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## General Certificate of Education

# Mathematics 6360

## MS2B Statistics 2B

# Mark Scheme

*2009 examination - June series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.



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**Key to mark scheme and abbreviations used in marking**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |

| | | | |
|---|---|---|---|
| √or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | or equivalent | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| –$x$ EE | deduct $x$ marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MS2B**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1** | $H_0: \mu = 768$<br>$H_1: \mu \neq 768$ | B1 | | (Both) |
| | Test statistic: $z = \dfrac{764.8 - 768}{8/\sqrt{18}}$ | M1 | | |
| | $= -1.70$ | A1 | | $(-1.697)$ |
| | $z_{crit} = \pm 1.96$ | B1 | | ( $z_{\text{crit}} = 1.96$ or $z_{\text{crit}} = -1.96$ ) |
| | $\Rightarrow$ Accept $H_0$ | A1 | | |
| | No evidence at the 5% level of significance, to deny Yvonne's claim. | E1 | 6 | |
| | **Total** | | **6** | |
| **2(a)(i)** | $X \sim \text{Po}(5.0)$<br>$\Rightarrow \; P(X < 4) = P(X \leq 3)$<br>$= 0.265$ | B2 | 2 | $(0.440 \text{ to } 0.441)$ for B1<br>CAO |
| **(ii)** | $Y \sim \text{Po}(1.5)$ | | 2 | |
| | $\Rightarrow \; P(Y = 4) = \dfrac{e^{-1.5} \times (1.5)^4}{4!}$ | M1 | | |
| | $= 0.0471$ | A1 | 2 | (0.047 to 0.0471) |
| **2(b)(i)** | $T = X + Y \sim \text{Po}(6.5)$ | B1 | | |
| | $\Rightarrow \; P(T > 5) = 1 - P(T \leq 5)$ | B1 | | $(1 - 0.2237)$ **or** $(1 - 0.5265)$ |
| | $= 1 - 0.369$<br>$= 0.631$ | B1 | 3 | |
| **(ii)** | $p = {}^8C_7 \,(0.631)^7\,(0.369) +$<br>$(0.631)^8$ | M1ft | | ft on their $p$ from (b)(i)<br>Either part attempted |
| | $p = 0.11758 + 0.02513$ | A1ft | | (both parts correct) |
| | $= 0.143$ | A1 | 3 | AWFW 1.142 to 0.143 (CAO) |
| **(c)(i)** | Mean = 8 | B1 | | CAO |
| | Variance = $s^2 = 16.9$<br>$(\text{sample variance} = 15.2)$ | B1 | 2 | (AWRT) |
| **(ii)** | Poisson not a good model for data<br>Mean $\neq$ Variance | B1dep<br>B1 | 2 | |
| | **Total** | | **14** | |

**MS2B (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 3 | $H_0$ **:** no association between age and attitude to school reorganisation | B1 | | |
| | $H_1$ **:** association between age and attitude to school reorganisation | | | |
| | (tables of $O_i$ and $E_i$ below) | M1<br>A1 | | E's attempted correctly (at least 6 E's) |

| Age | Against | |
|---|---|---|
| | $O_i$ | $E_i$ |
| 16 - 17 | 9 | $6\,{}^{17}\!/_{65}$ |
| 18 - 21 | 17 | $15\,{}^{24}\!/_{65}$ |
| 22 - 49 | 115 | $116\,{}^{9}\!/_{13}$ |
| 50 - 65 | 41 | $42\,{}^{9}\!/_{13}$ |
| > 65 | 3 | $3\,{}^{64}\!/_{65}$ |
| Total | 185 | 185 |

| $E_i$ |
|---|
| 6.262 |
| 15.369 |
| 116.692 |
| 42.692 |
| 3.985 |

| Age | Not Against | |
|---|---|---|
| | $O_i$ | $E_i$ |
| 16 - 17 | 2 | $4\,{}^{48}\!/_{65}$ |
| 18 - 21 | 10 | $11\,{}^{41}\!/_{65}$ |
| 22 - 49 | 90 | $88\,{}^{4}\!/_{13}$ |
| 50 - 65 | 34 | $32\,{}^{4}\!/_{13}$ |
| > 65 | 4 | $3\,{}^{1}\!/_{65}$ |
| Total | 140 | 140 |

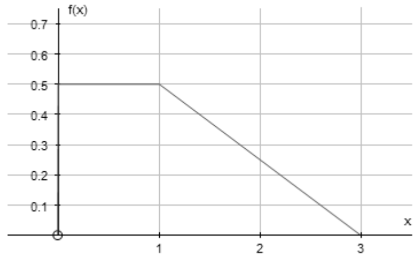
| $E_i$ |
|---|
| 4.738 |
| 11.631 |
| 88.308 |
| 32.308 |
| 3.015 |

| Solution | Marks | Total | Comments |
|---|---|---|---|
| Row totals: 11,27 205, 75.7 (325) | | | |
| Column totals: 185, 140 (325) | B1 | | Totals correct |
| $E_i{}'s < 5$ | | | |
| $\therefore$ combine cells 16 – 17 and 18 –21 **also** 50 – 65 and 'over 65' to give: | M1<br>A1 | | Attempt at combining rows Correctly |

| $O_i$ | $E_i$ | $\alpha = O_i - E_i$ | $\alpha^2/E_i$ |
|---|---|---|---|
| 26 | 21.63 | 4.369 | 0.8825 |
| 115 | 116.69 | –1.692 | 0.0245 |
| 44 | 46.68 | –2.677 | 0.1535 |
| 12 | 16.37 | –4.369 | 1.1662 |
| 90 | 88.31 | 1.692 | 0.0324 |
| 38 | 35.32 | 2.677 | 0.2029 |
| 325 | 325 | | 2.462 |

| Solution | Marks | Total | Comments |
|---|---|---|---|
| (final column of table) | ml | | Final column attempted (dep M1) |
| $X^2 = 2.462$ | A1 | | 2.4 to 2.5 |
| $v = 2$ | B1 | | |
| $\chi^2_{v=2}(0.95) = 5.991$ | B1ft | | On their $v$ |
| Accept $H_0$ | A1ft | | |
| No real evidence at 5% level of significance to suggest any association between age and attitude to school reorganisation. | E1ft | 12 | (context) |
| **Total** | | **12** | |

**MS2B (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4(a)** | Sketch:<br> | B3 | 3 | 1 for straight line $0 \le x \le 1$ from (0, 0.5) to (1, 0.5)<br><br>1 for straight line $1 \le x \le 3$ from (1, 0.5) to (3, 0)<br><br>1 for axes<br>[must have at least (0,0.5) (1,0) and (3,0) labelled] |
| **(b)** | $\mathrm{P}(X \le \eta) = \mathrm{F}(\eta) = 0.5$ | M1 | | |
| | $(\Rightarrow \quad \eta = 1$ (from graph)) | A1 | 2 | AG |
| **(c)** | $\mu = \mathrm{E}(X) = \int_0^1 \left(\frac{x}{2}\right) \mathrm{d}x + \int_1^3 x \left(\frac{3-x}{4}\right) \mathrm{d}x$ | M1 | | Both integrals stated |
| | $= \left[\frac{x^2}{4}\right]_0^1 + \frac{1}{4}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_1^3$ | A1 | | Either |
| | $= \frac{1}{4} + \frac{1}{4}\left[\left(\frac{27}{2} - 9\right) - \left(\frac{3}{2} - \frac{1}{3}\right)\right]$ | ml | | Correct limits used on both integrals +combined dep M1 |
| | $= \frac{1}{4} + \frac{5}{6} \qquad (0.25 + 0.83\dot{3})$ | | | |
| | $= 1\frac{1}{12}$ | A1 | 4 | (CAO) |
| **(d)** | Area of $\triangle$ | | | **Alternative:**<br>For $1 \le x \le 3$ |
| | $= \mathrm{P}\left(X > 2\frac{1}{4}\right) = \frac{1}{2} \times \frac{3}{4} \times \frac{3 - 2\frac{1}{4}}{4}$ | M1ft | | $\mathrm{F}(x) = 1 - \frac{1}{8}(3-x)^2$ M1ft |
| | $= \frac{3}{32} \times \frac{3}{4} = \frac{9}{128}$ | | | $\Downarrow$ |
| | $\therefore \quad \mathrm{P}\left(X < 2\frac{1}{4}\right) = 1 - \frac{9}{128}$ | M1ft | | $\mathrm{F}\left(2\frac{1}{4}\right) = 1 - \frac{1}{8} \times \frac{9}{16}$ M1ft |
| | $= \frac{119}{128} (0.9296875)$ | A1 | 3 | $= \frac{119}{128}$<br><br>CAO |

**MS2B (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4(d)** | **or** | | | **Alternative** |
| | $\int_{2\frac{1}{4}}^{3} \frac{3-x}{4}\,dx \left(=\frac{9}{128}\right)$ | M1 ft | | $f\left(2\frac{1}{4}\right)=\frac{3}{16}=0.1875$ |
| | $= 1\int_{2\frac{1}{4}}^{3} \frac{3-x}{4}\,dx$ | M1 ft | | $P(X<3\mu-\eta)=P\left(X<2\frac{1}{4}\right)$ |
| | $= 1-\frac{1}{4}\left[3x-\frac{x^2}{2}\right]_{2\frac{1}{4}}^{3}$ | | | $=\frac{1}{2}+\boxed{\frac{1}{2}\left(\frac{3}{16}+\frac{1}{2}\right)\times 1\frac{1}{4}}$ M1ft |
| | $= 1-\frac{1}{4}\left[9-\frac{9}{2}-\frac{27}{4}+\frac{81}{32}\right]$ | | | $=\frac{1}{2}+\frac{55}{128}(0.4296875)$ M1ft |
| | $= 1-\frac{1}{4}\times\frac{9}{32}=\frac{119}{128}$ <br> **or** $(1-0.0703125=0.9296875)$ | A1 | | $=\frac{119}{128}$ (0.930) A1 |
| | **Total** | | **12** | |
| **5(a)(i)** | $P(\text{GG or YY or RR})$ <br> $=\frac{2}{10}\times\frac{1}{9}+\frac{3}{10}\times\frac{2}{9}+\frac{4}{10}\times\frac{3}{9}$ | M1 | | |
| | $=\frac{2}{9}$ | A1 | 2 | (AG) |
| **(ii)** | $P(\text{B}\bar{\text{B}} \text{ or } \bar{\text{B}}\text{B})=\frac{1}{10}\times\frac{9}{9}+\frac{9}{10}\times\frac{1}{9}$ | M1 | | $\frac{1}{10}+\frac{9}{10}\times\frac{1}{9}$ |
| | $=\frac{1}{5}$ | A1 | 2 | (AG) |
| **(b)(i)** | \| \| Same \| 1 Blue \| Neither \| <br> \| $x$ \| 135 \| 145 \| $-45$ \| <br> \| $P(X=x)$ \| $\frac{2}{9}$ \| $\frac{1}{5}$ \| $\frac{26}{45}$ \| | B1 <br> B1 | 2 | |
| **(ii)** | $E(X)=135\times\frac{2}{9}+145\times\frac{1}{5}+(-45)\times\frac{26}{45}$ | M1 | | Multiply two rows of their table from (b)(i) |
| | $=29+30-26$ <br> $=33$ pence | A1 | 2 | AG |
| **(c)(i)** | $E(Y)=104-3E(X)$ | M1 | | |
| | $=104-3\times 33$ <br> $=5$ pence | A1 | | |
| | $\therefore$ Joanne would expect to win £5 | A1 | 3 | OE (eg 500p) |

**MS2B (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **5(c)(ii)** | $\text{E}\left(X^2\right) = 9425$ | B1 | | $(4205 + 4050 + 1170)$ |
| | $\text{Var}(X) = 9425 - 33^2 = \mathbf{8336}$ | | | $sd(X) = 91.30$ |
| | $\text{Var}(Y) = 9 \times \text{Var}(X)$ | B1 | | |
| | $= 9 \times 8336$ | | | $9 \times (\text{their Var}(X) > 0)$ |
| | $= 75024$ | M1 | | **or** $3 \times (\text{their sd}(X))$ |
| | $\Rightarrow$ standard deviation ($Y$) = 274 pence | A1 | 4 | 273.9p or £2.74 |
| | **Total** | | **15** | |
| **6(a)(i)** | $\bar{x} = 43.5$ | B1 | | |
| | $s = 2 \quad (s^2 = 4)$ | B1 | | |
| | Assumption: Weights of boxes are normally distributed | B1 | | |
| | $t_{0.975} = 2.365$ | B1 | | |
| | 95% CI for $\mu$: | | | |
| | $43.5 \pm 2.365 \times \frac{2}{\sqrt{8}}$<br>$43.5 \pm 1.6723$ | M1 | | |
| | $\Rightarrow \quad (41.8, 45.2)$ | A1 | 6 | (AWRT) |
| **(ii)** | CI contains mean (45) | B1 dep | | Must be clear use of 45 and |
| | Bishen's belief probably justified | B1 dep | | **not** 43.5 |
| | **or** | | | |
| | [Since 45 within CI] but close to upper limit, there is some evidence that Bishen's Belief is untrue | | | |
| | [but the evidence is not significant at 5 %.] | | | |
| | (75% of sample less than 45grams) | (B1) | 2 | |
| **6(b)(i)** | $\text{H}_0\!: \mu = 45$ | | | |
| | $\text{H}_0\!: \mu < 45$ | B1 | | (both) |
| | Test statistic: $t = \dfrac{43.5 - 45}{2/\sqrt{8}}$ | M1 | | |
| | $= -2.12$ | A1 | | $\text{P}(t_7 < -2.12.) = 0.035791$ |
| | $\nu = 7 \quad \Rightarrow \quad t_{crit} = -1.895$ | B1 | | $< 0.05$ |
| | $\Rightarrow \quad$ Reject $\text{H}_0$ | A1 | | |
| | Evidence at the 5% level of significance . to support Abi's claim that **mean** content < 45 grams | E1 | 6 | |
| **(ii)** | Type I error | B1 | | |
| | have/may have rejected $\text{H}_0$ when $\text{H}_0$ true | B1 | 2 | Clear statement |
| | **or** | | | |
| | No error | (B1) | | |
| | have/may have accepted $\text{H}_0$ when $\text{H}_0$ true | (B1) | | Clear statement |
| | **Total** | | **16** | |
| | **TOTAL** | | **75** | |