General Certificate of Education
Advanced Level Examination
January 2012

Mathematics MS2B

Unit Statistics 2B

Wednesday 25 January 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 Josephine accurately measures the widths of A4 sheets of paper and then rounds the widths to the nearest 0.1 cm. The rounding error, X centimetres, follows a rectangular distribution.

A randomly selected A4 sheet of paper is measured to be 21.1 cm in width.

(a) Write down the limits between which the true width of this A4 sheet of paper lies. *(1 mark)*

(b) Write down the value of $\mathrm{E}(X)$ and determine the **exact** value of the standard deviation of X. *(3 marks)*

(c) Calculate $\mathrm{P}(-0.01 \leqslant X \leqslant 0.03)$. *(1 mark)*

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2 (a) A particular bowling club has a large number of members. Their ages may be modelled by a normal random variable, X, with standard deviation 7.5 years.

On 30 June 2010, Ted, the club secretary, concerned about the ageing membership, selected a random sample of 16 members and calculated their mean age to be 65.0 years.

(i) Carry out a hypothesis test, at the 5% level of significance, to determine whether the mean age of the club's members has changed from its value of 61.4 years on 30 June 2000. *(6 marks)*

(ii) Comment on the likely number of members who were under the age of 25 on 30 June 2010, giving a numerical reason for your answer. *(1 mark)*

(b) During 2011, in an attempt to encourage greater participation in the sport, the club ran a recruitment drive.

After the recruitment drive, the ages of members of the bowling club may be modelled by a normal random variable, Y years, with mean μ and standard deviation σ. The ages, y years, of a random sample of 12 such members are summarised below.

$$\sum y = 702 \quad \text{and} \quad \sum (y - \bar{y})^2 = 88.25$$

(i) Construct a 90% confidence interval for μ, giving the limits to one decimal place. *(5 marks)*

(ii) Use your confidence interval to state, with a reason, whether the recruitment drive lowered the average age of the club's members. *(1 mark)*

3 (a) **Table 1** contains the observed frequencies, a, b, c and d, relating to the two attributes, X and Y, required to perform a χ^2 test.

Table 1

	Y	**Not *Y***	**Total**
X	a	b	m
Not *X*	c	d	n
Total	p	q	N

(i) Write down, in terms of m, n, p, q and N, expressions for the 4 expected frequencies corresponding to a, b, c and d. *(2 marks)*

(ii) Hence prove that the sum of the expected frequencies is N. *(3 marks)*

(b) Andy, a tennis player, wishes to investigate the possible effect of wind conditions on the results of his matches. The results of his matches for the 2011 season are represented in **Table 2**.

Table 2

	Windy	**Not windy**	**Total**
Won	15	18	33
Lost	12	5	17
Total	27	23	50

Conduct a χ^2 test, at the 10% level of significance, to investigate whether there is an association between Andy's results and wind conditions. *(8 marks)*

4 (a) A discrete random variable X has a probability function defined by

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad \text{for } x = 0, 1, 2, 3, 4, \ldots\ldots.$$

(i) State the name of the distribution of X. *(1 mark)*

(ii) Write down, in terms of λ, expressions for $E(3X - 1)$ and $\text{Var}(3X - 1)$. *(2 marks)*

(iii) Write down an expression for $P(X = x + 1)$, and hence show that

$$P(X = x + 1) = \frac{\lambda}{x + 1} P(X = x)$$ *(3 marks)*

(b) The number of cars and the number of coaches passing a certain road junction may be modelled by independent Poisson distributions.

(i) On a winter morning, an average of 500 cars per hour and an average of 10 coaches per hour pass this junction.

Determine the probability that a total of at least 10 such vehicles pass this junction during a particular 1-minute interval on a winter morning. *(3 marks)*

(ii) On a summer morning, an average of 836 cars per hour and an average of 22 coaches per hour pass this junction.

Calculate the probability that a total of at most 3 such vehicles pass this junction during a particular 1-minute interval on a summer morning. Give your answer to two significant figures. *(3 marks)*

5 (a) Joshua plays a game in which he repeatedly tosses an unbiased coin. His game concludes when he obtains either a head or 5 tails in succession.

The random variable N denotes the number of tosses of his coin required to conclude a game.

By completing **Table 3** below, calculate $\mathrm{E}(N)$. *(4 marks)*

(b) Joshua's sister, Ruth, plays a separate game in which she repeatedly tosses a coin that is **biased** in such a way that the probability of a head in a single toss of her coin is $\frac{1}{4}$. Her game also concludes when she obtains either a head or 5 tails in succession.

The random variable M denotes the number of tosses of her coin required to conclude her game.

Complete **Table 4** below. *(3 marks)*

(c) (i) Joshua and Ruth play their games simultaneously. Calculate the probability that Joshua and Ruth will conclude their games in an equal number of tosses of their coins. *(5 marks)*

(ii) Joshua and Ruth play their games simultaneously on 3 occasions. Calculate the probability that, on at least 2 of these occasions, their games will be concluded in an equal number of tosses of their coins. Give your answer to three decimal places. *(4 marks)*

Table 3

n	1	2	3	4	5
$\mathrm{P}(N=n)$			$\frac{1}{8}$		$\frac{1}{16}$

Table 4

m	1	2	3	4	5
$\mathrm{P}(M=m)$	$\frac{1}{4}$	$\frac{3}{16}$			

Turn over ▶

6 The random variable X has probability density function defined by

$$f(x) = \begin{cases} \frac{1}{40}(x+7) & 1 \leqslant x \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of f. *(2 marks)*

(b) Find the **exact** value of $E(X)$. *(3 marks)*

(c) Prove that the distribution function F, for $1 \leqslant x \leqslant 5$, is defined by

$$F(x) = \frac{1}{80}(x+15)(x-1)$$ *(4 marks)*

(d) Hence, or otherwise:

(i) find $P(2.5 \leqslant X \leqslant 4.5)$; *(2 marks)*

(ii) show that the median, m, of X satisfies the equation $m^2 + 14m - 55 = 0$. *(3 marks)*

(e) Calculate the value of the median of X, giving your answer to three decimal places. *(2 marks)*