General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MS2B

Unit Statistics 2B

Monday 20 June 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The number of cars passing a speed camera on a main road between 9.30 am and 11.30 am may be modelled by a Poisson distribution with a mean rate of 2.6 per minute.

(a) (i) Write down the distribution of X, the number of cars passing the speed camera during a 5-minute interval between 9.30 am and 11.30 am. *(1 mark)*

(ii) Determine $P(X = 20)$. *(2 marks)*

(iii) Determine $P(6 \leqslant X \leqslant 18)$. *(3 marks)*

(b) Give **two** reasons why a Poisson distribution with mean 2.6 may not be a suitable model for the number of cars passing the speed camera during a 1-minute interval between 8.00 am and 9.30 am on weekdays. *(2 marks)*

(c) When n cars pass the speed camera, the number of cars, Y, that exceed 60 mph may be modelled by the distribution $B(n, 0.2)$.

Given that $n = 20$, determine $P(Y \geqslant 5)$. *(2 marks)*

(d) Stating a necessary assumption, calculate the probability that, during a given 5-minute interval between 9.30 am and 11.30 am, exactly 20 cars pass the speed camera of which at least 5 are exceeding 60 mph. *(3 marks)*

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2 (a) The continuous random variable X has a rectangular distribution defined by the probability density function

$$f(x) = \begin{cases} 0.01\pi & u \leqslant x \leqslant 11u \\ 0 & \text{otherwise} \end{cases}$$

where u is a constant.

(i) Show that $u = \dfrac{10}{\pi}$. *(2 marks)*

(ii) Using the formulae for the mean and the variance of a rectangular distribution, find, in terms of π, values for $E(X)$ and $\text{Var}(X)$. *(2 marks)*

(iii) Calculate **exact** values for the mean and the variance of the circumferences of circles having diameters of length $\left(X + \dfrac{10}{\pi}\right)$. *(4 marks)*

(b) A machine produces circular discs which have an area of $Y\ \text{cm}^2$. The distribution of Y has mean μ and variance 25.

A random sample of 100 such discs is selected. The mean area of the discs in this sample is calculated to be $40.5\ \text{cm}^2$.

Calculate a 95% confidence interval for μ. *(3 marks)*

3 Emily believed that the performances of 16-year-old students in their GCSEs are associated with the schools that they attend. To investigate her belief, Emily collected data on the GCSE results for 2010 from four schools in her area.

The table shows Emily's collected data, denoted by O_i, together with the corresponding expected frequencies, E_i, necessary for a χ^2 test.

	$\geqslant$5 GCSEs		**1 $\leqslant$ GCSEs < 5**		**No GCSEs**	
	O_i	E_i	O_i	E_i	O_i	E_i
Jolliffe College for the Arts	187	193.15	93	90.62	30	26.23
Volpe Science Academy	175	184.43	97	86.52	24	25.05
Radok Music School	183	183.81	78	86.23	34	24.96
Bailey Language School	265	248.61	112	116.63	22	33.76

Emily used these values to correctly conduct a χ^2 test at the 1% level of significance.

(a) State the null hypothesis that Emily used. *(1 mark)*

(b) Find the value of the test statistic, X^2, giving your answer to one decimal place. *(3 marks)*

(c) State, in context, the conclusion that Emily should reach based on the results of her χ^2 test. *(3 marks)*

(d) Make **one** comment on the GCSE performances of 16-year-old students attending Bailey Language School. *(1 mark)*

(e) Emily's friend, Joanna, used the same data to correctly conduct a χ^2 test using the 10% level of significance.

State, with justification, the conclusion that Joanna should reach. *(2 marks)*

Turn over ▶

4 A discrete random variable X has the probability distribution

$$P(X = x) = \begin{cases} \dfrac{3x}{40} & x = 1, 2, 3, 4 \\ \dfrac{x}{20} & x = 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Calculate $E(X)$. *(2 marks)*

(b) Show that:

(i) $E\left(\dfrac{1}{X}\right) = \dfrac{7}{20}$; *(2 marks)*

(ii) $\text{Var}\left(\dfrac{1}{X}\right) = \dfrac{7}{160}$. *(4 marks)*

(c) The discrete random variable Y is such that $Y = \dfrac{40}{X}$.

Calculate:

(i) $P(Y < 20)$; *(3 marks)*

(ii) $P(X < 4 \mid Y < 20)$. *(3 marks)*

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5 (a) The lifetime of a new 16-watt energy-saving light bulb may be modelled by a normal random variable with standard deviation 640 hours. A random sample of 25 bulbs, taken by the manufacturer from this distribution, has a mean lifetime of 19 700 hours.

Carry out a hypothesis test, at the 1% level of significance, to determine whether the mean lifetime has changed from 20 000 hours. *(6 marks)*

(b) The lifetime of a new 11-watt energy-saving light bulb may be modelled by a normal random variable with mean μ hours and standard deviation σ hours.

The manufacturer claims that the mean lifetime of these energy-saving bulbs is 10 000 hours. Christine, from a consumer organisation, believes that this is an overestimate.

To investigate her belief, she carries out a hypothesis test at the 5% level of significance based on the null hypothesis $H_0 : \mu = 10\,000$.

(i) State the alternative hypothesis that should be used by Christine in this test. *(1 mark)*

(ii) From the lifetimes of a random sample of 16 bulbs, Christine finds that $s = 500$ hours.

Determine the range of values for the sample mean which would lead Christine **not** to reject her null hypothesis. *(5 marks)*

(iii) It was later revealed that $\mu = 10\,000$.

State which type of error, if any, was made by Christine if she concluded that her null hypothesis should **not** be rejected. *(1 mark)*

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6 The continuous random variable X has the probability density function defined by

$$f(x) = \begin{cases} \frac{3}{8}(x^2 + 1) & 0 \leqslant x \leqslant 1 \\ \frac{1}{4}(5 - 2x) & 1 \leqslant x \leqslant 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) The cumulative distribution function of X is denoted by $F(x)$.

Show that, for $0 \leqslant x \leqslant 1$,

$$F(x) = \frac{1}{8}x(x^2 + 3)$$ *(3 marks)*

(b) Hence, or otherwise, verify that the median value of X is 1. *(2 marks)*

(c) Show that the upper quartile, q, satisfies the equation $q^2 - 5q + 5 = 0$ and hence that $q = \frac{1}{2}(5 - \sqrt{5})$. *(5 marks)*

(d) Calculate the **exact** value of $P(q < X < 1.5)$. *(4 marks)*

END OF QUESTIONS