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General Certificate of Education

Mathematics 6360

MS2B Statistics 2B

Mark Scheme

2009 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Q	Solution	Marks	Total	Comments
1	H_0: No association between choice of subject and gender H_1: Association between choice of subject and gender	B1		
	Bul **Cl** **Fin** **Pol** **Total** **Male** 7 31 25 40 103 **Female** 2 24 22 19 67 **Total** 9 55 47 59 170	B1		Totals
	O_i / E_i 7 / 5.45 2 / 3.55 31 / 33.32 24 / 21.68 25 / 28.48 22 / 18.52 40 / 35.75 19 / 23.25 **170** / **170**	M1A1		E's attempted (correctly)
	One of the E_i's < 5 $\therefore$ combine cells	M1A1		Attempt at combining (correctly)
	O_i / E_i / $\alpha = (O_i - E_i)$ / α^2 / E_i 47 / 41.20 / 5.8 / 0.8165 21 / 26.80 / –5.8 / 1.2552 31 / 33.32 / –2.32 / 0.1615 24 / 21.68 / 2.32 / 0.2483 25 / 28.48 / –3.48 / 0.4252 22 / 18.52 / 3.48 / 0.6539	m1		Final column
	Test statistic: $X^2 = 3.56$ Critical value: = 4.605	A1 B1F		(AWFW 3.55 to 3.57) ft on their ν
	Accept H_0	A1F		
	Insufficient evidence to suggest that the choice of subject is associated with gender.	E1	11	
	Total		**11**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
2(a)	$H_0: \mu = 8.0$ $H_1: \mu \neq 8.0$	B1		
	$\bar{x} = \frac{84}{9} = 9.3\dot{3}$ or $9\frac{1}{3}$	B1		
	$z_{crit} = \pm 1.96$	B1		
	$z = \frac{9.33 - 8.0}{2.5/\sqrt{9}} = 1.60$	M1		$z = \frac{(\text{their } \bar{x}) - 8}{2.5/\sqrt{9}}$
		A1		AWFW 1.59 to 1.60
	$\lvert z \rvert < 1.96$ ∴ accept H_0	A1F		ft on incorrect $\bar{x}$
	Insufficient evidence to suggest that the mean completion time has changed from eight weeks.	E1F	7	
(b)	Neither a Type I nor a Type II error have occurred	B1		dependent
	Have accepted that $H_0: \mu = 8.0$, when $\mu = 8.0$.	B1	2	dependent on 'accept H_0' in (a)
	Total		**9**	
3(a)(i)	$P(X \leq 3) = 0.515$	B1	1	0.5152
(ii)	$P(Y = 5) = \frac{e^{-4.4} \times (4.4)^5}{5!}$	M1		$P(Y \leq 5) - P(Y \leq 4) = 0.7199 - 0.5512$ correct values seen
	$= 0.169$	A1	2	(0.1687)
(b)(i)	$T = \text{Po}(8.0)$	B1		
	X and Y are **independent** (Poisson random variables)	B1	2	
(ii)	$P(6 < T < 12) = P(T \leq 11) - P(T \leq 6)$	M1		
	$= 0.8881 - 0.3134$	A1		
	$= 0.575$	A1	3	(0.5747)
(iii)	$P(T > 14) = 1 - P(T \leq 14)$	M1		
	$= 1 - 0.9827$ $= 0.0173$	A1		CAO
	$p = (0.0173)^2$	M1		$[\text{their } P(T > 14)]^2$
	$= 0.0003$ (1sf)	A1F	4	ft if 0 < both p's <1
(iv)	$P(T \leq k) > 0.99$			$\begin{cases} P(T \leq 15) = 0.9918 \\ P(T \leq 14) = 0.9827 \end{cases}$
	$\Rightarrow \quad k \geq 15$	M1		
	∴ minimum number of devices that Joe should keep in stock = 15	A1	2	
	Total		**14**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\mathrm{P}\left(-\frac{3c}{4}<X<\frac{3c}{4}\right)$			
	$=\frac{\frac{3c}{4}+c}{4c}-\frac{\frac{-3c}{4}+c}{4c}$	M1		or $=\frac{3c}{2}\times\frac{1}{4c}$
	$=\frac{6c}{16c}$			
	$=\frac{3}{8}$ or 0.375	A1	2	CAO
(b)	For $-c\le x\le 3c$			
	$\mathrm{f}(x)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x+c}{4c}\right)$	M1		use of $\mathrm{f}(x)=\mathrm{F}'(x)$
	$=\frac{1}{4c}$			
	For $x>3c$ and $x<-c$			
	$\mathrm{f}(x)=\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{F})=0$	A1	2	for $\frac{1}{4c}$ and 0
(c)(i)	Rectangular distribution:			
	$\mathrm{E}(X)=\frac{1}{2}(-c+3c)=c$	B1	1	
(ii)	$\mathrm{Var}(X)=\frac{1}{12}(3c--c)^2=\frac{4c^2}{3}$	B1	1	Allow $\frac{16c^2}{12}$
	Total		**6**	
5(a)(i)	$\bar{x}=\frac{1}{2}(70.65+80.35)=75.5$	B1	1	AG
(ii)	Width of confidence interval $=80.35-70.65$			
	$=9.7$	B1	1	
(iii)	$t_{crit}=2.602$; $\nu=15$	B1		
	$\mathrm{w}=2t\times\frac{s}{\sqrt{n}}\ \Rightarrow\ \frac{s}{\sqrt{n}}=\frac{9.7}{2\times 2.602}$	M1		
	Estimate of s.e $=\frac{s}{\sqrt{n}}=1.86$	A1	3	(1.864)
(iv)	Unbiased estimate of $\sigma^2=1.86^2\times 16$	M1		
	$=55.6$ (3sf)	A1	2	AG (55.589)

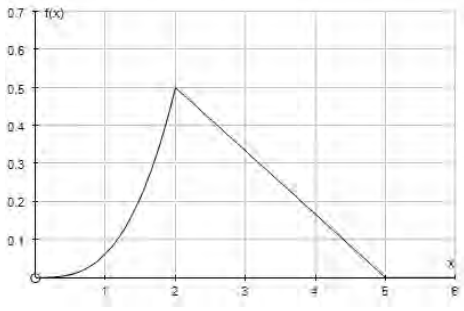
MS2B (cont)

Q	Solution	Marks	Total	Comments
5(b)	95% CI: $75.5 \pm 2.131 \times \frac{s}{\sqrt{n}}$ $= 75.5 \pm 3.972$ $= (71.5, 79.5)$	M1 A1	 2	$(71.5$ to 71.54, 79.4 to $79.5)$ CAO
(c)(i)	$(73.0, 78.0)$	B1	1	
(ii)	$w = 2t \times \frac{s}{\sqrt{n}} \Rightarrow t = \frac{5}{2 \times 1.864} = 1.341$ $\Rightarrow$ for $\nu = 15$ $P(X \leq 1.341) = 0.90$ $\Rightarrow$ $P(X \geq 1.341) = 0.10$ and $P(X \leq -1.341) = 0.10$ $\therefore$ $P(\lvert X \rvert \leq 1.341) = 0.80$ Percentage confidence interval = 80%	M1 M1 A1	 3	(AWFW 1.341 to 1.344)
	Total		**13**	
6(a)	<table><tr><td>***r***</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>**P(*R*=*r*)**</td><td>$2/3$</td><td>$2/9$</td><td>$2/27$</td><td>k</td></tr></table> $k + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} = 1 \Rightarrow k = \frac{1}{27}$	 M1 A1	 2	 AG
(b)	$P(R \geq 3) = \frac{2}{27} + \frac{1}{27} = \frac{1}{9}$	B1	1	Allow $\frac{3}{27}$ or 0.111
(c)(i)	$C = 27R + 5$ $E(R) = \left(1 \times \frac{2}{3}\right) + \left(2 \times \frac{2}{9}\right) + \left(3 \times \frac{2}{27}\right) + \left(4 \times \frac{1}{27}\right)$ $= 1\frac{13}{27}$ $\therefore$ $E(C) = 27 \times 1\frac{13}{27} + 5$ $= 45$	 B1 M1 A1F	 3	 (1.48) or $\frac{40}{27}$

MS2B (cont)

Q	Solution	Marks	Total	Comments
6(c)(ii)	$\mathrm{E}(R^2)=\left(1\times\frac{2}{3}\right)+\left(4\times\frac{2}{9}\right)+\left(9\times\frac{2}{27}\right)+\left(16\times\frac{1}{27}\right)$			
	$=2\frac{22}{27}$ or $\frac{76}{27}$	B1		(2.81)
	$\mathrm{Var}(R)=2\frac{22}{27}-\left(1\frac{13}{27}\right)^2$	M1		
	$=\frac{452}{729}$			(0.62)
	$\therefore$ St. dev$^{\mathrm{n}}(C)=27\times\sqrt{\frac{452}{729}}$	M1		$27\times\sqrt{\mathrm{Var}(R)}$ $[\mathrm{Var}(R)>0]$
	$=21.3$	A1	4	CAO (21.26)
				SC: $\mathrm{Var}(C)=452$ (CAO) (B1M1B1A0)
	Total		**10**	
	Alternative (c)			
	C: 32, 59, 86, 113 p: $\frac{2}{3}$, $\frac{2}{9}$, $\frac{2}{27}$, $\frac{1}{27}$			
	C: 32, 59, 86, 113 n: 18, 6, 2, 1			
	$\bar{x}=45$ and $\sigma=21.260$ from calculator			$\left(\bar{x}=\frac{\sum Cn}{27}\right)$

MS2B (cont)

Q	Solution	Marks	Total	Comments
7(a)		B1		for concave curve from (0 , 0) to (2 , 0.5)
		B1		for straight line from (2 , 0.5) to (5 , 0)
		B1	3	for axes [2 , 5 ; 0.5] seen
(b)	$\mathrm{P}(X \geq 2) = \frac{1}{2} \times 3 \times 0.5 = 0.75$			**Alternatives**:
	$\Rightarrow \mathrm{F}(2) = 0.25$			$\int \frac{1}{6}(5-x)\mathrm{d}x = \frac{1}{6} \times \frac{(5-x)^2 \times (-1)}{2}$
	$2 \leq x \leq 5$			$= -\frac{1}{12}(5-x)^2$
	$\mathrm{F}(x) = \mathrm{F}(2) + \int_2^x \frac{1}{6}(5-x)\mathrm{d}x$			Or
	$= 0.25 + \frac{1}{6}\left[5x - \frac{x^2}{2}\right]_2^x$	M1		$\mathrm{F}(x) = 1 - \text{Area } \triangle \text{ (base } x,5)$
	$= 0.25 + \frac{1}{6}\left(5x - \frac{x^2}{2}\right) - \frac{1}{6}(10-2)$	A1		$= 1 - \frac{1}{2}(5-x)\frac{1}{6}(5-x)$
	$= 0.25 - \frac{8}{6} + \frac{5x}{6} - \frac{x^2}{12}$	M1		$= 1 - \frac{1}{12}(5-x)^2$
	$= -\frac{1}{12}\left(x^2 - 10x + 13\right)$			
	$= 1 - \frac{1}{12}(5-x)^2$	A1	4	
(c)	$\mathrm{P}(X \leq 4) = \mathrm{F}(4)$			**Alternative:**
	$= 1 - \frac{1}{12}(5-4)^2 = \frac{11}{12}$ (0.916 to 0.917)	B1		$\mathrm{P}(X \geq 3 \mid X \leq 4)$
	$\mathrm{F}(3) = 1 - \frac{1}{12}(2)^2 = \frac{2}{3}$ (0.667)	B1		$= \frac{\mathrm{F}(4) - \mathrm{F}(3)}{\mathrm{F}(4)}$ (M1)
	$\mathrm{P}(X \geq 3 \text{ and } X \leq 4) = \mathrm{F}(4) - \mathrm{F}(3)$			$= 1 - \frac{\mathrm{F}(3)}{\mathrm{F}(4)}$
	$= \frac{11}{12} - \frac{2}{3} = \frac{1}{4}$ (0.25)	B1		$= 1 - \frac{2/3}{11/12}$
	$\mathrm{P}(X \geq 3 \mid X \leq 4) = \frac{\mathrm{F}(4) - \mathrm{F}(3)}{\mathrm{F}(4)}$	M1		$= 1 - \frac{8}{11}$ (B1) $\left(0.72\dot{7}\dot{2}\right)$
	$= \frac{1/4}{11/12} = \frac{3}{11}$	A1	5	$= \frac{3}{11}$ (AWFW 0.272 to 0.273)
	Total		**12**	
	TOTAL		**75**	