

**STATISTICS (A) UNIT 2****TEST PAPER 3**

1. Explain what is meant by
- (a) a population, (1 mark)
  - (b) a sampling unit. (1 mark)
- Suggest suitable sampling frames for surveys of
- (c) families who have holidays in Greece, (1 mark)
  - (d) mothers with children under two years old. (1 mark)
2. A continuous random variable  $X$  has the probability density function
- $$f(x) = k \quad 5 \leq x \leq 15,$$
- $$f(x) = 0 \quad \text{otherwise.}$$
- (a) Find  $k$  and specify the cumulative density function  $F(x)$ . (5 marks)
  - (b) Write down the value of  $P(X < 8)$ . (1 mark)
3. A coin is tossed 20 times, giving 16 heads.
- (a) Test at the 1% significance level whether the coin is fair, stating your hypotheses clearly. (6 marks)
  - (b) Find the critical region for the same test at the 0.1% significance level. (2 marks)
4. Alison and Gemma play table tennis. Alison starts by serving for the first five points. The probability that she wins a point when serving is  $p$ .
- (a) Show that the probability that Alison is ahead at the end of her five serves is given by  $p^3(6p^2 - 15p + 10)$ . (7 marks)
  - (b) Evaluate this probability when  $p = 0.6$ . (2 marks)
5. In a certain school, 32% of Year 9 pupils are left-handed. A random sample of 10 Year 9 pupils is chosen.
- (a) Find the probability that none are left-handed. (3 marks)
  - (b) Find the probability that at least two are left-handed. (4 marks)
  - (c) Use a suitable approximation to find the probability of getting more than 5 but less than 15 left-handed pupils in a group of 35 randomly selected Year 9 pupils. Explain what adjustment is necessary when using this approximation. (8 marks)

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6. A sample of radioactive material decays randomly, with an approximate mean of 1.5 counts per minute.
- (a) Name a distribution that would be suitable for modelling the number of counts per minute.  
Give any parameters required for the model. **(2 marks)**
- (b) Find the probability of at least 4 counts in a randomly chosen minute. **(3 marks)**
- (c) Find the probability of 3 counts or fewer in a random interval lasting 5 minutes. **(3 marks)**

More careful measurements, over 50 one-minute intervals, give the following data for  $x$ , the number of counts per minute:

$$\sum x = 84, \quad \sum x^2 = 226.$$

- (d) Decide whether these data support your answer to part (a). **(4 marks)**
- (e) Use the improved data to find probability of exactly two counts in a given one-minute interval. **(3 marks)**
7. Each day on the way to work, a commuter encounters a similar traffic jam. The length of time, in 10-minute units, spent waiting in the traffic jam is modelled by the random variable  $T$  with the cumulative distribution function:
- $$F(t) = 0 \quad t < 0,$$
- $$F(t) = \frac{t^2(3t^2 - 16t + 24)}{16} \quad 0 \leq t \leq 2,$$
- $$F(t) = 1 \quad t > 2.$$
- (a) Show that 0.77 is approximately the median value of  $T$ . **(3 marks)**
- (b) Given that he has already waited for 12 minutes, find the probability that he will have to wait another 3 minutes. **(5 marks)**
- (c) Find, and sketch, the probability density function of  $T$ . **(4 marks)**
- (d) Hence find the modal value of  $T$ . **(5 marks)**
- (e) Comment on the validity of this model. **(1 mark)**