4}\right)$ $(0 \leq x \leq 3)$,	B1 M1 A1	
	$F(x) = 1$ $(x > 3)$	B1	
	(d) For median m, $F(m) = 0{\cdot}5$ Now $F(1{\cdot}84) = 0{\cdot}498 < 0{\cdot}5$ and	M1 A1	
	$F(1{\cdot}85) = 0{\cdot}504 > 0{\cdot}5$, so $1{\cdot}84 < m < 1{\cdot}85$	A1 A1	
	(e) Mean < median < mode Negative skew	B1	15
7.	(a) $\int_2^{10} f(x)\, dx = 1$, so $k\int_2^{10} (-x^2 + 12x - 20)\, dx = 1$	M1 M1 A1	
	$k\left[-\frac{1}{3}x^3 + 6x^2 - 20x\right]_2^{10} = 1$ $\frac{256k}{3} = 1$ $k = \frac{3}{256}$	M1 A1	
	By symmetry of parabola, mean = 6	B1	
	(b) $E(X^2) = \frac{3}{256}\int_2^{10} (-x^4 + 12x^3 - 20x^2)\, dx$	M1 A1	
	$= \frac{3}{256}\left[-\frac{x^5}{5} + 3x^4 - \frac{20x^3}{3}\right]_2^{10} = 39{\cdot}2$ $\text{Var}(X) = 3{\cdot}2$	M1 A1 A1	
	Standard deviation = $\sqrt{3{\cdot}2} = 1{\cdot}789$, i.e. £1789	A1	
	(c) $P(X > 8) = \int_8^{10} f(x)\, dx = \frac{3}{256}\left[-\frac{1}{3}x^3 + 6x^2 - 20x\right]_8^{10} = 0{\cdot}156$	M1 A1 M1 A1	
	(d) $0{\cdot}156^4 = 0{\cdot}000592$	M1 A1	18