AQA Maths Statistics 2

Mark Scheme Pack

2006-2015

Version 1.0: 0106

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MS2B Statistics 2B

Mark Scheme

2006 examination – January series

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M	mark is for method
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B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
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No Method Shown

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Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Question	Solution	Marks	Total	Comments
1(a)(i)	$\mathrm{P}(X=2)=\dfrac{\mathrm{e}^{-1.5}\times(1.5)^2}{2!}=0.251$	M1A1	2	
(ii)	$p=(0.251)^3=0.0158$	M1A1√	2	on their p from (i)
(b)(i)	$Y\sim \mathrm{P}_o(9.0)$	B1	1	
(ii)	$\mathrm{P}(Y\geq 12)=1-\mathrm{P}(Y\leq 11)$ $=1-0.8030$ $=0.197$	M1 A1	2	
(c)	attacks patients: randomly (p constant) independently	B1 B1	2	mean of 1.5 $\Rightarrow p$ small (B1) (unless very few patients)
	Total		**9**	
2(a)	H_o : Choice independent of gender	B1		gender not associated with choice
	Squash Badminton Archery Hockey Male 5/3.5 16/14 30/24.5 19/28 Female 4/5.5 20/22 33/38.5 53/44	M1		
	Combine Squash and Badminton	M1		$\mathrm{E}_i<5$ (Similar categories)
	S & B Archery Hockey Male 21/17.5 30/24.5 19/28 Female 24/27.5 33/38.5 53/44	M1 M1		
	χ^2 values S & B Archery Hockey Male 0.7000 1.2347 2.8928 Female 0.4455 0.7857 1.8409	M1		
	$\chi^2_{\text{calc}}=7.90$	A1		(7.8 to 7.9)
	$\nu=2$	B1		
	$\chi^2_{5\%}(2)=5.991$	B1ft		(on their ν)
	Reject H_o Sufficient evidence, at the 1% level of significance, to support an association between the choice of sport and gender	A1ft	10	reject H_o and H_o stated **or** statement in context
(b)	**More females and fewer males** chose to participate in hockey **than expected**	B1 B1	2	
	Total		**12**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
3(a)	$\bar{x} = 8.0$	B1		
	$S = 2.121$	B1		
	$\nu = 8$	B1		
	$t = 1.860$	B1√		(on their ν)
	90% confidence interval for μ			
	$= 8 \pm 1.860\left(\frac{2.121}{3}\right)$	M1		
	$= 8 \pm 1.315$	A1ft		
	$= (6.68, 9.32)$	A1	7	(6.68 to 6.69, 9.31 to 9.32)
(b)	The Headteacher's claim seems to be slightly optimistic	E1ft		Headteacher's claim isn't supported by the evidence **and**
	because value of 5 outside the confidence interval	E1ft	2	It appears that the mean time to see a mathematics teacher is greater than 5 minutes
	Total		**9**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
4(a)(i)	$\text{Area} = k(b-a) = 1$			
	$\Rightarrow \quad k = \dfrac{1}{b-a}$	E1	1	AG
(ii)	$\mathrm{E}(X) = \int_a^b kx \, \mathrm{d}x$	M1		
	$= \left(\dfrac{kx^2}{2} \right) \Big\|_a^b$	A1		
	$= \dfrac{1}{2} k \left(b^2 - a^2 \right)$			
	$= \dfrac{1}{2} \times \dfrac{1}{(b-a)} \times (b-a)(a+b)$	M1A1		(factors shown)
	$= \dfrac{1}{2}(a+b)$		4	AG
(b)(i)	$\mu = 1$	B1	1	
(ii)	$\sigma^2 = \mathrm{Var}(X) = \dfrac{1}{12}(b-a)^2$			
	$= \dfrac{1}{12} \times 6^2$	M1		
	$= 3$			
	$\therefore \sigma = \sqrt{3}$	A1	2	1.7321
(iii)	$\mathrm{P}\left(X < \dfrac{2-\mu}{\sigma} \right) = \mathrm{P}\left(X < \dfrac{1}{\sqrt{3}} \right)$	M1√		(on their μ and σ)
	$= \dfrac{1}{6} \times 2.577$	M1√		
	$= 0.430$	A1	3	cao
	Total		**11**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
5(a)	$\mathrm{E}(X) = \sum_{\text{all } x} x\ \mathrm{P}(X = x)$			
	$= 50$	B1		(cao)
	$\mathrm{E}(X^2) = \sum_{\text{all } x} x^2 \mathrm{P}(X = x)$			
	$= 2602.6(0)$	M1		
	$\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$			
	$= 2602.6 - 50^2$	M1		
	$= 102.6(0)$			
	$\Rightarrow$ standard deviation (X) = 10.13	A1	4	(to nearest 1p)
(b)	$\mathrm{E}(Y) = \mu = \mathrm{E}(10X + 250)$			
	$= 10 \times \mathrm{E}(X) + 250$			
	$= 750$	B1√		(on their $\mathrm{E}(X)$)
	$\mathrm{s.d}(Y) = 10 \times 10.1$			
	$= 101$	B1√	2	(on their $\mathrm{sd}(X)$)
	Total		**6**	
6(a)	$\mathrm{H}_o : \mu = 65$			
	$\mathrm{H}_1 : \mu < 65$	B1		1-tailed test
	$\bar{X} \sim \mathrm{N}\left(65, \frac{81}{35}\right)$			
	$z_{crit} = -1.6449$	B1		
	$z = \frac{61.5 - 65}{9/\sqrt{35}} = -2.30$	M1A1		for σ^2/n used
	Reject H_o at 5% level of significance	A1√		(on their z-values)
	Evidence to suggest students may be under-achieving	E1	6	
(b)	Reject H_o when H_o true	E1		
	$\Downarrow$			
	Conclude that students are under-achieving when in fact they are not	E1	2	
	Total		**8**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
7(a)	$\mathrm{E}(T) = \int_0^1 t\ \mathrm{f}(t)\ \mathrm{d}t$			
	$= \int_0^1 4t^2\left(1-t^2\right)\mathrm{d}t$	M1		
	$= \left(\frac{4t^3}{3} - \frac{4t^5}{5}\right)\Big\|_0^1$	A1		
	$= \frac{4}{3} - \frac{4}{5}$	A1		
	$= \frac{8}{15}$		3	AG
(b)(i)	$\mathrm{F}(t) = \mathrm{P}(T \le t) = \int_0^t \mathrm{f}(t)\,\mathrm{d}t$			
	$= \int_0^t 4t\left(1-t^2\right)\mathrm{d}t$	M1		
	$= \left(2t^2 - t^4\right)\Big\|_0^t$			
	$= 2t^2 - t^4$	A1	2	
(ii)	$\mathrm{P}(\mu < T < \mathrm{m}) = \mathrm{F}(\mathrm{m}) - \mathrm{F}(\mu)$	M1		
	$\Downarrow$			
	$\mathrm{F}(m) = 0.5$	B1		
	$\mathrm{F}(\mu) = \mathrm{F}\left(\frac{8}{15}\right) = 0.4880$	B1		
	$\therefore \quad \mathrm{P}(\mu < T < \mathrm{m}) = 0.5 - 0.4880$	M1√		0.5 – their F(μ)
	$= 0.012$	A1	5	
	Total		**10**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
8	$H_o : \mu = 1000$ $H_1 : \mu \neq 1000$	B1		2-tailed test
	$\bar{x} = \frac{12036}{12} = 1003$	B1		
	$S = 5.444$	B1		($S^2 = 29.6$)
	$\nu = 12 - 1 = 11$	B1		
	$t = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{1003 - 1000}{5.444/\sqrt{12}} = 1.91$	M1 A1ft A1		
	$t_{crit} = \pm 2.201$	B1√		(on their ν)
	Accept H_o	A1√		(on their t-values)
	Insufficient evidence to indicate a change in the mean content of sherry in a bottle	E1√	10	
	Total		**10**	
	TOTAL		**75**	

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1(a)	For a 1-year period The number of A grades ~ $\mathrm{Po}(3)$ For a 5-year period Number of A grades ~ $\mathrm{Po}(15)$	B1		
	$\mathrm{P}(\text{Total A-grades} > 18)$ $= 1 - (\text{Total} \leq 18)$ $= 1 - 0.8195$ $= 0.1805$	M1		
	$= 0.181$	A1	3	AWFW 0.180 to 0.181
(b)(i)	$X + Y \sim \mathrm{Po}(10)$	B1		
	$\mathrm{P}(X + Y \leq 14) = 0.917$	M1A1	3	AWFW 0.916 to 0.917 incl
(ii)	X and Y are independent variables.	E1	1	
	Total		**7**	
2(a)	$\bar{x} = \frac{254}{5} = 50.8$	B1		
	$s = 4.55$	B1		
	$\nu = 5 - 1 = 4$	B1		
	$t_{\text{crit}} = 2.776$	B1		
	95% confidence interval $= 50.8 \pm 2.776 \times \frac{4.55}{\sqrt{5}}$	M1√		ft their values
	$= 50.8 \pm 5.648$ $= (45.2, 56.4)$	A1	6	
(b)	0.05	B1	1	
	Total		**7**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\mathrm{E}(R) = \sum_{\text{all } r} r\ \mathrm{P}(R = r)$			
	$= \left(1 \times \frac{7}{16}\right) + \left(2 \times \frac{5}{16}\right) + \left(3 \times \frac{3}{16}\right) + \left(4 \times \frac{1}{16}\right)$			
	$= \frac{30}{16}$			
	$= 1\frac{7}{8}$	B1		(1.875)
	$\mathrm{E}(R^2) = \sum_{\text{all } r} r^2\ \mathrm{P}(R = r)$			
	$= \frac{70}{16}$ or $4\frac{3}{8}$	B1		(4.375)
	$\mathrm{Var}(R) = 4\frac{3}{8} - \left(1\frac{7}{8}\right)^2$	M1		
	$= \frac{220}{256}$ or $\frac{55}{64}$	A1	4	(0.859375)
(b)(i)	$32 \times \frac{1}{4} = 8$	B1	1	
(ii)	$= \left(32 \times \frac{7}{16} \times \frac{1}{5}\right) + \left(32 \times \frac{5}{16} \times \frac{1}{2}\right) + 8 \times \frac{9}{10}$	M1		
	$= 2.8 + 5 + 7.2$			A0 if these numbers rounded before adding
	$= 15$	A1	2	
	Total		**7**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)				
		B1 B1	2	for A values for B values
(ii)				
	H_0 : no association between area and age profile H_1 : association between area and age profile	B1		At least H_0
		M1		Attempt at Row & Column totals
		M1		Attempt at E_i
		M1		Attempt at $\frac{(O_i - E_i)^2}{E_i}$
		M1		Attempt at χ^2
		A1		AWFW 12.5 to 12.6 provided correct method used
	$\nu = (3-1)(2-1) = 2$	B1		
	$\chi^2_{1\%}(2) = 9.210 < 12.554$	B1√		ft on their ν and χ^2
	Reject H_0 The evidence suggests that the area within which a school is situated seems to have an effect on the age-profile of the staff employed.	E1√	9	ft on χ^2 and calculated value depends on H_0 correct, if stated
(b)	There seems to be fewer staff employed in 22 - 34 age group than expected in school A and more than expected in school B	E1 E1	2	
	Total		**13**	

	A	B	Total
22-34	21	32	53
35-39	72	36	108
40-59	27	12	39
Total	120	80	200

O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$
24	31.8	3.6679
72	64.8	0.8000
24	23.4	0.5538
32	21.2	5.5019
36	43.2	1.2000
12	15.6	0.8308
$\sum O_i = 200$	$\sum E_i = 200$	$\chi^2 = 12.554$

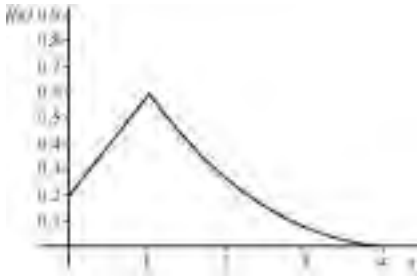
MS2B (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$\mathrm{E}(X)=\frac{1}{2}b$	B1	1	
(ii)	$\mathrm{E}(X^2)=\int_0^b \frac{1}{b}x^2\mathrm{d}x$	M1		
	$=\frac{1}{b}\left[\frac{x^3}{3}\right]_0^b$	A1		For correct integration
	$=\frac{1}{b}\left(\frac{b^3}{3}\right)$			
	$=\frac{1}{3}b^2$	A1		OE
	$\mathrm{Var}(X)=\frac{1}{3}b^2-\left(\frac{b}{2}\right)^2$	m1		Depending on using integration to get $\mathrm{E}(X^2)$
	$=\frac{1}{3}b^2-\frac{1}{4}b^2$			
	$=\frac{1}{12}b^2$	A1	5	AG
(b)	$\mathrm{P}(\lvert T\rvert>0.02)=1-\mathrm{P}(-0.02<T<0.02)$	M1		
	$=1-0.04\times 5$	M1		
	$=0.8$	A1	3	
	Total		**9**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\bar{x} = \frac{471}{5} = 94.2$	B1		
	$s = 6.058$	B1		Or $s^2 = 36.7$
	$\nu = 4$ 1-tailed test	B1		
	$t_{\text{crit}} = -2.132$	B1		Or on diagram
	$H_0 : \mu = 100$ $H_1 : \mu < 100$	B1		
	$t = \frac{94.2 - 100}{6.058/\sqrt{5}} = -2.14$	M1A1		$\frac{\text{their } \bar{x} - 100}{(\text{their } s)/\sqrt{5}}$
	Reject H_0 at 5% level of significance	A1√		On their t and critical value
	Evidence at the 5% level of significance to support the members' belief that the batteries last less than 100 hours.	E1√	9	
(b)	$\bar{x} = \frac{8080}{80} = 101$			
	$s^2 = \frac{6399}{79} = 81$ (or $\frac{6399}{80} = 79.9875$) $s = 9$ (or $s = 8.944$)	B1		For s (or s^2) and $\bar{x}$
	$H_0 : \mu = 100$ $H_1 : \mu \neq 100$	B1		
	$\bar{X} \sim N\left(100, \frac{81 \text{ (or } 79.9875)}{80}\right)$ under H_0	B1		Or 100, $\frac{9}{\sqrt{80}}$ used
	$z = \frac{101 - 100}{9/\sqrt{80}} = 0.99$	M1 A1		Allow use of t method AWFW 0.99 to 1.00 (allow 1)
	2-tailed test $z_{\text{crit}} = \pm 1.96$	B1		Or z = 1.96
	Accept H_0 at 5% level of significance.	A1√		On their z and critical value Or t
	Sufficient evidence at the 5% level of significance to support the manufacturer's belief.	E1√	8	
	Total		**17**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
7(a)		B2	2	B1 for line segment (0,0.2) to (1,0.6) B1 for correctly shaped curve (1,0.6) to (4,0)
(b)(i)	for $0 \le x \le 1$			
	$\mathrm{F}(x) = \int_0^x \frac{1}{5}(2x+1)\,\mathrm{d}x$	M1		Ignore limits
	$= \left[\frac{1}{5}\left(x^2+x\right)\right]_0^x$	A1		Ignore limits
	$= \frac{1}{5}x(x+1)$	A1	3	
(ii)	$\mathrm{P}(X \le 1) = \mathrm{F}(1)$ $= \frac{2}{5}$	B1	1	
(iii)	$\mathrm{P}(X \ge x) = \frac{17}{20} \quad \Rightarrow \quad \mathrm{F}(x) = \frac{3}{20}$	M1		
	$\frac{1}{5}x(x+1) = \frac{3}{20}$	m1		
	$x(x+1) = \frac{3}{4}$			
	$x^2 + x - \frac{3}{4} = 0$	A1		
	$\left(x - \frac{1}{2}\right)\left(x + \frac{3}{2}\right) = 0$	m1		Any valid method attempted
	$x = \frac{1}{2}$	A1	5	CAO
(iv)	Since $\mathrm{F}(1) = 0.4$, q lies in $0 \le r \le 1$			
	$\mathrm{F}(q) = \frac{1}{5}\left(q^2+q\right) = 0.25$	M1		
	$\Rightarrow \quad q^2 + q = 1.25$			
	$q^2 + q - 1.25 = 0$	A1		
	$\Rightarrow \quad q = \frac{-1 \pm \sqrt{1 - 4 \times (-1.25)}}{2}$	m1		
	$q = \frac{1}{2}\left(\sqrt{6} - 1\right) \quad (q > 0)$	A1	4	AWFW (0.724 to 0.725)
	Total		**15**	
	TOTAL		**75**	

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Otherwise we require evidence of a correct method for any marks to be awarded.

Jan 07

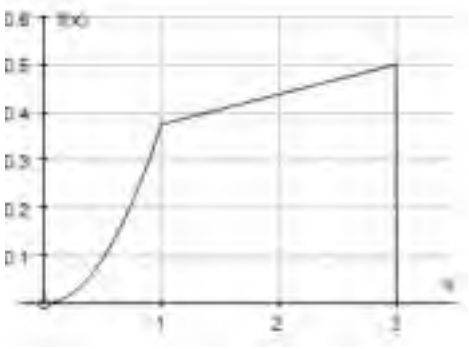
MS2B

Q	Solution	Marks	Total	Comments
1	$\bar{x} = 39.5 \quad s = 4.84 \quad (s^2 = 23.4)$	B1B1		$\sigma = 4.53 \quad (\sigma^2 = 20.5)$
	$t_{\text{crit}} = 2.365$	B1		
	95% CI for μ			
	$= \bar{x} \pm t_{\text{crit}} \times \dfrac{s}{\sqrt{n}}$			
	$= 39.5 \pm 2.365 \times \dfrac{4.84}{\sqrt{8}}$	M1		$39.5 \pm 2.365 \times \dfrac{4.53}{\sqrt{7}}$
	$= 39.5 \pm 4.05$			
	$= (35.5, 43.5)$	A1√	5	
	Total		**5**	
2(a)(i)	$\text{P}(A = 4) = \dfrac{\text{e}^{-3.5} \times (3.5)^4}{4!} = 0.189$	M1A1	2	
(ii)	$\text{P}(B \leq 6) = 0.762$	B1	1	
(iii)	$T = A + B \sim \text{Po}(8.5)$			
	$\text{P}(T \text{ fewer than } 10) = \text{P}(T < 10)$	M1		Use of Po (8.5)
	$= \text{P}(T \leq 9)$	M1		$T \leq 9$ attempted
	$= 0.653$	A1	3	CAO
(b)	$X \sim \text{B}(5, 0.653)$	B1		$X \sim \text{B}(5, \text{their } p)$
	$\text{P}(X \geq 4) = \binom{5}{4}(0.653)^4(0.347)$			
	$+ (0.653)^5$	M1		
	$= 0.31547 + 0.11873$			
	$= 0.434$	A1√	3	On their p from (a)(iii)
(c)(i)	$\bar{x} = 9.2$	B1		
	$s^2 = 9.29$	B1	2	$\sigma^2 = 8.36$
(ii)	Mean and variance have similar values	B1√		
	which suggests that Poisson distribution may be appropriate	B1√	2	
	Total		**13**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
3	$\overline{x} = 83.5$	B1		
	$s^2 = \frac{1}{99}(15321) = 154.76$			$(154 < s^2 \leq 155)$
	$s = 12.44$	B1		$(12.4 \leq s \leq 12.45)$
	$H_0 : \mu = 85.9$			
	$H_1 : \mu \neq 85.9$	B1		
	Under H_0, $\overline{X} \sim N\left(85.9, \frac{(12.44^2)}{100}\right)$			
	$z_{crit} = \pm 1.96$	B1		$z = 1.96$ + 2 tail test used
	$z = \frac{83.5 - 85.9}{12.44/10} = -1.929$	M1		$\frac{(\text{their } \overline{x}) - 85.9}{(\text{their } s)/10}$
		A1		AWFW –1.94 to 1.92
	accept H_0, reject the claim	A1√		On their z
	Insufficient evidence to suggest that the mean has changed from 85.9 at the 5% level of significance.	E1√	8	
	Total		**8**	
4(a)	$\sum p = 1$			
	$k = 1 - (0.01 + 0.05 + 0.14 + 0.30 + 0.12)$			
	$k = 0.38$	B1	1	
(b)(i)	$E(X) = \sum_{\text{all } x} x\ P(X = x) = 4.35$	B1	1	$\frac{87}{20}$
(ii)	$\text{Var}(X) = \sum_{\text{all } x} x^2\ P(X = x) - \mu^2$	M1		$E(X^2)$ attempted
	$= 20.09 - 18.9225$	M1		$\sum x^2 P(X = x) - \mu^2$
	$= 1.1675$	A1	3	$\frac{467}{400}$ (AWFW 1.16 – 1.17)
(c)(i)	$E(Y) = 5E(X) + 2$			
	$= 5 \times 4.35 + 2$	M1	1	Their (b)(i) × 5 + 2
	$= 23.75$			
(ii)	$\text{Var}(Y) = 25\text{Var}(X)$	M1		Their (b)(ii) × 25
	$= 29.1875$			
	Standard deviation $= 5.40$	m1		$\sqrt{}$
		A1	3	$(5.40 - 5.41)$
	Total		**9**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
5(a)	$H_0 : \mu = 30$			
	$H_1 : \mu > 30$	B1		
	$\bar{x} = 33.5$ and $s = 4.25 \left(s^2 = 18.06\right)$	B1B1		σ=4.03 $\left(\sigma^2 = 16.25\right)$
	Under H_0 $\bar{X} \sim N\left(30, \frac{\left(4.25^2\right)}{10}\right)$			$\downarrow$
	$t = \frac{33.5 - 30}{4.25/\sqrt{10}} = 2.60$	M1A1		$\frac{33.5-30}{4.03/\sqrt{9}}$ $(2.6 - 2.61)$
	$t_{crit} = 2.821$	B1		
	do not reject H_0 Insufficient evidence at the 1% level of significance that Jasmine's teacher is underestimating the time that it takes to complete the homework assignments.	E1√	7	
(b)	Times are Normally distributed	B1	1	
	Total		**8**	
6(a)		B1 B1 B1	3	for curve for line for axes
(b)	$P(T \geq 1) = \frac{1}{2} \times \frac{7}{8} \times 2 = \frac{7}{8}$	M1A1	2	OE

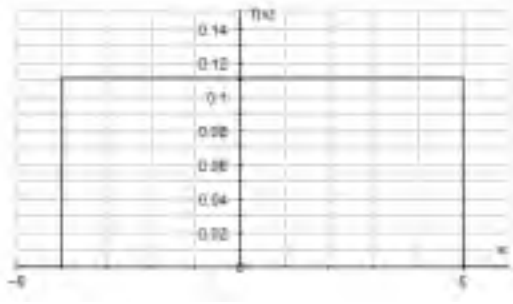
MS2B (cont)

Q	Solution	Marks	Total	Comments
6(c)(i)	For $1 \le t \le 3$			
	$\int_1^t \frac{1}{16}(t+5)\,\mathrm{d}t = \left[\frac{1}{32}t^2 + \frac{5}{16}t\right]_1^t$	M1A1		
	$\mathrm{F}(1) = \frac{1}{8}$	B1		
	$\mathrm{F}(t) = \frac{1}{8} + \frac{1}{32}t^2 + \frac{5}{16}t - \frac{11}{32}$	M1		Use of: $\mathrm{F}(t) = \mathrm{F}(1) + \int_1^t \frac{1}{16}(t+5)\,\mathrm{d}t$
	$\mathrm{F}(t) = \frac{1}{32}\left(t^2 + 10t - 7\right)$	A1	5	**AG**
	Alternative:			
	$\int \frac{1}{16}(t+5)\,\mathrm{d}t$			
	$= \frac{1}{16}\left(\frac{1}{2}t^2 + 5t + c\right)$	(M1) (A1)		
	$\mathrm{F}(1) = \frac{1}{8}$	(B1)		
	$\Rightarrow\ c = -3.5$	(M1)		
	$\mathrm{F}(t) = \frac{1}{32}\left(t^2 + 10t - 7\right)$	(A1)		
(ii)	$\frac{1}{32}\left(m^2 + 10m - 7\right) = 0.5$	M1		
	$m^2 + 10m - 23 = 0$	A1		
	$m = \frac{-10 \pm \sqrt{192}}{2} = -5 \pm \sqrt{48}$ $= -5 \pm 4\sqrt{3}$	m1		(or any valid method)
	$(m > 0)$			
	$m = 4\sqrt{3} - 5 = 1.93$	A1	4	(1.9282)
	Total		**14**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
7(a)	H_0 : No association between the performances at KS3 and GCE	B1		
	O_i | E_i | $O_i - E_i$ | X^2 60 | 63.55 | −3.55 | 0.1983 55 | 44.64 | 10.36 | 2.4043 40 | 46.81 | −6.81 | 0.9907 55 | 51.25 | 3.75 | 0.2744 32 | 36.00 | −4.00 | 0.4444 38 | 37.75 | 0.25 | 0.0017 47 | 46.33 | 0.67 | 0.0097 31 | 32.54 | −1.54 | 0.0733 35 | 34.13 | 0.87 | 0.0222 43 | 43.87 | −0.87 | 0.0173 26 | 30.82 | −4.82 | 0.7527 38 | 32.31 | 5.69 | 1.0005	M1 M1 M1 M1		E_i O_i - E_i $(O_i - E_i)^2 / E_i$ $\sum$
	$X^2 = 6.1897$	A1		AWFW 6.05 – 6.35
	$\nu = 3 \times 2 = 6 \Rightarrow \chi^2_{90\%} = 10.645$	B1B1√		on their ν
	Do not reject H_0 No evidence to suggest an association between KS3 results and GCE grades at 10% level of significance.	E1√	9	
(b)	More of the students achieving level 7 at KS3 gain grade A's at GCE than expected.	E1	1	
	Total		**10**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
8(a)	$f(x)=\begin{cases}\frac{1}{9} & -4\le x\le 5\\ 0 & \text{otherwise}\end{cases}$	M1 A1	2	
(b)		B1 B1	2	horizontal line from –4 to 5 for drawn at $\frac{1}{9}$
(c)	$P(X>2)=\frac{1}{9}\times 3$ $=\frac{1}{3}$	M1 A1	2	$F(5)-F(2)$ $=1-\frac{2}{3}$ $=\frac{1}{3}$
(d)	Mean $=\frac{1}{2}$ Variance $=\frac{1}{12}\times 81$ $=6.75$	B1 B1	2	
	Total		**8**	
	TOTAL		**75**	

Version 1.0 0607

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MS2B Statistics 2

Mark Scheme

2007 examination - June series

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Registered address: AQA, Devas Street, Manchester M15 6EX
Dr Michael Cresswell Director General

Key to mark scheme and abbreviations used in marking

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m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	C	candidate
PI	possibly implied	Sf	significant figure(s)
SCA	substantially correct approach	Dp	decimal place(s)

No Method Shown

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June 07

MS2B

Q	Solution	Marks	Total	Comments
1	H_0: condition independent of treatment H_1: condition dependent upon treatment	B1		
	Totals: 66, 84, 75, 75	B1		
	O | E | $\lvert O-E\rvert-0.5$ | $\frac{(\lvert O-E\rvert-0.5)^2}{E}$ 20 | 33 | 12.5 | 4.7348 55 | 42 | | 3.7202 46 | 33 | | 4.7348 29 | 42 | | 3.7202	M1A1 M1 M1		for E_i attempted, correctly for use of Yates' correction final column
	$\chi^2 = 16.91$	A1		allow 16.9 If no Yates' correction: possible M1A1M0M1A0 If 0.5 incorrectly used: possible M1A1M1M1A0
	$\chi^2_{5\%}(1) = 3.841 < 16.91$ Reject H_0 Evidence to suggest that the condition of the patients may be dependent upon the treatment that they received	B1√ A1√ E1√	10	for χ^2 on their ν iff H_0 stated correctly dependent on third M1
	Total		**10**	
2(a)(i)	$P(X=3) = \frac{e^{-3.5}\times(3.5)^3}{3!} = 0.216$	M1 A1	2	
(ii)	$P(Y \geq 5) = 1 - P(Y \leq 4)$ $= 1 - 0.2851$ $= 0.715$	M1 A1	2	used
(b)(i)	$T \sim Po(9.5)$	B1	1	
(ii)	$P(7 \leq T \leq 10) = P(T \leq 10) - P(T \leq 6)$ $= 0.6453 - 0.1649$ $= 0.480$	M1 A1 A1	3	Accept 0.48
(iii)	$p = (0.4804)^3 = 0.111$	M1 A1√	2	
	Total		**10**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
3	$H_0 : \mu = 36$			
	$H_1 : \mu < 36$	B1		
	$\bar{x} = \frac{1730}{50} = 34.6$	B1		
	$s^2 = \frac{784}{49} = 16$	B1		
	Test statistic: $z = \frac{34.6 - 36}{4/\sqrt{50}} = -2.47$	M1 A1		(–2.48 to –2.47)
	$z_{crit} = -2.3263$	B1		
	Reject H_0	A1√		
	Sufficient evidence at the 1% level of significance to support David's claim	E1√	8	
	Total		**8**	
4(a)	For a Rectangular Distribution $f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$			
	$(-0.05, 0.05) \Rightarrow$	B1		(explain error ± 0.05)
	$\frac{1}{b-a} = \frac{1}{0.05-(-0.05)} = \frac{1}{0.1} = 10$ (Area $= 10 \times 0.1 = 1$)	M1 A1	3	
(b)	$P(-0.01 < X < 0.02) = 0.03 \times 10 = 0.3$	M1 A1	2	
(c)	Mean $= 0$	B1		CAO
	Standard deviation = 0.0289	B1	2	$\frac{1}{20\sqrt{3}}$ OE
	Total		**7**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
5(a)	Assumption that the speeds of the cars passing through the village are normally distributed	B1		
	$\bar{x} = 35.6$	B1		
	$s^2 = 38.27 \quad (s = 6.186)$	B1		$(\sigma^2 = 34.44 \quad (\sigma = 5.869))$
	99% Confidence Interval for μ			
	$= 35.6 \pm 3.250 \times \dfrac{6.186}{\sqrt{10}}$	B1		or use of $\dfrac{\sqrt{34.44}}{3}$
	$= 35.6 \pm 6.36$	M1		
		A1√		on their mean and standard deviation
	$= (29.2, 42.0)$	A1	7	CAO (29.24, 41.96)
(b)	Confidence interval includes 30 mph	B1√		
	80% of sample exceed 30 mph limit	B1		
	Speed limit **not** adhered to	B1	3	dependent on previous B1
	Total		**10**	
6(a)(i)	$\mathrm{E}\left(\dfrac{1}{X}\right) = \int_0^1 \dfrac{1}{x} 3x^2 \, \mathrm{d}x = \int_0^1 3x \, \mathrm{d}x$	M1		
	$= \left[\dfrac{3x^2}{2}\right]_0^1 = 1.5$	A1		
		A1	3	CAO
(ii)	$\mathrm{E}\left(\dfrac{1}{X^2}\right) = \int_0^1 \dfrac{1}{x^2} 3x^2 \, \mathrm{d}x = \int_0^1 3 \, \mathrm{d}x$	M1		
	$= [3x]_0^1 = 3.0$	A1		
	$\mathrm{Var}\left(\dfrac{1}{X}\right) = 3.0 - (1.5)^2$	m1		dependent on previous M1
	$= 0.75$	A1√	4	[on their (i)] and Var > 0
(b)	$\mathrm{E}\left(\dfrac{5+2X}{X}\right) = \mathrm{E}\left(\dfrac{5}{X} + 2\right)$	M1		
	$= 5\mathrm{E}\left(\dfrac{1}{X}\right) + 2$	M1		
	$= 5 \times 1.5 + 2$			
	$= 9.5$	A1		CAO
	$\mathrm{Var}\left(\dfrac{5+2X}{X}\right) = \mathrm{Var}\left(\dfrac{5}{X} + 2\right)$			
	$= 25 \times \mathrm{Var}\left(\dfrac{1}{X}\right)$	M1		
	$= 25 \times 0.75$			
	$= 18.75$	A1	5	CAO
	Total		**12**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\begin{array}{\|c\|c\|c\|}\hline \boldsymbol{x} & 4 & -1 \\ \hline \mathbf{P}(\boldsymbol{X}=\boldsymbol{x}) & \frac{1}{5} & \frac{4}{5} \\ \hline \end{array}$	B1	1	
(ii)	$\mathrm{E}(X)=\left(4\times\frac{1}{5}\right)+\left(-1\times\frac{4}{5}\right)=0$	M1 A1	2	$\left(p>0, \sum p=1\right)$
(b)	$\begin{array}{\|c\|c\|c\|}\hline \boldsymbol{x} & 4 & -1 \\ \hline \mathbf{P}(\boldsymbol{X}=\boldsymbol{x}) & \frac{1}{3} & \frac{2}{3} \\ \hline \end{array}$	B1		
	$\mathrm{E}(X)=\left(4\times\frac{1}{3}\right)+\left(-1\times\frac{2}{3}\right)=\frac{2}{3}$	B1		$\left(p>0, \sum p=1\right)$
	$\mathrm{E}(24X)=24\times\mathrm{E}(X)$			
	$=24\times\frac{2}{3}$	M1		
	$=16$	A1	4	
	Total		**7**	
8(a)	$\bar{x}=225.25$	B1		
	$s=5.06 \quad \left(s^2=25.6\right)$	B1		$(\sigma=4.74)$, $(\sigma^2=22.4)$
	$\mathrm{H}_0: \mu=230$			
	$\mathrm{H}_1: \mu\neq 230$	B1		both
	$\nu=8-1=7$	B1		
	$t_{\text{crit}}=\pm 2.365$	B1		accept $t_{\text{crit}}=-2.365$
	Test statistic:			
	$t=\frac{225.25-230}{5.064/\sqrt{8}}=-2.65$	M1		$\frac{225.25-230}{4.74/\sqrt{7}}=-2.65$
		A1		(–2.66 to –2.65)
	Reject H_0 at 5% level	A1√		
	No evidence to support the producer's claim	E1√	9	
(b)	We have rejected H_0 when in fact H_0 may be true. This indicates that a Type I error may have been made.	B2	2	
	Total		**11**	
	TOTAL		**75**	

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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MS2B Statistics 2B

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2008 examination - January series

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B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
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MS2B

Q	Solution	Marks	Total	Comments
1	$H_0: \mu = 5.0$ $H_1: \mu > 5.0$	B1		Both H_0 and H_1: correct
	$z = \dfrac{5.5 - 5}{\sqrt{1.31}/\sqrt{40}}$	M1		
	$z = 2.76$	A1		(AWFW 2.76 to 2.78)
	$z_{crit} = 2.3263$	B1ft		on their H_1: $(t_{crit} = 2.426)$
	Reject H_0	A1		(dep M1)
	sufficient evidence to support David's claim at 1% level	E1	6	
	Total		**6**	
2(a)(i)	$X \sim \text{Po}(9.0) \Rightarrow$ standard deviation $= 3$	B1	1	
(ii)	$\text{P}(6 < X < 12)$ $= \text{P}(X \le 11) - \text{P}(X \le 6)$ $= 0.8030 - 0.2068$ $= 0.5962$	M1 M1ft A1	3	CAO
(b)(i)	$T \sim \text{Po}(11.5)$	B1	1	CAO
(ii)	$\text{P}(T \le 1) = \text{P}(T = 0) + \text{P}(T = 1)$ $= e^{-11.5} + 11.5e^{-11.5}$ $= 0.000127$	M1 M1 A1	3	Use of $T = 0$ and 1 Substitute correctly into formula AWFW 0.000126 and 0.00013
(c)	$\bar{x} = 12.0$ and $s^2 = 19.3$ (s = 4.40)	B1		$\sigma^2 = 17.4$ $(\sigma = 4.17)$
	Mean and variance very different $\Rightarrow$ $\text{Po}(12.0)$ not a suitable model	E1 E1	3	dep on s^2 (or σ^2) (dep E1)
	Total		**11**	

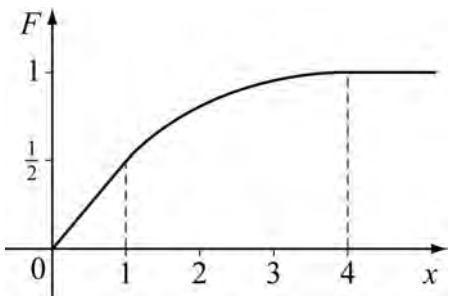
MS2B (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$k = \dfrac{1}{a+b}$	B1	1	
(ii)	$\mathrm{E}(T) = \int_{-a}^{b} ktdt$	M1		
	$= \left[\dfrac{kt^2}{2}\right]_{-a}^{b}$	A1		
	$= \dfrac{1}{2} \times \dfrac{1}{(a+b)} \times \left[b^2 - a^2\right]$			
	$= \dfrac{1}{2} \times \dfrac{1}{(a+b)} \times (b-a)(a+b)$	M1		Factorise
	$= \dfrac{1}{2}(b-a)$	A1	4	**AG**
(b)(i)	$\mathrm{E}(T) = 1$	B1	1	CAO
(ii)	$\mathrm{P}(T < -3 \text{ or } T > 3)$			Alternative
	$= \mathrm{P}(T < -3) + P(T > 3)$	M1		$1 - \mathrm{P}(-3 < T < 3)$
	$= 0.1 + 0.3$			
	$= 0.4$	A1	2	$1 - (0.3 + 0.3) = 0.4$
	Total		**8**	
4(a)	$\bar{v} = \dfrac{1179}{10} = 117.9$	B1		
	$s^2 = \dfrac{1014.9}{9} = 112.8 \Rightarrow s = 10.6$	B1		$\sigma^2 = 101.5 \quad (\sigma = 10.08)$
	$t_{0.995} = 3.250$	B1		
	99% confidence interval:			or use of $\dfrac{\sqrt{101.5}}{3} = 3.359$
	$117.9 \pm \dfrac{10.6}{\sqrt{10}} \times 3.250$	M1		$(\text{their } \bar{v}) \pm \dfrac{(\text{their } s)}{\sqrt{10}} \times t_9$
	$= 117.9 \pm 10.9$	A1ft		$(\text{on } \bar{x},\ s \text{ and } t_9 = 3.25)$
	$= (106.98, 128.82)$ $= (107, 129)$	A1		AWRT (107, 129)
	Assumption: **Speeds** form a Normal Distribution	B1	7	
(b)	John's claim is unlikely since 130 mph lies outside the confidence interval.	E1	1	
	Total		**8**	

MS2B (Cont)

Q	Solution	Marks	Total	Comments
5(a)	$\mathrm{P}(X \geq 5) = \mathrm{P}(X = 5) + \mathrm{P}(X = 6)$			
	$= \frac{5}{20} + \frac{6}{24}$	M1		
	$= \frac{1}{2}$	A1	2	
(b)(i)	$\mathrm{E}\left(\frac{1}{X}\right) = \sum \frac{1}{x} \times \mathrm{P}(X = x) =$			
	$\left(1 \times \frac{1}{20}\right) + \left(\frac{1}{2} \times \frac{2}{20}\right) + \left(\frac{1}{3} \times \frac{3}{20}\right) + \left(\frac{1}{4} \times \frac{4}{20}\right) + \left(\frac{1}{5} \times \frac{5}{20}\right)$ $+ \left(\frac{1}{6} \times \frac{6}{24}\right)$	M1		Use of $\sum \frac{1}{x} \times p$
	$= \frac{1}{4} + \frac{1}{24}$			
	$= \frac{7}{24}$	A1	2	**AG**
(ii)	$\mathrm{E}\left(\frac{1}{X^2}\right) = \frac{109}{900}$	M1 A1		Use of $\sum \frac{1}{x^2} \times p$ or 0.21
	$\mathrm{Var}\left(\frac{1}{X}\right) = \frac{109}{900} - \left(\frac{7}{24}\right)^2$	A1	3	**AG**
	$= 0.036$ **(3dp)**			
(c)	$A = \frac{1}{X}(X + 3)$ $A = 1 + \frac{3}{X}$	B1		(either)
	$\mathrm{E}(A) = 1 + 3\mathrm{E}\left(X^{-1}\right)$ $\mathrm{E}(A) = 1 + 3 \times \frac{7}{24}$	M1		(either)
	$= 1\frac{7}{8}$	A1		(1.875)
	$\mathrm{Var}(A) = \mathrm{Var}\left(1 + \frac{3}{X}\right) = 9\mathrm{Var}\left(X^{-1}\right)$	M1		
	$= 9 \times \frac{173}{4800}$			allow 9×0.036
	$= 0.324$ **or** $\frac{519}{1600}$	A1	5	0.324375
	Total		**12**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
6(a)	H_0: no association between education and salary	B1		
	O | E | $\alpha = \lvert O - E \rvert - 0.5$ | α^2/E 78 | 70.2 | 7.3 | 0.7591 57 | 64.8 | | 0.8224 52 | 59.8 | | 0.8911 63 | 55.2 | | 0.9654 | | | 3.4380	M1 A1 M1 M1		E attempted, correctly Yates' correction attempted α^2/E attempted (final col)
		A1		AWRT 3.4
	$\chi^2(10\%) = 2.706$	B1		
	Reject H_0 at 10% level	A1ft		
	Evidence to suggest an association between salary and having a university education.	E1ft	9	
(b)	Rejecting H_0 when H_0 correct	E1		
	Stating that there is an association between salary and education when there is not.	E1	2	
	Total		**11**	
7(a)(i)		B4	4	B1 for axes 0 to 4 & 0 to 1 B1 for straight line 0 – $\left(1, \frac{1}{2}\right)$ B1 for convex curve from $\left(1, \frac{1}{2}\right)$ to $(4,1)$ B1 for at least the straight line for $x > 4$
(ii)	$F(q_1) = 0.25 \Rightarrow \frac{1}{2}q_1 = 0.25$	M1		From sketch, or from $F(x)$, Median = 1.
	$\Rightarrow \quad q_1 = \frac{1}{2}$	A1	2	$\frac{1}{2}x$ is linear on (0 , 0) to $\left(1, \frac{1}{2}\right)$ $\therefore q_1 = \frac{1}{2}$ **AG**

Q	Solution	Marks	Total	Comments
7(iii)	$\mathrm{F}(1.6) = 0.744$ $\mathrm{F}(1.7) = 0.775$	M1		
	$\mathrm{F}(q_3) = 0.75$	M1		
	$\Rightarrow \quad 1.6 < q_3 < 1.7$	A1	3	**AG**
(b)(i)	$\mathrm{f}(x) = \mathrm{F}'(x)$	M1		$\boxed{\mathrm{f}(1) = \alpha = 9\beta}$ B1
	$\Rightarrow \quad \mathrm{f}(x) = \frac{1}{2}$ for $0 \le x \le 1$	A1		$\int_{-\infty}^{\infty} \mathrm{f}(x)\,dx = 1 \Rightarrow$
	$\Rightarrow \quad \alpha = \frac{1}{2}$			$[\alpha x]_0^1 + \left[\frac{\beta(x-4)^3}{3}\right]_1^4 = 1$ M1
	$\Rightarrow$ for $1 \le x \le 4$			
	$\mathrm{f}(x) = \frac{1}{54}(3x^2 - 24x + 48)$	A1		$\Rightarrow \quad \boxed{\alpha + 9\beta = 1}$ A1
	$= \frac{3}{54}(x^2 - 8x + 16)$	M1		Solving: M1
	$= \frac{1}{18}(x-4)^2$			$\alpha = \frac{1}{2}$ and $\beta = \frac{1}{18}$ A1
	$\Rightarrow \quad \beta = \frac{1}{18}$	A1	5	
(ii)	$\mathrm{E}(X) = \int_0^1 \frac{1}{2} x\,dx + \int_1^4 \frac{1}{18}(x^3 - 8x^2 + 16x)\,dx$	M1		Both seen
	$= \left[\frac{1}{4}x^2\right]_0^1 + \frac{1}{18}\left[\frac{x^4}{4} - \frac{8x^3}{3} + 8x^2\right]_1^4$	A1A1		
		m1		Dependent on M1
	$= \frac{1}{4} + \frac{7}{8}$	A1	5	CAO
	$= 1\frac{1}{8}$			
	Total		**19**	
	TOTAL		**75**	

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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MS2B Statistics 2B

Mark Scheme

2008 examination – June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

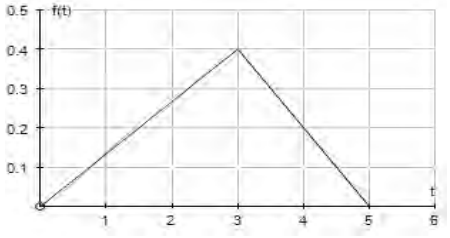
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Q	Solution	Marks	Total	Comments
1(a)	O_i | E_i | $\lvert O_i - E_i \rvert - 0.5$ | $\frac{7.5^2}{E_i}$ 52 | 44 | 7.5 | 1.2784 58 | 66 | 7.5 | 0.8523 28 | 36 | 7.5 | 1.5625 62 | 54 | 7.5 | 1.0417 4.7349	M1 M1 M1 A1		E attempted Yates' correction attempted χ^2 attempted AWFW 4.73 to 4.74
	H_0: No association between incidence of asthma and volume of traffic H_1: Association	B1		at least H_0 stated correctly
	$\nu = 1$ $\chi^2_{\text{crit}} = 3.841 < 4.7349$	B1		critical value
	Reject H_0 at 5% level	A1ft		
	Evidence to suggest an association between the incidence of asthma in children and the volume of traffic where they live	E1ft	8	
(b)	More than expected had asthma	E1	1	dep on statement of association
	Total		**9**	
2(a)	$P(X = 8) = P(X \le 8) - P(X \le 7)$ $= 0.8472 - 0.7440$	M1		$P(X = 8) = \frac{e^{-6}(6^8)}{8!}$
	$= 0.103$	A1	2	
(b)(i)	$\lambda = 9$	B1	1	
(ii)	$P(Y > 9) = 1 - P(Y \le 9)$ $= 1 - 0.5874$	M1		
	$= 0.4126$	A1ft	2	AWFW 0.412 to 0.413
(c)(i)	$T \sim \text{Po}(15)$	B1ft	1	
(ii)	$P(T \le 20) = 0.917$	B1ft	1	
(iii)	$P(T \text{ at least } 21) = 0.083$	B1ft		
	$p = 15 \times (0.083)^4 (0.917)^2$	M1		B(6, (iii)) used
	$= 0.000599$	A1	3	CAO; AWFW 0.000598 to 0.0006
	Total		**10**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
3	$H_0: \mu = 34.5$ $H_1: \mu \neq 34.5$	B1		
	$z_{crit} = \pm 1.96$	B1		
	$z = \dfrac{35.1 - 34.5}{2.5/\sqrt{50}} = 1.70$	M1A1		(1.697)
	Accept H_0	A1		
	Insufficient evidence, at 5% level of significance, to suggest that the mean weight has changed	E1	6	or … to confirm Alan's belief
	Total		**6**	
4(a)		B1 B1 B1	3	line segment on 0 – 3 line segment on 3 – 5 scales (0 – 0.4 vertical; 0 – 5 horizontal)
(b)(i)	$P(T \leq 2) = \dfrac{1}{2} \times 2 \times \dfrac{4}{15}$ $= \dfrac{4}{15}$	M1 A1	2	(0.267)
(ii)	$P(2 < T < 4) = 1 - (P(T < 2) + P(T > 4))$ $= 1 - \left(\dfrac{4}{15} + \dfrac{1}{2} \times \dfrac{1}{5}\right)$ $= 1 - \dfrac{4}{15} - \dfrac{1}{10}$ $= \dfrac{19}{30}$	M1 A1 A1	3	for $P(T > 4) = \dfrac{1}{10}$ $\dfrac{1}{2} d\left[(f_2 + f_4) + 2f_3\right]$ $f_2 = \dfrac{4}{15};\ f_4 = \dfrac{1}{5};\ f_3 = \dfrac{2}{5};\ d = 1$ (0.633)
(c)	$E(T) = \int_0^3 \dfrac{2}{15} t^2 \, dt + \int_3^5 t\left(1 - \dfrac{1}{5}t\right) dt$ $= \left[\dfrac{2}{45}t^3\right]_0^3 + \left[\dfrac{1}{2}t^2 - \dfrac{1}{15}t^3\right]_3^5$ $= \dfrac{6}{5} + \dfrac{25}{6} - \dfrac{27}{10}$ $= 2\dfrac{2}{3}$	M1 B1B1 A1	4	both integrals seen OE
	Total		**12**	

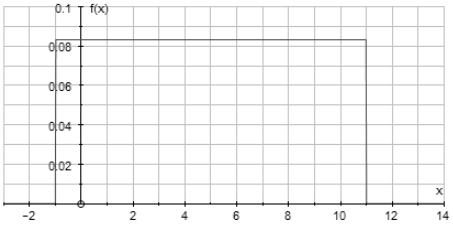
MS2B (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$\bar{x} = 3.19$ and $s^2 = \frac{1.849}{9} = 0.2054$	B1		both (s = 0.453)
	$t_9 = 3.250$	B1		
	99% confidence interval: $3.19 \pm 3.250 \times \frac{\sqrt{0.2054}}{\sqrt{10}}$	M1		$3.19 \pm (\text{their } t_9) \times \frac{\sqrt{0.2054}}{\sqrt{10}}$
	$= 3.19 \pm 0.4658$	A1ft		
	$= (2.72, 3.66)$	A1	5	(2.72 to 2.73, 3.65 to 3.66)
(ii)	Reasonable claim, with 3.5 within the 99% confidence interval	B1 E1	2	dep on correct CI in (a)(i)
(b)	$0.01 \times 200 = 2$	B1	1	
	Total		**8**	
6	$\bar{x} = 4.1$ $s = 0.392$ $(s^2 = 0.153)$	B1		both
	$H_0: \mu = 3.8$ $H_1: \mu > 3.8$	B1		both
	$t = \frac{4.1 - 3.8}{0.392/\sqrt{7}} = 2.03$	M1A1		AWFW 2.02 to 2.03
	$t_{crit} = 1.943$	B1ft		
	Reject H_0	A1		
	Evidence at 5% level of significance to support the doctor's belief that the cholesterol level is higher than the management's claim of 3.8	E1		
	Cholesterol levels normally distributed	B1	8	
	Total		**8**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\mathrm{E}(Y) = \sum y\ \mathrm{P}(Y = y)$ $= 5\times 0.1 + 15\times 0.2 + 25\times 0.3 + 35\times 0.4$ $= 25$	B1		
	$\mathrm{Var}(Y) = \mathrm{E}(Y^2) - [\mathrm{E}(Y)]^2$			
	$= 725 - 25^2$	M1		
	$= 100$	A1		CAO
	Standard deviation = 10	A1ft	4	ft on $\mathrm{Var}(Y) > 0$
(ii)	$C = 10Y + 5$ $\mathrm{E}(C) = 10\mathrm{E}(Y) + 5$ $= 10\times 25 + 5$ $= 255$ pence	B1	1	OE
(b)	$\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$			
	$= 75.25 - 8.35^2$	M1		
	$= 75.25 - 69.7225$ $= 5.5275$	A1		AWFW 5.52 to 5.53
	$T = 0.4X + 250$ $\mathrm{Var}(T) = \mathrm{Var}(0.4X + 250)$			
	$= 0.4^2 \times \mathrm{Var}(X)$	M1		$\mathrm{Var}(X) > 0$
	$= 0.16\times 5.5275$ $= 0.8844$	A1	4	AWFW 0.884 to 0.885
	Total		**9**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\mathrm{P}(X<0)=\mathrm{F}(0)$	M1		
	$=\dfrac{1}{k+1}$	A1	2	
(b)	$(q_1+1)\times\dfrac{1}{k+1}=\dfrac{1}{4}$	M1		alternative (from a sketch)
	$q_1+1=\dfrac{1}{4}(k+1)$	A1		
	$q_1=\dfrac{1}{4}(k+1)-1$	A1	3	OE
(c)	$\mathrm{f}(x)=\dfrac{\mathrm{d}}{\mathrm{d}x}(\mathrm{F}(x))$	M1		use of
	$=\dfrac{1}{k+1}\times\dfrac{\mathrm{d}}{\mathrm{d}x}(x+1)$ $=\dfrac{1}{k+1}\quad -1\le x\le k$ $=0\quad$ otherwise	A1	2	AG; $\dfrac{1}{k+1}$ clearly deduced
(d)(i)	$k=11\Rightarrow \mathrm{f}(x)=\begin{cases}\dfrac{1}{12} & -1\le x\le 11\\ 0 & \text{otherwise}\end{cases}$			
	Rectangular distribution:			
		B1		horizontal line on $[-1,11]$
		B1	2	at $\mathrm{f}=\dfrac{1}{12}$
(ii)	$\mathrm{E}(X)=\dfrac{1}{2}(-1+11)=5$	B1		
	$\mathrm{Var}(X)=\dfrac{1}{12}(11--1)^2=12$	B1	2	
(iii)	$\mathrm{P}(q_1<X<\mathrm{E}(X))=\mathrm{P}(2<X<5)$			
	$=(5-2)\times\dfrac{1}{12}$	M1		
	$=0.25$	A1	2	AG
	Total		**13**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MS2B Statistics 2B

Mark Scheme

2009 examination - January series

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Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Q	Solution	Marks	Total	Comments
1	H_0: No association between choice of subject and gender H_1: Association between choice of subject and gender	B1		
	Bul **Cl** **Fin** **Pol** **Total** **Male** 7 31 25 40 103 **Female** 2 24 22 19 67 **Total** 9 55 47 59 170	B1		Totals
	O_i / E_i 7 / 5.45 2 / 3.55 31 / 33.32 24 / 21.68 25 / 28.48 22 / 18.52 40 / 35.75 19 / 23.25 **170** / **170**	M1A1		E's attempted (correctly)
	One of the E_i's < 5 $\therefore$ combine cells	M1A1		Attempt at combining (correctly)
	O_i / E_i / $\alpha = (O_i - E_i)$ / α^2 / E_i 47 / 41.20 / 5.8 / 0.8165 21 / 26.80 / –5.8 / 1.2552 31 / 33.32 / –2.32 / 0.1615 24 / 21.68 / 2.32 / 0.2483 25 / 28.48 / –3.48 / 0.4252 22 / 18.52 / 3.48 / 0.6539	m1		Final column
	Test statistic: $X^2 = 3.56$ Critical value: = 4.605	A1 B1F		(AWFW 3.55 to 3.57) ft on their ν
	Accept H_0	A1F		
	Insufficient evidence to suggest that the choice of subject is associated with gender.	E1	11	
	Total		**11**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
2(a)	$H_0: \mu = 8.0$	B1		
	$H_1: \mu \neq 8.0$			
	$\bar{x} = \frac{84}{9} = 9.3\dot{3}$ or $9\frac{1}{3}$	B1		
	$z_{crit} = \pm 1.96$	B1		
	$z = \frac{9.33 - 8.0}{2.5/\sqrt{9}} = 1.60$	M1		$z = \frac{(\text{their } \bar{x}) - 8}{2.5/\sqrt{9}}$
		A1		AWFW 1.59 to 1.60
	$\lvert z \rvert < 1.96$ ∴ accept H_0	A1F		ft on incorrect $\bar{x}$
	Insufficient evidence to suggest that the mean completion time has changed from eight weeks.	E1F	7	
(b)	Neither a Type I nor a Type II error have occurred	B1		dependent
	Have accepted that $H_0: \mu = 8.0$, when $\mu = 8.0$.	B1	2	dependent on 'accept H_0' in (a)
	Total		**9**	
3(a)(i)	$P(X \leq 3) = 0.515$	B1	1	0.5152
(ii)	$P(Y = 5) = \frac{e^{-4.4} \times (4.4)^5}{5!}$	M1		$P(Y \leq 5) - P(Y \leq 4) = 0.7199 - 0.5512$ correct values seen
	$= 0.169$	A1	2	(0.1687)
(b)(i)	$T = \text{Po}(8.0)$	B1		
	X and Y are **independent** (Poisson random variables)	B1	2	
(ii)	$P(6 < T < 12) = P(T \leq 11) - P(T \leq 6)$	M1		
	$= 0.8881 - 0.3134$	A1		
	$= 0.575$	A1	3	(0.5747)
(iii)	$P(T > 14) = 1 - P(T \leq 14)$	M1		
	$= 1 - 0.9827$			
	$= 0.0173$	A1		CAO
	$p = (0.0173)^2$	M1		$[\text{their } P(T > 14)]^2$
	$= 0.0003$ (1sf)	A1F	4	ft if 0 < both p's <1
(iv)	$P(T \leq k) > 0.99$			$\begin{cases} P(T \leq 15) = 0.9918 \\ P(T \leq 14) = 0.9827 \end{cases}$
	$\Rightarrow \quad k \geq 15$	M1		
	∴ minimum number of devices that Joe should keep in stock = 15	A1	2	
	Total		**14**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\mathrm{P}\left(-\frac{3c}{4} < X < \frac{3c}{4}\right)$			
	$= \frac{\frac{3c}{4}+c}{4c} - \frac{\frac{-3c}{4}+c}{4c}$	M1		or $= \frac{3c}{2} \times \frac{1}{4c}$
	$= \frac{6c}{16c}$			
	$= \frac{3}{8}$ or 0.375	A1	2	CAO
(b)	For $-c \le x \le 3c$			
	$\mathrm{f}(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x+c}{4c}\right)$	M1		use of $\mathrm{f}(x) = \mathrm{F}'(x)$
	$= \frac{1}{4c}$			
	For $x > 3c$ and $x < -c$			
	$\mathrm{f}(x) = \frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{F}) = 0$	A1	2	for $\frac{1}{4c}$ and 0
(c)(i)	Rectangular distribution:			
	$\mathrm{E}(X) = \frac{1}{2}(-c+3c) = c$	B1	1	
(ii)	$\mathrm{Var}(X) = \frac{1}{12}(3c--c)^2 = \frac{4c^2}{3}$	B1	1	Allow $\frac{16c^2}{12}$
	Total		**6**	
5(a)(i)	$\bar{x} = \frac{1}{2}(70.65+80.35) = 75.5$	B1	1	AG
(ii)	Width of confidence interval $= 80.35 - 70.65$			
	$= 9.7$	B1	1	
(iii)	$t_{crit} = 2.602$; $\nu = 15$	B1		
	$\mathrm{w} = 2t \times \frac{s}{\sqrt{n}} \Rightarrow \frac{s}{\sqrt{n}} = \frac{9.7}{2 \times 2.602}$	M1		
	Estimate of s.e $= \frac{s}{\sqrt{n}} = 1.86$	A1	3	(1.864)
(iv)	Unbiased estimate of $\sigma^2 = 1.86^2 \times 16$	M1		
	$= 55.6$ (3sf)	A1	2	AG (55.589)

MS2B (cont)

Q	Solution	Marks	Total	Comments
5(b)	95% CI: $75.5 \pm 2.131 \times \frac{s}{\sqrt{n}}$	M1		
	$= 75.5 \pm 3.972$			
	$= (71.5, 79.5)$	A1	2	$(71.5$ to $71.54, 79.4$ to $79.5)$ CAO
(c)(i)	$(73.0, 78.0)$	B1	1	
(ii)	$w = 2t \times \frac{s}{\sqrt{n}} \Rightarrow t = \frac{5}{2 \times 1.864} = 1.341$	M1		(AWFW 1.341 to 1.344)
	$\Rightarrow$ for $\nu = 15$ $P(X \leq 1.341) = 0.90$			
	$\Rightarrow$ $P(X \geq 1.341) = 0.10$ and $P(X \leq -1.341) = 0.10$	M1		
	$\therefore$ $P(\lvert X \rvert \leq 1.341) = 0.80$			
	Percentage confidence interval = 80%	A1	3	
	Total		**13**	
6(a)				

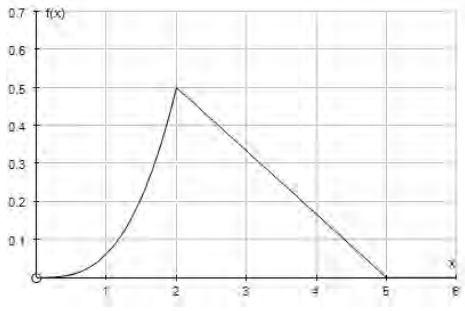
r	1	2	3	4
P(*R*=*r*)	$2/3$	$2/9$	$2/27$	k

$k + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} = 1 \Rightarrow k = \frac{1}{27}$	M1 A1	2	AG
(b) $P(R \geq 3) = \frac{2}{27} + \frac{1}{27} = \frac{1}{9}$	B1	1	Allow $\frac{3}{27}$ or 0.111
(c)(i) $C = 27R + 5$			
$E(R) = \left(1 \times \frac{2}{3}\right) + \left(2 \times \frac{2}{9}\right) + \left(3 \times \frac{2}{27}\right) + \left(4 \times \frac{1}{27}\right)$			
$= 1\frac{13}{27}$	B1		(1.48) or $\frac{40}{27}$
$\therefore$ $E(C) = 27 \times 1\frac{13}{27} + 5$	M1		
$= 45$	A1F	3	

MS2B (cont)

Q	Solution	Marks	Total	Comments
6(c)(ii)	$\mathrm{E}(R^2)=\left(1\times\frac{2}{3}\right)+\left(4\times\frac{2}{9}\right)+\left(9\times\frac{2}{27}\right)+\left(16\times\frac{1}{27}\right)$			
	$=2\frac{22}{27}$ or $\frac{76}{27}$	B1		(2.81)
	$\mathrm{Var}(R)=2\frac{22}{27}-\left(1\frac{13}{27}\right)^2$	M1		
	$=\frac{452}{729}$			(0.62)
	$\therefore$ St. dev$^{\mathrm{n}}(C)=27\times\sqrt{\frac{452}{729}}$	M1		$27\times\sqrt{\mathrm{Var}(R)}$ $[\mathrm{Var}(R)>0]$
	$=21.3$	A1	4	CAO (21.26)
				SC: $\mathrm{Var}(C)=452$ (CAO) (B1M1B1A0)
	Total		**10**	
	Alternative (c)			
	C: 32, 59, 86, 113 p: $\frac{2}{3}$, $\frac{2}{9}$, $\frac{2}{27}$, $\frac{1}{27}$			
	C: 32, 59, 86, 113 n: 18, 6, 2, 1			
	$\bar{x}=45$ and $\sigma=21.260$ from calculator			$\left(\bar{x}=\frac{\sum Cn}{27}\right)$

MS2B (cont)

Q	Solution	Marks	Total	Comments
7(a)		B1		for concave curve from (0 , 0) to (2 , 0.5)
		B1		for straight line from (2 , 0.5) to (5 , 0)
		B1	3	for axes [2 , 5 ; 0.5] seen
(b)	$\mathrm{P}(X \geq 2) = \frac{1}{2} \times 3 \times 0.5 = 0.75$			**Alternatives**:
	$\Rightarrow \mathrm{F}(2) = 0.25$			$\int \frac{1}{6}(5-x)\mathrm{d}x = \frac{1}{6} \times \frac{(5-x)^2 \times (-1)}{2}$
	$2 \leq x \leq 5$			$= -\frac{1}{12}(5-x)^2$
	$\mathrm{F}(x) = \mathrm{F}(2) + \int_2^x \frac{1}{6}(5-x)\mathrm{d}x$			Or
	$= 0.25 + \frac{1}{6}\left[5x - \frac{x^2}{2}\right]_2^x$	M1		$\mathrm{F}(x) = 1 - \text{Area } \triangle \text{ (base } x,5)$
	$= 0.25 + \frac{1}{6}\left(5x - \frac{x^2}{2}\right) - \frac{1}{6}(10-2)$	A1		$= 1 - \frac{1}{2}(5-x)\frac{1}{6}(5-x)$
	$= 0.25 - \frac{8}{6} + \frac{5x}{6} - \frac{x^2}{12}$	M1		$= 1 - \frac{1}{12}(5-x)^2$
	$= -\frac{1}{12}(x^2 - 10x + 13)$			
	$= 1 - \frac{1}{12}(5-x)^2$	A1	4	
(c)	$\mathrm{P}(X \leq 4) = \mathrm{F}(4)$			**Alternative:**
	$= 1 - \frac{1}{12}(5-4)^2 = \frac{11}{12}$ (0.916 to 0.917)	B1		$\mathrm{P}(X \geq 3 \mid X \leq 4)$
	$\mathrm{F}(3) = 1 - \frac{1}{12}(2)^2 = \frac{2}{3}$ (0.667)	B1		$= \frac{\mathrm{F}(4) - \mathrm{F}(3)}{\mathrm{F}(4)}$ (M1)
	$\mathrm{P}(X \geq 3 \text{ and } X \leq 4) = \mathrm{F}(4) - \mathrm{F}(3)$			$= 1 - \frac{\mathrm{F}(3)}{\mathrm{F}(4)}$
	$= \frac{11}{12} - \frac{2}{3} = \frac{1}{4}$ (0.25)	B1		$= 1 - \frac{2/3}{11/12}$
	$\mathrm{P}(X \geq 3 \mid X \leq 4) = \frac{\mathrm{F}(4) - \mathrm{F}(3)}{\mathrm{F}(4)}$	M1		$= 1 - \frac{8}{11}$ (B1) $(0.72\dot{7}\dot{2})$
	$= \frac{1/4}{11/12} = \frac{3}{11}$	A1	5	$= \frac{3}{11}$ (AWFW 0.272 to 0.273)
	Total		**12**	
	TOTAL		**75**	

Version : 1.0 0609

General Certificate of Education

Mathematics 6360

MS2B Statistics 2B

Mark Scheme

2009 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Q	Solution	Marks	Total	Comments
1	$H_0: \mu = 768$ $H_1: \mu \neq 768$	B1		(Both)
	Test statistic: $z = \dfrac{764.8 - 768}{8/\sqrt{18}}$	M1		
	$= -1.70$	A1		(-1.697)
	$z_{crit} = \pm 1.96$	B1		($z_{crit} = 1.96$ or $z_{crit} = -1.96$)
	$\Rightarrow$ Accept H_0	A1		
	No evidence at the 5% level of significance, to deny Yvonne's claim.	E1	6	
	Total		**6**	
2(a)(i)	$X \sim \text{Po}(5.0)$ $\Rightarrow \text{P}(X < 4) = \text{P}(X \leq 3)$			$(0.440$ to $0.441)$ for B1
	$= 0.265$	B2	2	CAO
(ii)	$Y \sim \text{Po}(1.5)$		2	
	$\Rightarrow \text{P}(Y = 4) = \dfrac{e^{-1.5} \times (1.5)^4}{4!}$	M1		
	$= 0.0471$	A1	2	(0.047 to 0.0471)
2(b)(i)	$T = X + Y \sim \text{Po}(6.5)$	B1		
	$\Rightarrow \text{P}(T > 5) = 1 - \text{P}(T \leq 5)$	B1		$(1 - 0.2237)$ **or** $(1 - 0.5265)$
	$= 1 - 0.369$ $= 0.631$	B1	3	
(ii)	$p = {}^8C_7 (0.631)^7 (0.369) +$ $(0.631)^8$	M1ft		ft on their p from (b)(i) Either part attempted
	$p = 0.11758 + 0.02513$	A1ft		(both parts correct)
	$= 0.143$	A1	3	AWFW 1.142 to 0.143 (CAO)
(c)(i)	Mean = 8	B1		CAO
	Variance = $s^2 = 16.9$ $(\text{sample variance} = 15.2)$	B1	2	(AWRT)
(ii)	Poisson not a good model for data	B1dep		
	Mean $\neq$ Variance	B1	2	
	Total		**14**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
3	H_0 **:** no association between age and attitude to school reorganisation	B1		
	H_1 **:** association between age and attitude to school reorganisation			
		M1		E's attempted
		A1		correctly (at least 6 E's)

Age	Against	
	O_i	E_i
16 - 17	9	$6\frac{17}{65}$
18 - 21	17	$15\frac{24}{65}$
22 - 49	115	$116\frac{9}{13}$
50 - 65	41	$42\frac{9}{13}$
> 65	3	$3\frac{64}{65}$
Total	185	185

E_i
6.262
15.369
116.692
42.692
3.985

Age	Not Against	
	O_i	E_i
16 - 17	2	$4\frac{48}{65}$
18 - 21	10	$11\frac{41}{65}$
22 - 49	90	$88\frac{4}{13}$
50 - 65	34	$32\frac{4}{13}$
> 65	4	$3\frac{1}{65}$
Total	140	140

E_i
4.738
11.631
88.308
32.308
3.015

Row totals: $\widehat{11,27}$ 205, $\widehat{75.7}$ (325)

Column totals: 185, 140 (325) — B1 — Totals correct

$E_i's < 5$

∴ combine cells 16 – 17 and 18 –21 **also** 50 – 65 and 'over 65' to give: — M1 Attempt at combining rows; A1 Correctly

O_i	E_i	$\alpha = O_i - E_i$	α^2 / E_i
26	21.63	4.369	0.8825
115	116.69	−1.692	0.0245
44	46.68	–2.677	0.1535
12	16.37	–4.369	1.1662
90	88.31	1.692	0.0324
38	35.32	2.677	0.2029
325	325		2.462

ml — Final column attempted (dep M1)

$X^2 = 2.462$ — A1 — 2.4 to 2.5

$\nu = 2$ — B1

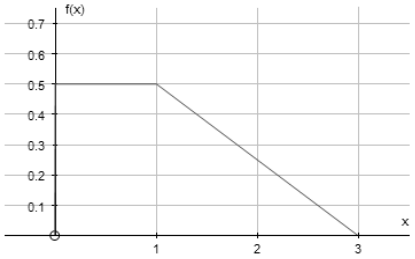
$\chi^2_{\nu=2}(0.95) = 5.991$ — B1ft — On their *v*

Accept H_0 — A1ft

No real evidence at 5% level of significance to suggest any association between age and attitude to school reorganisation. — E1ft — 12 — (context)

	Total		**12**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
4(a)	Sketch:  	B3	3	1 for straight line $0 \le x \le 1$ from (0, 0.5) to (1, 0.5) 1 for straight line $1 \le x \le 3$ from (1, 0.5) to (3, 0) 1 for axes [must have at least (0,0.5) (1,0) and (3,0) labelled]
(b)	$\mathrm{P}(X \le \eta) = \mathrm{F}(\eta) = 0.5$ $(\Rightarrow \quad \eta = 1$ (from graph))	M1 A1	2	AG
(c)	$\mu = \mathrm{E}(X) = \int_0^1 \left(\frac{x}{2}\right) \mathrm{d}x + \int_1^3 x\left(\frac{3-x}{4}\right) \mathrm{d}x$ $= \left[\frac{x^2}{4}\right]_0^1 + \frac{1}{4}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_1^3$ $= \frac{1}{4} + \frac{1}{4}\left[\left(\frac{27}{2} - 9\right) - \left(\frac{3}{2} - \frac{1}{3}\right)\right]$ $= \frac{1}{4} + \frac{5}{6} \quad (0.25 + 0.83\dot{3})$ $= 1\frac{1}{12}$	M1 A1 ml A1	4	Both integrals stated Either Correct limits used on both integrals +combined dep M1 (CAO)
(d)	Area of $\triangle$ $= \mathrm{P}\left(X > 2\frac{1}{4}\right) = \frac{1}{2} \times \frac{3}{4} \times \frac{3 - 2\frac{1}{4}}{4}$ $= \frac{3}{32} \times \frac{3}{4} = \frac{9}{128}$ $\therefore \quad \mathrm{P}\left(X < 2\frac{1}{4}\right) = 1 - \frac{9}{128}$ $= \frac{119}{128}(0.9296875)$	M1ft M1ft A1	3	**Alternative:** For $1 \le x \le 3$ $\mathrm{F}(x) = 1 - \frac{1}{8}(3-x)^2$ M1ft $\Downarrow$ $\mathrm{F}\left(2\frac{1}{4}\right) = 1 - \frac{1}{8} \times \frac{9}{16}$ M1ft $= \frac{119}{128}$ CAO

MS2B (cont)

Q	Solution	Marks	Total	Comments
4(d)	**or**			**Alternative**
	$\int_{2\frac{1}{4}}^{3} \frac{3-x}{4}\,dx \left(= \frac{9}{128}\right)$ M1 ft			$f\left(2\frac{1}{4}\right) = \frac{3}{16} = 0.1875$
	$= 1 - \int_{2\frac{1}{4}}^{3} \frac{3-x}{4}\,dx$ M1 ft			$P\left(X < 3\mu - \eta\right) = P\left(X < 2\frac{1}{4}\right)$
	$= 1 - \frac{1}{4}\left[3x - \frac{x^2}{2}\right]_{2\frac{1}{4}}^{3}$			$= \frac{1}{2} + \boxed{\frac{1}{2}\left(\frac{3}{16} + \frac{1}{2}\right) \times 1\frac{1}{4}}$ M1ft
	$= 1 - \frac{1}{4}\left[9 - \frac{9}{2} - \frac{27}{4} + \frac{81}{32}\right]$			$= \frac{1}{2} + \frac{55}{128}(0.4296875)$ M1ft
	$= 1 - \frac{1}{4} \times \frac{9}{32} = \frac{119}{128}$ A1			$= \frac{119}{128}$ (0.930) A1
	or $(1 - 0.0703125 = 0.9296875)$			
	Total		**12**	
5(a)(i)	$P(\text{GG or YY or RR})$ $= \frac{2}{10} \times \frac{1}{9} + \frac{3}{10} \times \frac{2}{9} + \frac{4}{10} \times \frac{3}{9}$	M1		
	$= \frac{2}{9}$	A1	2	(AG)
(ii)	$P(B\bar{B} \text{ or } \bar{B}B) = \frac{1}{10} \times \frac{9}{9} + \frac{9}{10} \times \frac{1}{9}$	M1		$\frac{1}{10} + \frac{9}{10} \times \frac{1}{9}$
	$= \frac{1}{5}$	A1	2	(AG)
(b)(i)				
		B1		
		B1	2	
(ii)	$E(X) = 135 \times \frac{2}{9} + 145 \times \frac{1}{5} + (-45) \times \frac{26}{45}$	M1		Multiply two rows of their table from (b)(i)
	$= 29 + 30 - 26$ $= 33$ pence	A1	2	AG
(c)(i)	$E(Y) = 104 - 3E(X)$ $= 104 - 3 \times 33$ $= 5$ pence	M1 A1		
	∴ Joanne would expect to win £5	A1	3	OE (eg 500p)

	Same	1 Blue	Neither
x	135	145	−45
$P(X = x)$	$\frac{2}{9}$	$\frac{1}{5}$	$\frac{26}{45}$

MS2B (cont)

Q	Solution	Marks	Total	Comments
5(c)(ii)	$\mathrm{E}(X^2) = 9425$	B1		$(4205 + 4050 + 1170)$
	$\mathrm{Var}(X) = 9425 - 33^2 = \mathbf{8336}$			$\mathrm{sd}(X) = 91.30$
	$\mathrm{Var}(Y) = 9 \times \mathrm{Var}(X)$	B1		
	$= 9 \times 8336$			$9 \times (\text{their } \mathrm{Var}(X) > 0)$
	$= 75024$	M1		**or** $3 \times (\text{their } \mathrm{sd}(X))$
	$\Rightarrow$ standard deviation (Y) = 274 pence	A1	4	273.9p or £2.74
	Total		**15**	
6(a)(i)	$\bar{x} = 43.5$	B1		
	$s = 2 \ (s^2 = 4)$	B1		
	Assumption: Weights of boxes are normally distributed	B1		
	$t_{0.975} = 2.365$	B1		
	95% CI for μ : $43.5 \pm 2.365 \times \frac{2}{\sqrt{8}}$ / 43.5 ± 1.6723	M1		
	$\Rightarrow \quad (41.8, 45.2)$	A1	6	(AWRT)
(ii)	CI contains mean (45)	B1 dep		Must be clear use of 45 and
	Bishen's belief probably justified	B1 dep		**not** 43.5
	or			
	[Since 45 within CI] but close to upper limit, there is some evidence that Bishen's Belief is untrue			
	[but the evidence is not significant at 5 %.]			
	(75% of sample less than 45grams)	(B1)	2	
6(b)(i)	$\mathrm{H}_0: \mu = 45$			
	$\mathrm{H}_0: \mu < 45$	B1		(both)
	Test statistic: $t = \frac{43.5 - 45}{2/\sqrt{8}}$	M1		
	$= -2.12$	A1		$\mathrm{P}(t_7 < -2.12.) = 0.035791$
	$\nu = 7 \ \Rightarrow \ t_{crit} = -1.895$	B1		< 0.05
	$\Rightarrow$ Reject H_0	A1		
	Evidence at the 5% level of significance . to support Abi's claim that **mean** content < 45 grams	E1	6	
(ii)	Type I error	B1		
	have/may have rejected H_0 when H_0 true	B1	2	Clear statement
	or			
	No error	(B1)		
	have/may have accepted H_0 when H_0 true	(B1)		Clear statement
	Total		**16**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MS2B Statistics 2B

Mark Scheme

2010 examination - January series

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It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

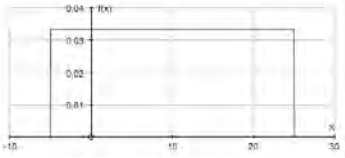
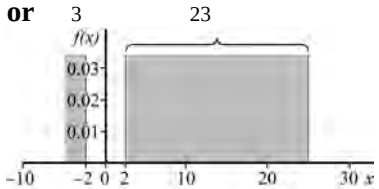
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Q	Solution	Marks	Total	Comments
1	$H_0: \mu = 45$ $H_1: \mu > 45$	B1		
	$z = \dfrac{45.8 - 45}{\sqrt{4.8/30}} = \dfrac{0.8}{0.4} = 2.0$	M1A1		AWRT
	$z_{crit} = 2.3263$	B1		$t_{29} = 2.462$
	Do not reject H_0 Insufficient evidence at 1% level of significance to support Roger's claim.	E1	5	
	Total		**5**	
2(a)(i)	$E(T) = \frac{1}{2}(25 + -5) = 10$	B1	1	CAO
(ii)	$Var(T) = \frac{1}{12}(25 - -5)^2$ $= 75$	B1	1	CAO
(b)	$P(-2 < T < 2) = \frac{2}{15}$ (OE)	B1		Diagram (optional) 
	$P(\text{magnitude at least 2 minutes})$ $= 1 - P(-2 < T < 2)$ $= 1 - \frac{4}{30}$	M1		
	$= \frac{13}{15}$ (OE) = 0.867	A1	3	CAO (AWRT)
	or  $\frac{1}{30}(3 + 23) = \frac{26}{30} = \frac{13}{15}$			**Alternative** $P(T > 2) = \frac{23}{30}$ $(0.766\dot{})$ **or** $P(T < -2) = \frac{1}{10}$ } B1
	or $\int_{-5}^{-2} \frac{1}{30} dt + \int_{2}^{25} \frac{1}{30} dt = \frac{1}{10} + \frac{23}{30} = \frac{13}{15}$			$P(\text{magnitude at least 2 minutes})$ $= P(T < -2) + P(T > 2)$ $= \frac{13}{15}$ for M1A1
	or $1 - \int_{-2}^{2} \frac{1}{30} dt = 1 - \left[\frac{t}{30}\right]_{-2}^{2}$ $= 1 - \frac{4}{30} = \frac{26}{30} = \frac{13}{15}$			
	Total		**5**	

MS2B (cont)

Q	Solution	Mark	Total	Comments
3	Assume that lengths of shots are normally distributed	B1		$\begin{cases} s_n^2 = 124;\ s_n = 11.1 \\ \text{iff } \frac{s_n}{3} \text{ used} \end{cases}$
	$\left.\begin{array}{l} \bar{x} = 184 \\ s^2 = \frac{1240}{9} = 137.\dot{7} \quad (s = 11.7) \end{array}\right\}$	B1		CAO $\begin{cases} \text{AWFW } 137.7 \text{ to } 138 \\ \text{both } \bar{x} \text{ and } s^2 \text{ (or } s) \end{cases}$
	$H_0: \mu = 190$ $H_1: \mu \neq 190$	B1		Both
	$t = \dfrac{184 - 190}{\sqrt{1240/9 \times 10}}$	M1		$t = \dfrac{\text{their } \bar{x} - 190}{\frac{\text{their } s_{n-1}}{\sqrt{10}}}$ **or** $\dfrac{\text{their } \bar{x} - 190}{\frac{\text{their } s_n}{\sqrt{9}}}$
	$t = -1.62$	A1		AWRT
	$\nu = 9 \quad \Rightarrow \quad t_{\text{crit}} = \pm\ 2.821$	B1		(accept 2.82)
	$-2.821 < -1.62 < 2.821$ accept H_0			
	Evidence to support Lorraine's belief at 2% level of significance	E1	7	
	Total		**7**	

MS2B (cont)

Q	Solution	Mark	Total	Comments
4(a)	H_0: no association between age and first time performance in driving test H_1: association between age and first time performance in driving test	B1		
	<table><tr><th></th><th colspan="2">Pass</th><th colspan="2">Fail</th><th></th></tr><tr><th>Age</th><th>O</th><th>E</th><th>O</th><th>E</th><th>Total</th></tr><tr><td>17-18</td><td>28</td><td>19.2</td><td>20</td><td>28.8</td><td>48</td></tr><tr><td>19-30</td><td>2</td><td>6.4</td><td>14</td><td>9.6</td><td>16</td></tr><tr><td>31-39</td><td>12</td><td>18.0</td><td>33</td><td>27.0</td><td>45</td></tr><tr><td>40-60</td><td>6</td><td>4.4</td><td>5</td><td>6.6</td><td>11</td></tr><tr><td>Total</td><td>48</td><td>48</td><td>72</td><td>72</td><td>120</td></tr></table>	M1 A1		E's attempted Correctly
	<table><tr><th>O</th><th>E</th><th>$(O-E)^2/E$</th></tr><tr><td>28</td><td>19.20</td><td>4.0333</td></tr><tr><td>2</td><td>6.40</td><td>3.0250</td></tr><tr><td>18</td><td>22.40</td><td>0.8643</td></tr><tr><td>20</td><td>28.80</td><td>2.6889</td></tr><tr><td>14</td><td>9.6</td><td>2.0167</td></tr><tr><td>38</td><td>33.6</td><td>0.5762</td></tr><tr><td></td><td></td><td>13.20</td></tr></table>	M1 A1 m1 A1		Attempt at combining Correctly Final column attempted For X^2 correct
	$\nu = 2 \quad \Rightarrow \quad \chi^2(2) = 9.210$	B1ft		(on $\nu = 2$ or $\nu = 3$ only)
	Reject H_0 Evidence to support Julie's belief at 1% level of significance.	E1ft	9	
(b)	More students than expected in the age group17-18 pass their test first time.	E1	1	Fewer than expected fail
	Total		**10**	

MS2B (cont)

Q	Solution	Mark	Total	Comments
5(a)	X = no. with blood disorder for $X \sim \mathrm{B}(25,0.7)$ $\mathrm{P}(X>15)=\mathrm{P}(X\geq 16)$ Consider $X' \sim \mathrm{B}(25,0.3)$ then: $\mathrm{P}(X\geq 16)=\mathrm{P}(X'\leq 9)$ $=0.8106$	B3,2,1	3	**Alternative:** $X \sim \mathrm{B}(25,0.7)$ $\mathrm{P}(X>15)=1-\mathrm{P}(X\leq 15)$ $=1-0.18943$ $=0.81057$ **B3** $0.81\leq p\leq 0.811$ **B2** for $0.902\leq p\leq 0.9022$ **B1** for $0.5\leq p\leq 0.95$
5(b)(i)	$X \sim \mathrm{P}_0(2.6)$ $\mathrm{P}(X\leq 5)=0.951$	B1	1	AWRT
(ii)	$Y \sim \mathrm{P}_0(4.9)$ $\mathrm{P}(Y=10)=\dfrac{e^{-4.9}\times(4.9)^{10}}{10!}$ $=0.0164$	B1 M1 A1	3	$\lambda=4.9$ stated or used in poisson expression AWFW 0.016 to 0.0165
(iii)	$T \sim \mathrm{P}_0(7.5)$ $\mathrm{P}(T>16)=1-\mathrm{P}(T\leq 16)$ $=1-0.9980$ $=0.002$	B1ft M1 A1	3	2.6 + (their mean in (ii)) (for 0.9980) CAO (0.00196)
	Total		**10**	

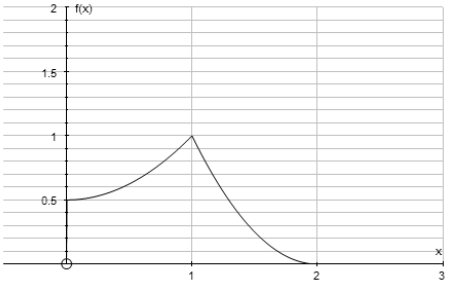
MS2B (cont)

Q	Solution	Mark	Total	Comments
6(a)(i)	$a = \frac{25}{63}$ (OE)	B1	1	$\left(\frac{100}{252}\text{ or }\frac{50}{126}\text{ or }0.397\right)$
(ii)	$\mathrm{E}(X) = 2.5$ (symmetry)	B1	1	
(iii)	$\mathrm{E}(X^2) = \left(1\times\frac{25}{252}\right)+\left(4\times\frac{25}{63}\right)+\left(9\times\frac{25}{63}\right)$ $+\left(16\times\frac{25}{252}\right)+\left(25\times\frac{1}{252}\right)$	M1		$\sum x^2 \times p$ attempted
	$\mathrm{E}(X^2) = \frac{125}{18}$	A1		$\left(6\frac{17}{18}\text{ or }6.94\right)$
	$\mathrm{Var}(X) = \frac{125}{18} - \frac{25}{4}$	m1		$\begin{cases}\left[\text{their E}(X^2) - (\text{their E}(X))^2\right] \\ \text{dep } \sum x^2 \times p \text{ used}\end{cases}$
	$= \frac{25}{36}$	A1		0.694 [Var > 0]
	$\mathrm{sd}(X) = \frac{5}{6}$	A1ft	5	$0.83\dot{3}$ $\left(\sqrt{\text{their Var}(X)}\right)$ (dep **m1)**
(b)(i)	$\mathrm{E}(\text{Pay}) = \frac{4}{9}\times 90$ pence $= 40$ pence $\Rightarrow$ Joanne expected to make a loss (loss of 10p per game)	**M1** **A1**		Alternative: $\frac{5}{9} > \frac{2}{9}+\frac{2}{9} \Rightarrow$ loss (for B1) then M1A1
(ii)	$\mathrm{E}(\text{Loss}) = 100\times 10$ pence $= £10$	**B1ft**	3	$100\times(\text{their loss/game})$
	Total		**10**	

MS2B (cont)

Q	Solution	Mark	Total	Comments
7(a)(i)	$d^2 = \frac{93}{12}$ $= 7.75$	**M1** **A1**	2	$d = \sqrt{\frac{93}{12}} = \sqrt{7.75}$ $\Rightarrow d^2 = 7.75$
(ii)	80% CI: $= 64.8 \pm 1.363 \times \sqrt{7.75}$ $= 64.8 \pm 3.79$ $= (61.0, 68.6)$	**B1** **M1** **A1**	3	$t_{11} = 1.363$ or 1.36 $64.8 \pm t_{11}\sqrt{7.75}$ iff $t_{11} = 1.363$ or 1.796 AWRT
(b)(i)	$(64.8 - 5, 64.8 + 5)$ $= (59.8, 69.8)$	**B1**	1	AWRT
(ii)	$w = 2\sqrt{7.75} \times t = 10$ $\Rightarrow \quad t = 1.796$	**M1** **A1**		t = 1.79 to 1.80
	$\mathrm{P}(X \geq 1.796) = 0.05$ $\mathrm{P}(X \leq -1.796) = 0.05$ $\Rightarrow \quad \mathrm{P}(\lvert X \rvert \leq 1.796) = 0.90$ 90% Confidence Level	**M1** **A1**	4	iff $t = 1.796$ correct
	Total		**10**	

MS2B (cont)

Q	Solution	Mark	Total	Comments
8(a)		B3	3	B1 for axes B1 for curve from (0, 0.5) to (1, 1) B1 for curve from (1, 1) to (2, 0)
(b)	$\mathrm{P}(X \le 1) = \int_0^1 \frac{1}{2}\left(x^2+1\right) dx$	M1		
	$= \left[\frac{x^3}{6} + \frac{x}{2}\right]_0^1$	A1		
	$= \left[\frac{1}{6} + \frac{1}{2}\right] = \frac{2}{3}$	A1	3	0.667
(c)	$\mathrm{E}\left(X^2\right) = \int_0^1 x^2 \times \frac{1}{2}\left(x^2+1\right) dx$ $+ \int_1^2 x^2 (x-2)^2 \, dx$	M1		both integrals seen
	$= \left[\frac{x^5}{10} + \frac{x^3}{6}\right]_{x=0}^{x=1} + \left[\frac{x^5}{5} - x^4 + \frac{4x^3}{3}\right]_{x=1}^{x=2}$	A1A1		
	$= \left(\frac{1}{10} + \frac{1}{6}\right) + \left(\left[\frac{32}{5} - 16 + \frac{32}{3}\right] - \left[\frac{1}{5} - 1 + \frac{4}{3}\right]\right)$	m1		dep(M1)
	$= \frac{4}{5}$	A1	5	AG
(d)(i)	$\mathrm{E}(X) = \frac{19}{24}$ and $k\mathrm{Var}(X) = 499$ $\mathrm{Var}(X) = \mathrm{E}\left(X^2\right) - \mathrm{E}^2(X)$ $= \frac{4}{5} - \left(\frac{19}{24}\right)^2$	M1		
	$= \frac{499}{2880}$ (0.173)	A1		
	$\Rightarrow \qquad k = 2880$	A1	3	CAO

MS2B (cont)

Q	Solution	Mark	Total	Comments
8(d)(ii)	$\mathrm{E}\left(5X^2+24X-3\right)$			
	$=5\mathrm{E}\left(X^2\right)+24\mathrm{E}\left(X\right)-3$	M1		
	$=5\times\frac{4}{5}+24\times\frac{19}{24}-3$			
	$=20$	A1	2	CAO
(iii)	$\mathrm{Var}\left(12X-5\right)=144\mathrm{Var}\left(X\right)$	M1		
	$=144\times\frac{499}{2880}$			
	$=\frac{499}{20}$ or (24.95)	A1	2	CAO (AWFW 24.9 to 25)
	Total		**18**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education
June 2010

Mathematics MS2B

Statistics 2B

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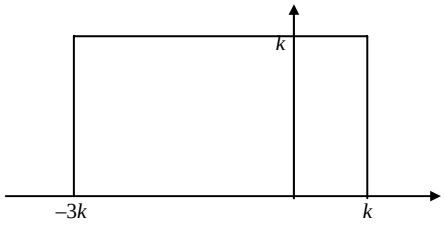
MS2B

Q	Solution	Marks	Total	Comments
1	$\bar{x} = 82 \ ; \ s^2 = 31.1 \ (s = 5.58)$	B1B1		
	Assumption: The number of customers served daily at the post office counter forms a Normal distribution.	B1		
	$H_0: \mu = 79$ $H_1: \mu > 79$	B1		
	$t = \dfrac{82 - 79}{5.58/\sqrt{12}}$	M1		$\dfrac{\text{their } \bar{x} - 79}{\text{their } s/\sqrt{12}}$
	$t = 1.86$	A1		(AWRT)
	$\nu = 11 \ \Rightarrow \ t_{\text{crit}} = 1.796$	B1		
	Reject H_0	A1		
	Sufficient evidence at 5% level of significance to support Judith's belief.	E1	9	Iff $t_{\text{calc}} > t_{\text{crit}}$
	Total		**9**	
2				
	O_i | E_i | $(\lvert O_i - E_i \rvert - 0.5)$ (α) | α^2/E_i 24 | 28 | 3.5 | 0.4375 56 | 52 | 3.5 | 0.2356 11 | 7 | 3.5 | 1.7500 9 | 13 | 3.5 | 0.9423 | | | 3.3654	M1 M1 M1 A1		*E* attempted Yates' correction attempted χ^2 attempted AWFW 3.36 to 3.37
	H_0: No association between drug and prevention of sickness	B1		(at least H_0 stated correctly)
	H_1: Association between drug and prevention of sickness			
	$\chi^2_{5\%} = 3.841$	B1		CAO
	Accept H_0	A1ft		
	No evidence at the 5% level of significance to support the claim that the drug is effective against sickness.	E1ft	8	
	Total		**8**	

MS2B

Q	Solution	Marks	Total	Comments
3(a)(i)		B2,1	2	Horizontal line $f(x)=k$ From $-3k$ to k If $\frac{1}{2}$ then max. B1
(ii)	Area $=4k\times k=1$ $k^2=\frac{1}{4}$ $k=\frac{1}{2}\quad (k>0)$	M1 A1	2	**SC** If use $k=\frac{1}{2}$ to show that the Area = 1 then $\Rightarrow$B1 **AG**
(b)	$E(X)=\frac{1}{2}(-3k+k)$ $=-k$ $=-\frac{1}{2}$	B1		CAO
	$\text{Var}(X)=\frac{1}{12}(k--3k)^2=\frac{16k^2}{12}=\frac{4k^2}{3}$ $=\frac{1}{3}$	M1		CAO
	st. dev$(X)=\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ or $\sqrt{\frac{1}{3}}$	A1	3	OE (**exact**)
(c)(i)	$P\left(X\geq-\frac{1}{4}\right)=\frac{1}{2}\times\frac{3}{4}$	M1		
	$=\frac{3}{8}\quad (0.375)$	A1	2	
(ii)	$P\left(X\neq-\frac{1}{4}\right)=1$	B1	1	
	Total		**10**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
4				
	$\bar{x} = \frac{0.35}{10} = 0.035$ $s^2 = \frac{0.12705}{9} = 0.0141$ or $(s = 0.1188)$	B1		both
	$t_{\text{crit}} = 3.250$	B1		
	99% CI for μ:			
	$0.035 \pm 3.25 \times \frac{\sqrt{0.0141}}{\sqrt{10}}$ 0.035 ± 0.1221	M1 A1		$0.035 \pm 3.25 \times \frac{\sqrt{0.012705}}{\sqrt{9}}$ Iff $\bar{x}, s$ and $t_9 = 3.25$ all correct in expression
	$(-0.087, 0.157)$	A1	5	CAO (3dp only)
	Total		**5**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$X \sim \text{Po}(7)$ $\text{P}(X \leq 5) = 0.301$	B1	1	AWFW 0.300 and 0.301
(ii)	$\text{P}(X = 7) = \dfrac{\text{e}^{-7} \times 7^7}{7!}$ $= 0.149$	M1 A1	2	$\text{P}(X \leq 7) - \text{P}(X \leq 6)$ $= 0.5987 - 0.4497$ (M1) $= 0.149$ (A1)
(iii)	$0.65 \leq p \leq 0.66$	B3		$\text{P}(X \leq 9) - \text{P}(X \leq 4)$
	$0.72 \leq p \leq 0.73$ **or** $0.52 \leq p \leq 0.53$	(B2)		$\begin{cases}\text{P}(X \leq 10) - \text{P}(X \leq 4) \\ \text{P}(X \leq 9) - \text{P}(X \leq 5)\end{cases}$
	0.60	(B1)	3	$\text{P}(X \leq 10) - \text{P}(X \leq 5)$
(b)	No. telephone calls received per hour $= Y \sim \text{P}_0(0.875)$	B1	1	
(c)(i)	Maximum number = 4	B1	1	
(ii)	$\text{P}(Y < 4) = \text{P}(Y = 0,1,2,3)$ $= \text{e}^{-0.875}\left(1 + \dfrac{7}{8} + \dfrac{49}{128} + \dfrac{343}{3072}\right)$ $= 0.4169(1 + 0.875 + 0.3828 + 0.1117)$ $= 0.987740443$	B2		Any correct expression (B2) **or** AWFW 0.987 to 0.988
	$\text{P}(Y \geq 4) = 1 - 0.9877$ $= 0.0123$	M1 A1	4	$1 - (\text{their P}(Y < 4))$ AWFW 0.0122 and 0.0123
				SC $\text{P}(Y \leq 4) = 0.997$ to 0.998 **or** any correct expression B2 $\text{P}(Y > 4) = 0.002$ to 0.003 M1A0
(d)	λ probably not constant The number of calls in any time interval of 1 hour is likely to vary throughout the day.	E1	1	'System Down' $\Rightarrow$ not independent
	Total		**13**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\mathrm{P}(R \geq 5) = 0.3 + 0.25 + 0.1 + 0.05$ $= 0.70$	B1	1	CAO
(ii)	$\mathrm{E}(R) = \sum rp$ $= 3\times0.1 + 4\times0.2 + 5\times0.3 +$ $6\times0.25 + 7\times0.1 + 8\times0.05$	M1		
	$= 5.2$	A1	2	
(iii)	$\mathrm{E}(R^2) = 9\times0.1 + 16\times0.2$ $+ 25\times0.3 + 36\times0.25$ $+ 49\times0.1 + 64\times0.05$	M1A1		(Correct expression **or** 28.7)
	$(= 28.7)$			
	$\mathrm{Var}(R) = 28.7 - 5.2^2$	M1		
	$= 1.66$	A1	4	**AG**
(b)(i)	$\mathrm{P}(R+S=6) = 0.1\times0.15 = 0.015$	B1		$\mathrm{P}(R=3 \text{ and } S\leq5) = 0.1\times0.85 = 0.085$
	$\mathrm{P}(R+S=7) = 0.1\times0.4 + 0.2\times0.15$ $= 0.04 + 0.03$ $= 0.07$	B1		$\mathrm{P}(R=4 \text{ and } S\leq4) = 0.2\times0.55 = 0.110$
	$\mathrm{P}(R+S=8) = 0.2\times0.4 + 0.1\times0.3$ $+ 0.3\times0.15$ $= 0.08 + 0.03 + 0.045$ $= 0.155$	B1		$\mathrm{P}(R=5 \text{ and } S\leq3) = 0.3\times0.15 = 0.045$
	$\mathrm{P}(R+S\leq8) = 0.015 + 0.07 + 0.155$	M1		$\mathrm{P}(R+S\leq8) = 0.085 + 0.110 + 0.045$
	$= 0.24$	A1	5	$=0.24$ **(AG)**
(ii)	$p = {}^5C_4(0.24)^4(0.76)$ $+ (0.24)^5$ $= 0.0126 + 0.000796$	M1 M1		First term correct + correct second term **or** correct numerical values (**must** use $p = 0.24$)
	$= 0.0134$	A1	3	$(0.013 \text{ to } 0.0135)$
(iii)				**Alternative:** (using (b)(i))
	$\mathrm{P}(R=4 \mid R+S\leq8)$ $= \dfrac{\mathrm{P}(R=4 \text{ and } R+S\leq8)}{\mathrm{P}(R+S\leq8)}$			$= \dfrac{\mathrm{P}(R=4 \text{ and } S\leq4)}{\mathrm{P}(R+S\leq8)}$ $= \dfrac{0.11}{0.24} = \dfrac{11}{24}$
	$= \dfrac{0.03+0.08}{0.24}$	B1 M1		(numerator) **or** 0.11 seen ($\div 24$ iff numerator < 0.24)
	$= \dfrac{11}{24}$ (0.458)	A1	3	CAO
	Total		**18**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
7(a)	Median $=1$	B1		
	Lower quartile $=\frac{1}{2}$	B1	2	
(b)	$\mathrm{F}(1)=\frac{1}{2}$			
	For $1\le x\le 4$			
	$\int\frac{1}{18}(x-4)^2\mathrm{d}x$	M1		ignore limits
	$=\left[\frac{1}{54}(x-4)^3\right]_1^x$ $=\left[\frac{1}{54}(x-4)^3+\frac{1}{2}\right]$	A1		Correct integration + correct limits seen or used
	$\mathrm{F}(x)=\left[\frac{1}{54}(x-4)^3+\frac{1}{2}\right]+\frac{1}{2}$	m1		adding $\frac{1}{2}$ **or** $\mathrm{F}(1)$
	$=1+\frac{1}{54}(x-4)^3$	A1	4	CAO (AG)
	Alternative			**Alternative**
	$\int\frac{1}{18}(x-4)^2\mathrm{d}x=\frac{1}{54}(x-4)^3+c$	(M1)		$\int\frac{1}{18}(x-4)^2\mathrm{d}x$ (M1)
	$\mathrm{F}(1)=\frac{1}{2}\ \Rightarrow\ c=1$	(m1) (A1)		$=\int_1^x\frac{1}{18}\left(x^2-8x+16\right)\mathrm{d}x$
	$\mathrm{F}(x)=1+\frac{1}{54}(x-4)^3$	(A1)		$=\frac{1}{18}\left[\frac{x^3}{3}-4x^2+16x\right]_1^x$ (A1) $\mathrm{F}(x)=\frac{1}{2}+\frac{1}{54}\left[x^3-12x^2+48x\right]_1^x$ (m1) $=\frac{1}{2}+\frac{1}{54}\left(x^3-12x^2+48x-37\right)$ $=1+\frac{1}{54}\left(x^3-12x^2+48x-64\right)$ $=1+\frac{1}{54}(x-4)^3$ (A1)
(c)	$\mathrm{P}(2\le X\le 3)=\frac{53}{54}-\frac{46}{54}$	M1		$\mathrm{F}(3)-\mathrm{F}(2)$
	$=\frac{7}{54}$ (0.130)	A1	2	$0.1\dot{2}9\dot{6}$
(d)(i)	$\mathrm{F}(q)=\frac{3}{4}$	M1		**use** of $\mathrm{F}(q)=\frac{3}{4}$
	$1+\frac{1}{54}(q-4)^3=\frac{3}{4}$ $\frac{1}{54}(q-4)^3=-\frac{1}{4}$	M1		(either)
	$(\times 54)\ \Rightarrow\ (q-4)^3=-13.5$	A1	3	**AG**
(ii)	$q-4=\sqrt[3]{-13.5}=-2.3811$ $q=1.619$ (3dp)	B1	1	CAO
	Total		**12**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
2011

Mathematics MS2B

(Specification 6360)

Statistics 2B

Mark Scheme

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Further copies of this Mark Scheme are available from: aqa.org.uk

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

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Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Question	Solution	Marks	Total	Comments
1(a)	$Y \sim \mathrm{N}(\mu_Y, 4)$ $n = 16, \bar{y} = 450$ (known variance) $\Rightarrow$ use z			
	For 95% CI $z_{crit} = 1.96$	B1		
	$450 \pm 1.96 \times \frac{2}{\sqrt{16}}$ 450 ± 0.98	M1		
	$(449, 451)$	A1	3	awrt
(b)(i)	$X \sim \mathrm{N}(\mu_X, \sigma^2)$ (unknown variance) $\Rightarrow$ use t_{n-1}			
	$n = 9$ & $\bar{x} = \frac{4950}{9} = 550$ $s_{n-1}^2 = \frac{334}{8} = 41.75$ $(s_{n-1} = 6.461)$	B1		both
	For 90% CI $t_{crit} = 1.860$	B1		
	$550 \pm 1.860 \times \frac{\sqrt{41.75}}{\sqrt{9}}$	M1		their $\bar{x} \pm t_8 \times \frac{\text{their } s_{n-1}}{\sqrt{9}}$ (must have a t_8-value)
	550 ± 4.0	A1ft		
	$(546, 554)$	A1	5	awrt
(ii)	545 not in 90% CI $\therefore$ Reject claim Evidence to suggest that **mean** content of a bottle of tomato sauce is not 545 grams and hence to reject Holly's claim at **10% level of significance**.	B1ft (dep on (b)(i)) E1ft (dep on (b)(i))		**Alternative** $H_0: \mu_X = 545$ $H_1: \mu_X \neq 545$ $t = \frac{550 - 545}{\sqrt{41.75/9}} = 2.32$ $t_{crit} = 1.86 < 2.32$, Reject H_0 **B1** Comment in context **E1**
		B1	3	**10% significance level** **B1**
	Alternatives (such as): Claim justified at 1% level of significance			$t_{crit} = 3.355 > 2.32$ Accept H_0 1% significance level
	Total		**11**	

MS2B(cont)

Question	Solution	Marks	Total	Comments
2(a)				
		B1		
		B1	2	For each correct row
(b)	H_0: No association between gender and the way students vote			
	H_1: Association between gender and the way students vote	B1		For at least H_0 correct
		M1		Attempt at E_i
		M1		Attempt at $(O_i - E_i)^2 / E_i$
		M1		Attempt at $\sum\left(\frac{(O_i - E_i)^2}{E_i}\right)$
		A1		awfw 10.2 to 10.3 (A1 dependent on all 3 method marks)
	$\nu = 3 \Rightarrow \chi^2_{crit} = 11.345$	B1		
		B1ft		ft on their ν value
	$X^2 < \chi^2_{crit}$ ∴ accept H_0	A1		
	Accept claim at 1% level. Evidence to suggest that the way students vote is independent of gender.	E1	9	
	Total		**11**	

O_i	E_i	$(O_i - E_i)^2 / E_i$
156	175.06	2.075
216	196.94	1.844
144	131.29	1.230
135	147.71	1.093
120	107.29	1.505
108	120.71	1.337
60	66.35	0.608
81	74.65	0.541
		$X^2 = 10.233$

	C	L	LD	OP	Tot
M	156	144	120	60	480
F	216	135	108	81	540
Tot	372	279	228	141	1020

MS2B(cont)

Question	Solution	Marks	Total	Comments
3(a)(i)	$X \sim \mathrm{P}_{\mathrm{o}}(0.6)$ $\mathrm{P}(X \leq 1) = 0.8781$	B1	1	Awrt 0.878
(ii)	For matches : The number of run outs: $Y \sim \mathrm{P}_0(0.15)$ $\mathrm{P}(Y \geq 1) = 1 - \mathrm{P}(Y = 0)$			
	$= 1 - e^{-0.15}$ $= 1 - 0.8607$	M1		must **use** $\mathrm{P}_0(0.15)$
	$= 0.1393$	A1		awrt 0.139
	$\mathrm{P}(X \leq 1 \text{ and } Y \geq 1) = 0.8781 \times 0.1393$ $= 0.122$	M1 A1	4	their (a)(i) × their $\mathrm{P}(Y \geq 1)$ awrt
(b)	X and Y are independent. Number of catches and runouts independent	B1	1	
(c)(i)	For Season: $S \sim \mathrm{P}_{\mathrm{o}}(9.6)$ $\mathrm{P}(S = 10) = \dfrac{e^{-9.6} \times 9.6^{10}}{10!}$	M1		Use of $\lambda = 9.6$ in correct Poisson expression
	$= 0.124$	A1	2	
(ii)	$T \sim \mathrm{P}_{\mathrm{o}}(9.6 + 2.4) = \mathrm{P}_{\mathrm{o}}(12)$	B1		$\mathrm{P}_{\mathrm{o}}(12)$ used or seen
	$\mathrm{P}(T \geq 15) = 1 - \mathrm{P}(T \leq 14)$ $= 1 - 0.7720$ $= 0.228$	B2,1	3	(1-0.8444 = 0.155 to 0.156) **B1**
	Total		**11**	

MS2B(cont)

Question	Solution	Marks	Total	Comments
4(a)(i)	$\mathrm{E}(X)=\sum_x x\times \mathrm{P}(X=x)=2.8$ $\mathrm{E}(X^2)=\sum_x x^2\times \mathrm{P}(X=x)=9$	B1		
	$\mathrm{Var}(X)=9-2.8^2$	M1		$\left(\text{their } \mathrm{E}(X^2)-\text{their } \mathrm{E}^2(X)\right)$
	$=1.16$	A1	**3**	cao
(ii)	$\mathrm{E}(S)=3\times \mathrm{E}(X)=8.4$	B1ft		on their $\mathrm{E}(X)$
	$\mathrm{Var}(S)=3\times \mathrm{Var}(X)=3.48$	B1ft	**2**	$3\times$ their $\mathrm{Var}(X)$ from (i) >0 **NB** There was a problem with part 4(a)(ii) which affected the marking of this part. Please see the Report on the Examination for details.
(b)	$\mathrm{E}(Y)=3.5$ $\mathrm{E}(Y^2)=13$	B1		for E(Y)
	$\mathrm{Var}(Y)=13-3.5^2=0.75$	M1 A1ft		on their $\mathrm{E}(Y)$ and $\mathrm{E}(Y^2)$ Var (Y) > 0
	$\mathrm{E}(T)=3\times \mathrm{E}(Y)=10.5$	B1		cao
	$\mathrm{Var}(T)=3^2\times \mathrm{Var}(Y)$ $=9\times 0.75$	M1		$9\times$ their $\mathrm{Var}(Y)>0$
	$=6.75$	A1		cao
	Alternative:			
	T: 3, 6, 9, 12 T^2: 9, 36, 81, 144 P: 1/20, 2/20, 3/20, 14/20 E(T): 3/20, 12/20, 27/20, 168/20 → 10.5 E(T^2): 9/20, 72/20, 243/20, 2016/20 → 117	(M1A1) (M1A1)		
	$\mathrm{Var}(T)=\mathrm{E}(T^2)-\left[\mathrm{E}(T)\right]^2$	(M1)		(used)
	$=117-10.5^2$ $=6.75$	(A1)	**6**	

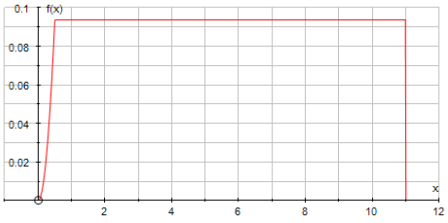
MS2B(cont)

Question	Solution	Marks	Total	Comments
4(c)(i)	$\mathrm{P}(X>1)=0.8$	B1	1	
(ii)	$\mathrm{P}(T=3)=\frac{1}{20}$ **and** $\mathrm{P}(T=3 \text{ or } 6)=\frac{3}{20}$ $\mathrm{P}(X+T\le 9 \text{ and } X>1)$ $=\mathrm{P}([2,3 \text{ or } 6],[3,3 \text{ or } 6],[4,3])$ $=0.1\times\frac{3}{20}+0.4\times\frac{3}{20}+0.3\times\frac{1}{20}$ $=0.015+0.06+0.015$ $=0.09$	B4	4	**Alternative:** $\mathrm{P}(T=3)=\frac{1}{20}$ and $\mathrm{P}(T=6)=\frac{2}{20}$ $\mathrm{P}(X+T\le 9 \text{ and } X>1)$ $=\mathrm{P}([(2,3,4),3],[(2,3),6])$ $=0.8\times\frac{1}{20}+0.5\times\frac{2}{20}$ $=0.04+0.05=0.09$ **sc** [any 4 correct p's from table B**2**] [0.12 **B3]** **[**0.096 **or** 0.072 **B2]** $\begin{cases}\mathrm{P}(T=3)=\frac{1}{20} \text{ **and** } \mathrm{P}(T=3 \text{ or } 6)=\frac{3}{20}\\ \mathrm{P}(T=3)=\frac{1}{20} \text{ **and** } \mathrm{P}(T=6)=\frac{2}{20}\end{cases}$ **B1**
(iii)	$\mathrm{P}(X+T\le 9 \mid X>1)=\frac{0.09}{0.80}$ $=\frac{9}{80}$ (0.1125)	M1 A1	2	$\frac{\text{their (c)(ii)}}{0.80}$ $(0<p<1)$ cao
	Total		**18**	

MS2B(cont)

Question	Solution	Marks	Total	Comments
5(a)(i)	$H_0: \mu = 165$ $H_1: \mu > 165$	B1	1	
(ii)				
	David (5%) | James (1%)			
	$z = \frac{167.1 - 165}{\sqrt{101.2}/10}$ $= 2.09$	M1 A1		awfw 2.08 to 2.09
	$z_{crit} = 1.6449$ $(t_{crit} = 1.660)$ | $z_{crit} = 2.3263$ $(t_{crit} = 2.364)$	B1		(both)
	Reject H_0 | Accept H_0	A1		(both) dependent on M1
	Evidence to suggest that the **mean** height of students in final year has **increased** at 5% level | No evidence to suggest an **increase** in the **mean** height of final year students at 1% level	E1 E1	6	
(iii)	**Population not stated as being Normal / not known.** Heights of all students may not be Normal/ Known	B1	1	Large sample size of 100 indicates that the distribution of the sample mean is very likely to be Normal even though **the parent population not given as being Normal**. Hence $\bar{X} \sim N\left(\mu, s^2/n\right)$
(b)(i)	David: $\mu = 165$ $\therefore$ rejected H_0 when H_0 correct $\Rightarrow$ Type I error	M1 A1		
(ii)	James: $\mu = 165$ $\therefore$ accepted H_0 when H_0 correct $\Rightarrow$ No error	M1 A1	4	
	Total		**12**	

MS2B(cont)

Question	Solution	Marks	Total	Comments
6(a)		B3	3	B1 for concave curve from (0 , 1) to $\left(\frac{1}{2},\frac{3}{32}\right)$ B1 for horizontal straight line $f=\frac{3}{32}$ from $\left(\frac{1}{2},\frac{3}{32}\right)$ to $\left(\frac{1}{2},\frac{3}{32}\right)$ B1 for correct axes
(b)(i)	$P\left(X\geq 8\frac{1}{3}\right)=\left[\frac{3}{32}\times\left(11-8\frac{1}{3}\right)\right]$ $=\frac{3}{32}\times\frac{8}{3}$ $=\frac{1}{4}$	M1 A1		Any correct method attempted in either part AG
(ii)	$P(X\geq 3)=\frac{3}{32}\times(11-3)$ $=\frac{3}{4}$	A1	3	Any correct method attempted AG
(c)(i)	Interquartile Range = $5\frac{1}{3}$	B1		cao
(ii)	Median = $5\frac{2}{3}$ **Alternative** : $\frac{1}{64}+\frac{3}{32}\left(m-\frac{1}{2}\right)=\frac{1}{2}$ $\Rightarrow 3\left(m-\frac{1}{2}\right)=15.5\Rightarrow m=5\frac{2}{3}$	B2	3	cao **sc** if B0 then: M1 for correct method seen $\frac{1}{2}\left(8\frac{1}{3}+3\right)$ **or** $\frac{1}{2}\times 11\frac{1}{3}$ **or** $\frac{3}{32}(11-m)=\frac{1}{2}\Rightarrow 11-5\frac{1}{3}$
(d)	$P\left[(X<m)\cap(X\geq 3)\right]=\frac{1}{4}$ $P(X<m\mid X\geq 3)=\frac{1/4}{3/4}=\frac{1}{3}$	B1 M1 A1	3	$\left(\frac{3}{4}-\frac{1}{2}\right)$ **attempted** $(\text{their } p)/_{3/4}$ for $0<p<1$ **cao** **Alternative:** (Ratio of relevant two areas) $P(X<m\mid X\geq 3)=\frac{2\frac{2}{3}}{8}=\frac{1}{3}$ cao
	Total		**12**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
June 2011

Mathematics MS2B

(Specification 6360)

Statistics 2B

Final

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Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Q	Solution	Marks	Total	Comments
1(a)(i)	$X \sim \text{Po}(13)$	B1	1	Both Poisson and $\lambda = 13$
(ii)	$\text{P}(X = 20) = \text{P}(X \leq 20) - \text{P}(X \leq 19)$ $= 0.975(0) - 0.957(3)$ [allow $0.975 - 0.957$]	M1		Must use $\lambda = 13$ otherwise M0A0
	$= 0.0177$ (3sf)	A1	2	AWFW 0.0176 to 0.018 **or** $\text{P}(X = 20) = \frac{e^{-13} \times 13^{20}}{20!}$ M1 $= 0.0177$ A1
(iii)	$\text{P}(6 \leq X \leq 18) = \text{P}(X \leq 18) - \text{P}(X \leq 5)$ $= 0.930(2)$	M1		
	$-(0.0107$ **or** $0.0259)$	M1		
	$= 0.920$ (3sf)	A1	3	AWFW 0.919 to 0.92
(b)	Cars not random Cars not independent Mean and Variance of cars different / not equal	B1		Allow (number of) cars not random / not independent B1 for any one of these 3 statements Must indicate a reference to ***cars***
	Mean / Average / λ / 2.6			Correct comment about value of $\lambda \neq 2.6$
	greater / less / smaller / different / variable / not constant / too small / too large			Any combination (one from each group):
	Any contextual reason that suggests a change in traffic flow, eg due to: rush hour / congestion / traffic jams / accidents / work traffic / school traffic / peak time	B1	2	eg mean greater *due to* rush hour, **or** λ smaller *due to* congestion, **or** 2.6 too small *due to* school traffic
(c)	$Y \sim \text{Bin}(20, 0.2)$ $\text{P}(Y \geq 5) = 1 - \text{P}(Y \leq 4)$			**or:** $1 - (0.01153 + 0.05765 + 0.13691 + 0.20536 + 0.21820)$
	$= 1 - 0.6296$	M1		$1 - 0.6296$ (Allow $1 - 0.8042$ seen for M1)
	$= 0.37(0)$ (3sf)	A1	2	AWFW 0.37 to 0.3704
(d)	X and Y independent	B1		Any statement which indicates two / both events are independent
	$p = 0.0177 \times 0.3704$	M1		[their (c)] $\times$ [their (a)(ii)]
	$= 0.00656$ (3sf)	A1	3	AWFW 0.0065 and 0.0067
			13	

MS2B (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	Area / $F(x) = 10u \times 0.01\pi$ (OE)	B1		Shown by any correct method
	$= 1 \Rightarrow u = \dfrac{10}{\pi}$	Bdep1	2	**Alternatives:** $f = \dfrac{1}{10u}$ B1
	or			
	$u = \dfrac{10}{\pi} \Rightarrow F(x) = 1$	(Bdep1)		Show $u = \dfrac{10}{\pi}$ **or** show $\dfrac{1}{10u} = 0.01\pi$ Bdep1
(ii)	$E(X) = \dfrac{1}{2}(11u + u) = 6u = 6 \times \dfrac{10}{\pi} = \dfrac{60}{\pi}$	B1	1	Must be in terms of π (eg $60\pi^{-1}$)
	$\text{Var}(X) = \dfrac{1}{12}(b-a)^2$			
	$\text{Var}(X) = \dfrac{1}{12}(11u-u)^2$			**Alternatives:**
	$= \dfrac{1}{12} \times 100 \times \dfrac{100}{\pi^2} = \dfrac{100^2}{12\pi^2}$	B1	1	$\dfrac{10000}{12\pi^2} = \dfrac{5000}{6\pi^2} = \dfrac{2500}{3\pi^2} = \left(\dfrac{50}{\pi\sqrt{3}}\right)^2 = \dfrac{(\text{AWRT } 833)}{\pi^2}$ Must be in terms of π
(iii)	$C = \pi\left(X + \dfrac{10}{\pi}\right)$			
	$E(C) = \begin{cases} \pi \times [\text{their } E(X)] + 10 \\ \pi \times \dfrac{60}{\pi} + 10 \end{cases}$	M1		Their **numerical** value of $E(X)$ used correctly Must have a multiplier of π or 2π
	$= 70$	A1		CAO
	$\text{Var}(C) = \pi^2 \times \dfrac{100^2}{12\pi^2} = \dfrac{100^2}{12}$	M1		$\pi^2 \times [\text{their Var}(X) > 0]$ Must have a multiplier of π^2 or $4\pi^2$
	$= 833\dfrac{1}{3}$ $(833.\dot{3})$	A1	4	**Alternatives:** $\dfrac{10000}{12} = \dfrac{5000}{6} = \dfrac{2500}{3}$ Must be exact: 833.3 gets A0
(b)	$n = 100$ and $\bar{a} = 40.5$			
	95% CI for $\mu = \begin{cases} 40.5 \pm z \times \dfrac{\sqrt{25}}{\sqrt{100}} \\ 40.5 \pm 1.0 \end{cases}$	B1		For $z = 1.96$
		M1		$z = 1.96$ or 1.64 to 1.65 only
	$= (39.5, 41.5)$	A1	3	AWRT
	Total		**11**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
3(a)	H_0: no association (between type of school and performance of 16 year olds in their GCSEs)	B1	1	H_0: type of school and performance of 16 year olds in their GCSEs independent
(b)	$\frac{(O-E)^2}{E}$ 0.195819311 0.482160711 0.003569447 1.080536181 0.062507172 1.269422099 0.785491128 0.183802623 0.541856652 0.044011976 3.274102564 4.096492891	M1		Attempt at $\frac{(O-E)^2}{E}$ (≥ 4 correct values seen to 2dp)
	$X^2 = \sum \frac{(O-E)^2}{E}$	m1		Attempt to add ≥ 8 terms
	$= 12.01977275$ $= 12.0$ (1dp)	A1	3	Allow $11.9 \leq X^2 \leq 12.1 \Rightarrow$ M1m1 CAO
(c)	$\nu = 6 \Rightarrow \chi^2_{1\%} = 16.8(12)$	B1,B1		$\nu = 6$ can be implied by $\chi^2_{1\%} = 16.8(12)$
	No (significant evidence to suggest an) **association between** (type of) **school and** (GCSE) **performance** (of 16 year olds)	Adep1	3	Insufficient/no evidence to support Emily's belief. School and performance are independent. Correct conclusion **in context** Dep on B1M1m1B1B1 given in (a), (b), (c) **and** $11.9 \leq X^2 \leq 12.1$
(d)	More than expected gained at least / more than 5 GCSEs Fewer than expected gained at least / more than 1 GCSE but less than 5 GCSEs Fewer than expected gained no GCSEs	B1	1	Since conclusion of ***no association*** between school and GCSE performance, it may be misleading to look at individual differences in any great detail Any one of these 4 comments seen
(e)	$\chi^2_{10\%} = 10.6(45)$	B1		Correct value of χ^2 only
	Reject H_0 at 10% level of significance. (Evidence to suggest) an **association between** (type of) **school and** (GCSE) **performance**	Bdep1	2	Evidence to support Emily's (Joanne's) belief. (Type of) school + (GCSE) performance dependent. Dep on B1M1m1 **and** $11.9 \leq X^2 \leq 12.1$ **and** B1 in (e)
	Total		**10**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\mathrm{E}(X)=\sum xp$ $=\frac{3}{40}+\left(2\times\frac{6}{40}\right)+\left(3\times\frac{9}{40}\right)+\left(4\times\frac{12}{40}\right)+\left(5\times\frac{5}{20}\right)=3.5$	B2,1	2	
(b)(i)	$\mathrm{E}\left(\frac{1}{X}\right)=\sum\frac{1}{x}\times p$ $=\left(1\times\frac{3}{40}\right)+\left(\frac{1}{2}\times\frac{6}{40}\right)+\left(\frac{1}{3}\times\frac{9}{40}\right)+\left(\frac{1}{4}\times\frac{12}{40}\right)+\left(\frac{1}{5}\times\frac{5}{20}\right)$	M1		At least 4 of these terms added (accept decimal equivalents)
	$=\frac{7}{20}$	A1	2	AG (allow 0.35 seen)
(ii)	$\mathrm{E}\left(\frac{1}{X^2}\right)=\sum\frac{1}{x^2}\times p$ $=\left(1\times\frac{3}{40}\right)+\left(\frac{1}{4}\times\frac{6}{40}\right)+\left(\frac{1}{9}\times\frac{9}{40}\right)+\left(\frac{1}{16}\times\frac{12}{40}\right)+\left(\frac{1}{25}\times\frac{5}{20}\right)$	M1		At least 4 of these terms added (accept decimal equivalents) (can be
	$=\frac{133}{800}\ (0.16625)$	A1		implied by $\frac{133}{800}$ seen with no other working shown)
	$\mathrm{Var}\left(\frac{1}{X}\right)=\frac{133}{800}-\frac{49}{400}$	m1		$\left[\text{their E}\left(\frac{1}{X^2}\right)\right]-\left(\frac{7}{20}\right)^2$
	$=\frac{7}{160}$	Adep1	4	AG (allow 0.04375 seen)
(c)(i)	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>40</td><td>20</td><td>13⅓</td><td>10</td><td>8</td></tr><tr><td>p</td><td>$\frac{3}{40}$</td><td>$\frac{6}{40}$</td><td>$\frac{9}{40}$</td><td>$\frac{12}{40}$</td><td>$\frac{10}{40}$</td></tr></table>			
	Identifying $X=(2),3,4,5$ **or** $Y=(20),13\frac{1}{3},10,8$	M1		**Alternative:** $Y<20\Rightarrow\frac{40}{X}<20\Rightarrow 40<20X\Rightarrow X>2$ M1 (allow < or ≤ **and** > or ≥ in above)
	$\mathrm{P}(X>2)=\frac{9}{40}+\frac{12}{40}+\frac{5}{20}$ $=\mathrm{P}(Y<20)$	A1		$\mathrm{P}(Y<20)=\mathrm{P}(X>2)$ $=1-\left(\frac{3}{40}+\frac{6}{40}\right)$ A1
	$=\frac{31}{40}\ (0.775)$	A1	3	$=\frac{31}{40}\ (0.775)$ A1
(ii)	$\frac{9}{40}$ seen irrespective of labelling	B1		As numerator **or** final answer (0.225)
	$\mathrm{P}(X<4\mid Y<20)=\frac{\frac{9}{40}}{\frac{31}{40}}=\frac{0.225}{0.775}$	M1		$=\frac{\frac{9}{40}}{(\text{their (c)(i)})}$ (or correct use of table)
	$=\frac{9}{31}(0.290)$	A1	3	AWFW 0.29 to 0.2904
	Total		**14**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
5(a)	$Y \sim \mathrm{N}(\mu_y, 640^2)$			
	$n = 25$ and $\bar{y} = 19700$			
	$\mathrm{H}_0: \mu_y = 20000$			
	$\mathrm{H}_1: \mu_y \neq 20000$ (both)	B1		**Alternative:** $\mathrm{P}(\bar{Y} < 19700) = \mathrm{P}(Z < -2.34375)$
	$\bar{Y} \sim \mathrm{N}\left(20000, \frac{640^2}{25}\right)$			$= 1 - 0.99036$
				$= 0.00964 \geq 0.005$ Accept H_0
	$z = \frac{19700 - 20000}{640/\sqrt{25}}$	M1		$(-2.35$ to $-2.34)$
	$= -2.34375$	A1		$(\pm 2.57$ to $\pm 2.58)$
	$z_{\text{crit}} = \pm 2.5758$	B1		Use of t $\Rightarrow$ max B1M1A1
	Accept H_0	Adep1		dep on B1M1B1
	Insufficient / **no evidence** (to suggest) that the **mean** (lifetime) has **changed** (from 20000 hours)	Edep1	6	dep on Adep1
	Mean (lifetime) has **not** changed at 1% level (of significance)			If incorrect hypotheses then B0 $\Rightarrow$ max M1A1B1 ie final Adep1Edep1 not available
(b)(i)	$\mu < 10000$	B1	1	
(ii)	$n = 16$ and $s = 500$; $t_{\text{crit}} = 1.753$	B1		For t_{crit} (ignore signs)
	$\mathrm{sd}(\bar{X}) = \frac{500}{\sqrt{16}}$ (125)	B1		Ignore notation
	Critical value is one of: $10000 \pm 1.753 \times \frac{500}{\sqrt{16}}$ (considered)	M1		M0 if only considered upper value No ft on incorrect t value
	Choose 9780 (3sf)	A1		AWFW 9780 to 9781 (ignore inequality)
	($\Rightarrow$ critical region: $\bar{x} < 9780$)			If z used then max B0B1M0A0A0
	$\therefore$ Range of values for $\bar{x}$ which leads Christine **not** to reject $\mathrm{H}_0: \mu = 10000$ is: $\bar{x} > 9780$	A1	5	Allow $\bar{x} \geq 9780$ to 9781
(iii)	No error	B1	1	Ignore any subsequent statements
	Total		**13**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\mathrm{F}(x)=\int \frac{3}{8}\left(x^2+1\right)\,\mathrm{d}x$	M1		Ignore limits
	$=\frac{3}{8}\left[\frac{x^3}{3}+x\right]$ or $=\frac{1}{8}x^3+\frac{3}{8}x$	A1		Either
	$=\frac{1}{8}x\left(x^2+3\right)$	A1	3	(including use of correct limits 0 and x **or** $+c$ used and evaluated) (AG)
(b)	$\mathrm{F}(m)=\frac{1}{2}$	B1		
	$\mathrm{F}(1)=\frac{1}{8}\times1\times4=\frac{1}{2}$	B1	2	AG
(c)	Upper quartile lies in range $1<x<2$			
	such that $\mathrm{F}(q)=\frac{3}{4}$			$\frac{1}{2}+\int_1^q \frac{1}{4}(5-2x)\,\mathrm{d}x=\frac{3}{4}$
	$\int_1^q \frac{1}{4}(5-2x)\,\mathrm{d}x=\frac{1}{4}$	M1		**Alternative:** $\int_q^2 \frac{1}{4}(5-2x)\,\mathrm{d}x=\frac{1}{4}$
	$\left[5x-x^2\right]_1^q=1$			$\left[5x-x^2\right]_q^2=1$
	$5q-q^2-4=1$			$(10-4)-\left(5q-q^2\right)=1$
				$6-5q+q^2=1$
	$q^2-5q+5=0$	A1		$q^2-5q+5=0$
	$q=\frac{5\pm\sqrt{25-20}}{2}$ or $\frac{1}{2}\left(5\pm\sqrt{5}\right)$	M1		Correct use of formula (OE) to give the two surd answers to given quadratic equation
	but $1<q<2$ [or $(q<2)$]	m1		
	$\therefore\quad q=\frac{1}{2}\left(5-\sqrt{5}\right)$	A1	5	Must qualify with a numerical comparison, not just quote the given answer; dep on M1; AG
(d)	$\mathrm{P}(X>1.5)=\frac{1}{2}\left[\frac{1}{2}+\frac{1}{4}\right]\times\frac{1}{2}$	M1		$\mathrm{P}(X<1.5)=0.5+\frac{1}{2}\left[\frac{3}{4}+\frac{1}{2}\right]\times\frac{1}{2}$ (M1)
	$=\frac{3}{16}$ (0.1875)	A1		$=\frac{1}{2}+\frac{1}{2}\times\frac{5}{4}\times\frac{1}{2}$
				$=\frac{1}{2}+\frac{5}{16}=\frac{13}{16}$ (A1)
	$\mathrm{P}(X>q)=\frac{1}{4}$ (0.25)	B1		$\mathrm{P}(X<q)=\frac{3}{4}$ (0.75) (B1)
	$\mathrm{P}(q<X<1.5)=\frac{1}{4}-\frac{3}{16}$			$\mathrm{P}(q<X<1.5)=\frac{13}{16}-\frac{3}{4}=\frac{1}{16}$ (A1)
	$=\frac{1}{16}$ (0.0625)	A1	4	(0.0625)

MS2B (cont)

Q	Solution	Marks	Total	Comments
6(d) cont	**OR** $\int_{1\frac{1}{2}}^{2} \frac{1}{4}(5-2x)\,dx = \frac{3}{16}$ etc (M1A1)			**OR** $\int_{q}^{1.5} \frac{1}{4}(5-2x)\,dx = \frac{1}{4}\left[5x-x^2\right]_q^{1.5}$ (M1) (correct integration and limits) Allow use of $q = 1.38$ to $q = 1.382$ in limits for M1 Whatever follows **must** be **exact**
				$= \frac{1}{4}\left[(7.5-2.25)-\left(5q-q^2\right)\right]$ (A1) for use of $5q-q^2=5$ **or** showing $5q-q^2=5$ by substituting $q=\frac{1}{2}\left(5-\sqrt{5}\right)$ (A1)
				$= \frac{1}{4}[5.25-5] = \frac{1}{16}$ (A1)
	NB statement $\mathrm{F}(1.5)-\frac{3}{4}=\frac{1}{16}$ (OE) scores 4 marks			
	Alternative: $\int_{q}^{1.5} \frac{1}{4}(5-2x)\,dx = \left[-\frac{1}{16}(5-2x)^2\right]_{\frac{5-\sqrt{5}}{2}}^{1.5}$ (M1) $= -\frac{1}{16}(4)-\left[-\frac{1}{16}\left(\sqrt{5}\right)^2\right]$ (sub) (A1) $= -\frac{4}{16}+\frac{5}{16}$ (A1) $= \frac{1}{16}$ (A1)			**Alternative using F(x):** for $1 \le x \le 2$ $\mathrm{F}(x) = \frac{1}{2}+\int_1^x \frac{1}{4}(5-2x)\,dx$ $= \frac{1}{2}+\frac{1}{4}\left[5x-x^2\right]_1^x$ $= \frac{1}{2}+\frac{1}{4}\left[\left(5x-x^2\right)-(5-1)\right]$ $= \frac{1}{4}\left(2+5x-x^2-4\right)$ $= \frac{1}{4}\left(5x-x^2-2\right)$ (seen or used) (M1) $\mathrm{F}(1.5) = \frac{1}{4}(7.5-2.25-2) = \frac{3.25}{4}$ $= 0.8125 = \frac{13}{16}$ (A1) $\mathrm{F}(q) = \frac{1}{16}\left(50-10\sqrt{5}-\left(25-10\sqrt{5}+5\right)-8\right)$ $= \frac{12}{16}$ OE (B1) $\mathrm{P}(q<X<1.5) = \frac{13}{16}-\frac{12}{16} = \frac{1}{16}$ (A1)
	Total		**14**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
January 2012

Mathematics MS2B

(Specification 6360)

Statistics 2B

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Question	Solution	Marks	Total	Comments
1(a)	21.05 and 21.15	B1	1	both (allow $21.04\dot{9}$ and $21.14\dot{9}$)
(b)	$\mathrm{E}(X)=0$ (symmetry)	B1		For $\mathrm{R}[-a,a]$: $\mathrm{E}(X)=0$ iff $a = 0.05,\ 0.1,\ 0.5$ then:
	$\mathrm{Var}(X)=\frac{1}{12}(0.05--0.05)^2=\frac{1}{12}\times\frac{1}{100}$	M1		$\mathrm{Var}(X)=\frac{1}{12}(a--a)^2$ **or** their a = 0.049 to 0.05 used for **M1**
	$\Rightarrow \quad sd(X)=\sqrt{\frac{1}{12}\times\frac{1}{100}}=\frac{1}{20\sqrt{3}}$	A1	3	or $\frac{\sqrt{3}}{60}$ or $\sqrt{\frac{1}{1200}}$ 0.0289 (3sf) A0
(c)	$\mathrm{P}(-0.01\le X\le 0.03)=0.04\times 10$ $=0.4$	B1	1	cao from correct value used $\int_{-0.01}^{0.03} 10\,dx=[10x]_{-0.01}^{0.03}=0.4$ oe
	Total		**5**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
2(a)(i)	$H_0: \mu = 61.4$ $H_1: \mu \neq 61.4$	B1		(both)
	$z_{calc} = \dfrac{65.0 - 61.4}{7.5/\sqrt{16}}$	M1		**Alternative:** $P(\bar{X} > 65.0) = P(Z > 1.92)$
	$= 1.92$	A1		$= 1 - 0.97257$ $= 0.02743$
	$z_{crit} = \pm 1.96$ **or** (shown in / implied by diagram)	B1		≥ 0.025 $\therefore$ Accept H_0 Use of $t \Rightarrow$ max(B1M1A1)
	Accept H_0	Adep1		dep(B1M1) but not A1B1
	Insufficient / No evidence (at 5% level) to suggest /show **mean** (age has) **changed** (from 61.4 years.)			If incorrect **or** no hypothesis then **B0** $\Rightarrow$ max(M1A1B1) i.e. final Adep1Edep1 **not** available
	Mean (age) has **not** *changed at 1% level* (of significance)	Edep1	6	dep(Adep1)
(ii)	$61.4 - 3 \times 7.5 = 38.9 > 25$ $\Rightarrow$ none under the age of 25 years. Very unlikely any members < 25 yrs.	B1	1	$z = \dfrac{25 - 61.4}{7.5} = -4.85$ $\Rightarrow P(Z < -4.85) \approx 0$ $\Rightarrow$ none aged under 25 included
(b)(i)	$\bar{y} = \dfrac{\sum y}{n} = \dfrac{702}{12} = 58.5$	B1		($s = 2.83$)
	$s^2 = \dfrac{\sum (y - \bar{y})^2}{n-1} = \dfrac{88.25}{11} = 8.02$	B1		($\sigma^2 = 7.35$ or $\sigma = 2.71$ iff $\sigma/\sqrt{11}$ used below)
	$t_{crit} = \pm 1.796$	B1		Ignore signs for t_{crit} If *z* used then max(B1B1B0M0A0)
	90% CI for μ: $58.5 \pm 1.796 \times \dfrac{s}{\sqrt{12}}$ 58.5 ± 1.4685 $= 57.03, 59.97$	M1		$(\text{their } \bar{y}) \pm t_{11} \times \dfrac{(\text{their } s)}{\sqrt{12}}$ **OR** $(\text{their } \bar{y}) \pm t_{11} \times \dfrac{(\text{their } \sigma)}{\sqrt{11}}$
	$= (57.0,\ 60.0)$	A1	5	
(ii)	upper limit < 61.4 $\Rightarrow$ recruitment drive lowered the average age of the club membership	B1ft	1	Must refer to 61.4 (on their CI)
	Total		**13**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
3(a)(i)	E_i: $\frac{mp}{N}$; $\frac{mq}{N}$; $\frac{np}{N}$; $\frac{nq}{N}$	B2,1	2	B1 any one correct B2 all correct (simplified)
(ii)	$\sum_i E_i = \frac{mp+mq+np+nq}{N}$ $= \frac{m(p+q)}{N} + \frac{n(p+q)}{N}$ (oe)	M1		$\sum_i E_i = \frac{mp+mq+np+nq}{N}$ $= \frac{m(p+q)+n(p+q)}{N}$ (or use of unsimplified forms)
	$= \frac{mN}{N} + \frac{nN}{N}$ $= m+n$ $= N$ (since p + q = m + n = N)	Mdep1 Adep1	3	$= \frac{(p+q)(m+n)}{N} = \frac{N \times N}{N} = N$ (AG)
(b)	H_0: No association between Andy's results and wind conditions	B1		
	E_i:			
	17.82 | 15.18 | 33 9.18 | 7.82 | 17 27 | 23 | 50	M1		Attempt E's
	$\Rightarrow \quad \lvert 0_i - E_i \rvert - 0.5 = 2.32$	M1		Yates' correction attempted
	$X^2 = 0.3020 + 0.3546 + 0.5863 + 0.6883$ = 1.93	M1 A1		Final column attempted awrt
	$\chi^2_{10\%}(1) = 2.706$	B1		correct value of χ^2 only (allow 2.71)
	$\Rightarrow$ Accept H_0	Adep1		dep (B1 for H_0)
	No association (between Andy's results and wind conditions)	Edep1	8	Appropriate conclusion dep(B1 for H_0 ; M1final column; $\chi^2_{10\%}$ = 2.706)
	Total		**13**	
(a)(ii)	An example of unsimplified values derived from $a = \frac{mp}{N}$: $\Rightarrow b = m - \frac{mp}{N}$; $c = p - \frac{mp}{N}$; $d = n - \frac{mp}{n}$ (oe)			

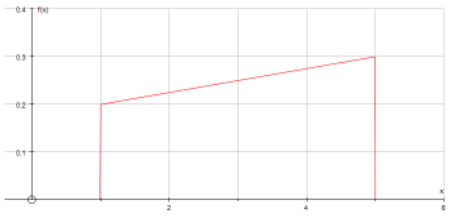
MS2B (cont)

Question	Solution	Marks	Total	Comments
4(a)(i)	Poisson	B1	1	
(ii)	$\mathrm{E}(3X-1)=3\lambda-1$	B1		
	$\mathrm{Var}(3X-1)=9\lambda$	B1	2	oe (allow $3^2\lambda$)
(iii)	$\mathrm{P}(X=x+1)=\dfrac{e^{-\lambda}\times\lambda^{x+1}}{(x+1)!}$	B1		
	$\mathrm{P}(X=x+1)=\dfrac{e^{-\lambda}\times\lambda^{x+1}}{(x+1)!}$ $=\dfrac{e^{-\lambda}\times\lambda^{x}\times\lambda}{(x+1)x!}$ $=\dfrac{\lambda}{x+1}\times\dfrac{e^{-\lambda}\times\lambda^{x}}{x!}$	Mdep1		dep(B1)
	$=\dfrac{\lambda}{x+1}\mathrm{P}(X=x)$	Adep1	3	AG
(b)(i)	$\lambda_{\text{car}}=500/\text{hour}$ $\lambda_{\text{coach}}=10/\text{hour}$			
	$\Rightarrow\ \lambda_{\text{vehicle}}=510/\text{hour}\ =8.5/\text{min}$	B1		for 8.5 stated / used special case: $\lambda=10\Rightarrow$ B1M0A0 B1 $\Rightarrow 1-0.458$ or 0.542
	$\mathrm{P}(V\geq 10)=1-0.6530$	M1		
	$=0.347$	A1	3	
(ii)	$\mu_{\text{car}}=836/\text{hour}$ $\mu_{\text{coach}}=22/\text{hour}$			
	$\Rightarrow\ \mu_{\text{vehicle}}=858/\text{hour}\ =14.3/\text{min}$	B1		for 14.3 stated /used
	$\mathrm{P}(V\leq 3)=\mathrm{P}(V=0,1,2,3)$			
	$=\begin{cases} e^{-14.3}\left[1+\dfrac{14.3}{1}+\dfrac{14.3^2}{2}+\dfrac{14.3^3}{6}\right] \\ e^{-14.3}\times 604.91283 \\ 0.0003726 \text{ to } 0.000373 \end{cases}$	M1		All 4 terms required for **any** $\lambda>0$ M0 for use of normal approximation
	$=0.00037$ (2sf)	Adep1	3	dep M1
	Total		**12**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
5(a)	<table><tr><th>*n*</th><th>Outcome</th><th>$P(N=n)$</th></tr><tr><td>1</td><td>H</td><td>**0.5** ($\frac{1}{2}$)</td></tr><tr><td>2</td><td>TH</td><td>**0.25** ($\frac{1}{4}$)</td></tr><tr><td>3</td><td>TTH</td><td>0.125 ($\frac{1}{8}$)</td></tr><tr><td>4</td><td>TTTH</td><td>**0.0625** ($\frac{1}{16}$)</td></tr><tr><td>5</td><td>TTTTA</td><td>0.0625 ($\frac{1}{16}$)</td></tr></table>	B2,1		B1 for **one** correct entry for n = 1, 2, 4 B2 for all 3 correct
	$\mathrm{E}(N)=\left(1\times\frac{1}{2}\right)+\left(2\times\frac{1}{4}\right)+\left(3\times\frac{1}{8}\right)+\left(4\times\frac{1}{16}\right)+\left(5\times\frac{1}{16}\right)$	M1		$\sum_{n=1}^{n=5} n\times \mathrm{P}(N=n)$ (all 5 terms attempted /seen/ implied)
	$=\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\frac{5}{16}=\frac{31}{16}$ $=1\frac{15}{16}$ (1.9375)	A1	4	Can be implied by correct $\mathrm{E}(N)$ (awfw 1.93 to 1.94)
(b)	<table><tr><td>m</td><td>**Outcome**</td><td>$P(M=m)$</td></tr><tr><td>1</td><td>H</td><td>$\frac{1}{4}$</td></tr><tr><td>2</td><td>TH</td><td>$\frac{3}{4}\times\frac{1}{4}$ $=\frac{3}{16}$</td></tr><tr><td>3</td><td>TTH</td><td>$\left(\frac{3}{4}\right)^2\times\frac{1}{4}$ $=\frac{9}{64}$</td></tr><tr><td>4</td><td>TTTH</td><td>$\left(\frac{3}{4}\right)^3\times\frac{1}{4}$ $=\frac{27}{256}$</td></tr><tr><td>5</td><td>TTTTA</td><td>$\left(\frac{3}{4}\right)^4\times 1$ $=\frac{81}{256}$</td></tr></table>	B3,2,1	3	(given) (given) (B1 any one correct) (B2 any 2 correct) (B3 all 3 correct)
(c)(i)	$\mathrm{P}(J,R)$:			
	$\mathrm{P}(1,1)=\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$ (oe) $\mathrm{P}(2,2)=\frac{1}{4}\times\frac{3}{16}=\frac{3}{64}$ (oe) $\mathrm{P}(3,3)=\frac{1}{8}\times\frac{9}{64}=\frac{9}{512}$ (oe)	M1		e.g 0.125 attempt at any $\mathrm{P}(n,n)$
	$\mathrm{P}(4,4)=\frac{1}{16}\times\frac{27}{256}=\frac{27}{4096}$ (oe)	A1		any 1 correct **to 3sf**
	$\mathrm{P}(5,5)=\frac{1}{16}\times\frac{81}{256}=\frac{81}{4096}$ (oe)	A1		all 5 correct **to 3sf**
	$p=\sum_{n=1}^{n=5}\mathrm{P}(n,n)$	m1		$\sum_{n=1}^{n=5}\mathrm{P}(n,n)$ with all 5 values attempted
	$\Rightarrow \quad p=\frac{221}{1024}$ (0.2158)	A1	5	(awfw 0.215 to 0.217)
(ii)	$=3\times\left(\frac{221}{1024}\right)^2\times\left(\frac{803}{1024}\right)$	M1		(either term with **their *p*** used) $(0<p<1)$
	$+\ \left(\frac{221}{1024}\right)^3$	M1		(second term with **their *p*** used) $(0<p<1)$
	$\mathrm{P}(X\geq 2)=\mathrm{P}(X=2)+\mathrm{P}(X=3)$	Mdep1		dep (M1M1)
	$=0.120$ (3dp)	A1	4	(allow 0.119; 0.12; 0.121)
	Total		**16**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
6(a)	 0 1 3 5	B2,1	2	B2 for st. line from $(1,0.2)$ to $(5,0.3)$ B1 st. line (m > 0) from $x = 1$ to $x = 5$.
(b)	$\mathrm{E}(X)=\frac{1}{40}\int_1^5 x(x+7)\,dx$	M1		Ignore limits
	$=\frac{1}{40}\left(\frac{x^3}{3}+\frac{7x^2}{2}\right)_1^5$	A1		Ignore limits
	$=\frac{1}{40}\left(\frac{125}{3}+\frac{175}{2}-\frac{1}{3}-\frac{7}{2}\right)$			
	$=3\frac{2}{15}$	A1	3	cao (accept $3.13\dot{3}$ or $\frac{47}{15}$ oe ***exact***)
(c)	$\mathrm{F}(x)=\int_1^x \frac{1}{40}(x+7)\,dx$	M1		$\mathrm{F}(x)=\int\left(\frac{x}{40}+\frac{7}{40}\right)dx$
	$=\frac{1}{40}\left[\frac{x^2}{2}+7x\right]_1^x$	A1		$=\frac{x^2}{80}+\frac{7x}{40}+c \Rightarrow$ (M1A1)
	$=\frac{1}{80}\left(x^2+14x-1-14\right)$			$\mathrm{F}(1)=0 \Rightarrow c=-\frac{1}{80}-\frac{7}{40}=-\frac{15}{80}$ **or** [use of F(5) = 1]
	$=\frac{1}{80}\left(x^2+14x-15\right)$	Adep1		$\Rightarrow \mathrm{F}(x)=\frac{1}{80}\left(x^2+14x-15\right)$
	$=\frac{1}{80}(x+15)(x-1)$	Adep1	4	$\mathrm{F}(x)=\frac{1}{80}(x+15)(x-1)$ (AG)
(d)(i)	$\mathrm{P}(2.5\le X\le 4.5)=\mathrm{F}(4.5)-\mathrm{F}(2.5)$			Trapezium Rule
	$=\frac{1}{80}(19.5\times 3.5-17.5\times 1.5)$	M1		$\frac{1}{2}\left(\frac{23}{80}+\frac{19}{80}\right)\times 2$
	$=\frac{42}{80}=\frac{21}{40}$ (0.525)	A1	2	$=\frac{42}{80}=\frac{21}{40}$
(ii)	$\mathrm{F}(m)=\frac{1}{2}$	B1		$\int_1^m \frac{1}{40}(x+7)dx=0.5$ (B1)
	$\Rightarrow \quad \frac{1}{80}\left(m^2+14m-15\right)=\frac{1}{2}$	M1		Correct equation formed
	$(\times 80) \quad \Rightarrow m^2+14m-15=40$ $m^2+14m-55=0$	Adep1	3	AG
(e)	$m=\frac{-14\pm\sqrt{196+220}}{2}=\frac{-14\pm 20.396}{2}$	M1		Correct attempt at solving quadratic (by formula, oe).
	$m=\frac{-14+20.396}{2}$ (since m > 1) $m=3.198$ (3dp)	A1	2	cao
	Total		**16**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
June 2012

Mathematics MS2B

(Specification 6360)

Statistics 2B

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Q	Solution	Marks	Total	Comments
1(a)	$\bar{x} = \frac{\sum x}{n} = \frac{546}{15} = \frac{182}{5} = 36.4$	B1		oe
	$s^2 = \frac{\sum (x-\bar{x})^2}{n-1} = \frac{1407.6}{14} = 100.54$ **(or** $s = 10.03$ **)**	B1		$\sigma^2 = 93.84$ or $\sigma = 9.687$ iff $\sigma/\sqrt{14}$ used below
	$t_{crit} = \pm 2.624$	B1		ignore signs for t_{crit} (allow $t = 2.62$) (if z used then **max**(B1B1B0 M0A0A0)
	98% CI for μ: $36.4 \pm 2.624 \times \frac{s}{\sqrt{15}}$	M1		their $\bar{x} \pm t_{14} \times \frac{\text{their } s}{\sqrt{15}}$ **or** their $\bar{x} \pm t_{14} \times \frac{\text{their } \sigma}{\sqrt{14}}$
	$(29.6,\ 43.2)$ 36.4 ± 6.8	A1ft		(allow any of the following for t_{14}: 1.345; 1.761; 2.145; 2.624; 2.977)
	$= 29.6, 43.2$	A1	6	cao
(b)	$40.0 \in$ C.I.	E1ft		Must refer to 40 (dep M1)
	$\Rightarrow$ no change	E1ft	2	Dep on previous mark
			8	
2(a)	$H_0: \mu = 4.0$ $H_1: \mu > 4.0$	B1		(both)
	$z_{calc} = \frac{4.2-4}{1.1/\sqrt{40}}$	M1		**Alternative:** $P(\bar{X} > 4.2) = P(Z > 1.15)$ **M1A1**
	$= 1.15$	A1		awrt
	$z_{crit} = 1.6449$	B1		$= 1 - 0.87493$ $= 0.125$ **B1**
				$0.125 > 0.05 \Rightarrow$ accept H_0 **Adep1**
	Accept H_0 [or Reject H_1]	A1		Dep on B1M1B1
	Insufficient evidence at 5% level to support Julian's claim	E1	6	Dep on previous mark
(b)	Type II error.	B1ft		Follow through on conclusion in (a)
	Accepted H_0 when H_0 was false (oe)	E1	2	Dep on previous mark
				If Reject H_0 in (a) then: No error (B1ft) Rejected H_0 when H_0 was false (oe) (E1)
	Total		**8**	

MS2B

Q	Solution	Marks	Total	Comments
3(a)	for $-5 \le x \le 15$ $f(x) = \frac{d}{dx}F(x) = \frac{d}{dx}\left(\frac{x+5}{20}\right) = \frac{1}{20}$	B1	1	AG
(b)(i)	$P(X \ge 7) = 1 - F(7)$ $= 1 - \frac{12}{20}$ $= \frac{2}{5}$ **or** $\left[\frac{8}{20}; \frac{4}{10}; 0.4\right]$	B1	1	**Alternative:** Use of $f(x) = \frac{1}{20}$ ***or*** graph $\Rightarrow$ $P(X \ge 7) = \frac{1}{20} \times (15-7) = \frac{2}{5}$ (oe)
(ii)	$P(X \ne 7) = 1$	B1	1	cao
(iii)	$E(X) = \frac{1}{2}(-5+15) = 5$	B1	1	**Alternative:** $E(X) = \int_{-5}^{15} \frac{x}{20} dx = \left[\frac{x^2}{40}\right]_{-5}^{15}$ $= \frac{1}{40}(225-25)$ $= \frac{1}{40} \times 200$ $= 5$ **B1** (cao)
(iv)	$E(3X^2) = \int_{-5}^{15} \frac{3x^2}{20} dx$ } (ignore limits)	M1		
	$\left[\frac{x^3}{20}\right]_{-5}^{15}$ $\frac{1}{20}(3375+125)$ $168\frac{3}{4} + 6\frac{1}{4}$ }	A1		correct limits seen / used
	$= 175$	A1	3	(cao) (allow $174.\dot{9}$)
	Alternative:			
	$\text{Var}(X) = \frac{1}{12}(15--5)^2 = \frac{400}{12}$ (oe)	(B1)		$E(3X^2) = 3E(X^2)$
	$E(3X^2) = 3 \times \left[\frac{400}{12} + 5^2\right]$	(M1)		$= 3 \times \left[\{\text{their Var}(X) > 0\} + \{\text{their E}(X)\}^2\right]$ ***used*** $(\Rightarrow$ M1$)$
	$= 175$	(A1)		
	Total		**7**	

MS2B

Q	Solution	Marks	Total	Comments
4(a)				
	r | 1 | 2 | 3 | 4 | 5 p | .5 | **.24** | **.144** | **.0864** | 0.0296			
	$0.4\times0.6=0.24$ $0.24\times0.6=0.144$ $0.144\times0.6=0.0864$	B2,1	2	B1 for any 1 correct (unsimplified) (B1) B2 all correct and simplified
(b)	$\mathrm{P}(\text{fewer than 3 bedrooms **are** rented})$ $=\mathrm{P}(R=1,2)\Rightarrow$			**Alternative:** P(fewer than 3 not rented) = P(0, 1 or 2 not rented) = P(5, 4 or 3 are rented)
	$\mathrm{P}(\text{fewer than 3 bedrooms **not** rented})$ $=1-\mathrm{P}(R=1,2)$	M1		= P(R = 3, 4, 5) **M1**
	$=1-\mathrm{P}(\text{1 or 2 rooms are rented})$ $=1-$ (0.5 + **0.24**) [their $0<p(2)\le0.4704$ value from table **used**]	m1		$p=0.4\times0.6^2+0.4\times0.6^3+0.0296$ = **0.144 + 0.0864** + 0.0296 **m1** [**or** their $p(3)+p(4)\le0.4704$ value from table **used**]
	$=1-0.74$ $=0.26$	A1	3	= 0.26 (cao) **A1** [SC 0.74 for B1]
(c)(i)	$\mathrm{E}(R)=0.5\times1+0.4\times0.6\times2$ $+0.4\times0.6^2\times3+0.4\times0.6^3\times4$ $+0.0296\times5$	M1		$\sum_1^5 r_i\times\mathrm{P}(R=r_i)$ from their table
	$=0.5\times1+0.24\times2+0.144\times3+0.0864\times4$ $+0.0296\times5$			
	= 0.5 + 0.48 + 0.432 + 0.3456 + 0.148 $\left[=\frac{1}{2}+\frac{12}{25}+\frac{54}{125}+\frac{216}{625}+\frac{37}{250}\right]$			$(0.5+1.2576+0.148)$
	$\therefore\ \mathrm{E}(R)=1.9056$	A1	2	[awfw 1.9 to 1.91] $\left[1\frac{566}{625}\right]$
(ii)	$\mathrm{E}(R^2)=0.5\times1^2+0.4\times0.6\times2^2$ $+0.4\times0.6^2\times3^2+0.4\times0.6^3\times4^2$ $+0.0296\times5^2$			$[0.5+0.96+1.296+1.3824+0.74]$
	$\mathrm{E}(R^2)=4.8784$	B1		AG
	$\mathrm{Var}(R)=4.8784-1.9056^2$ **(=** 1.24708864)	M1		4.8784 – their $\mathrm{E}^2(R)$
	$=1.25$ **(3sf)**	A1	3	(awfw 1.23 to 1.25)

MS2B

Q	Solution	Marks	Total	Comments
(d)	$\mathrm{E}(M) = 1250\mathrm{E}(R) - 282$ $= 1250 \times 1.9056 - 282$ $= 2100$	B1		cao
	$\mathrm{Var}(M) = 1250^2 \times \left[4.8784 - 1.9056^2 \right]$	M1		$1250^2 \times$ their $\mathrm{Var}(R) > 0$ in (c)(ii) (1 948 473 to 1 953 125)
	$\mathrm{sd}(M) = 1250 \times \sqrt{1.24708864}$ $= 1395.91$	A1	3	$\mathrm{sd}(M) = \sqrt{1948437} = 1395.9$ $\left(\sqrt{1953125} = 1397.5\right)$ (awfw 1395 to 1400)
	Total		**13**	

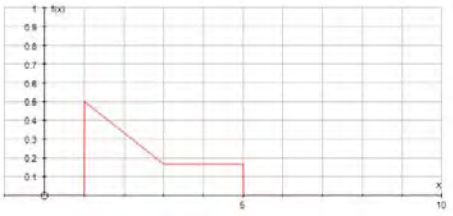
MS2B

	Solution	Marks	Total	Comments
5(a)(i)	$P(X \geq 9) = 1 - P(X \leq 8)$ $= 1 - 0.5231$ $= 0.4769$	B2,1	2	$1 - 0.6530 = 0.347$ (B1) awfw 0.476 and 0.477
(ii)	$P(5 < X < 10) = P(X \leq 9) - P(X \leq 5)$ $= 0.653 - 0.1496$ $= 0.5034$	B3,2,1	3	awfw 0.503 to 0.504 $0.7634 - 0.1496 = 0.613$ to 0.614 (B2) $0.6530 - 0.2562 = 0.397$ to 0.398 (B2) $0.7634 - 0.2562 = 0.507$ to 0.508 (B1) $\alpha - 0.1496$ or $0.653 - \alpha$ (B1) iff $0 < p < 1$
(b)	$P(Y < 2) = P(Y \leq 1) = P(Y = 0 \text{ or } Y = 1)$ $= e^{-1.5} + e^{-1.5} \times 1.5$ $[0.2231 + 0.3347]$ $= 0.5578254$ $= 0.558$	M1 A1	2	0.8 to 0.81 (B1) (both) awfw 0.557 to 0.56
(c)(i)	$\lambda = 8.5 + 1.5 = 10$	B1	1	Allow P(10) or $Po(10)$
(ii)	$P(T > 16) = 1 - P(T \leq 16)$ $= 1 - 0.9730$ $= 0.027$	M1 A1	2	
(iii)	$p = {}^3C_2\, 0.027^2 \times 0.973$ $+ 0.027^3$ $= 0.002128 + 0.00001968$ $= 0.0021$ [4 dp]	M1 M1 A1	3	for either term correct for addition of the two correct terms 0.0021 or 0.0022 [iff M1M1 (+ 4dp)]
	Alternative: $p = 1 - P(X \leq 1)$ $P(X = 0) + P(X = 1)$ $= 0.973^3 + 3 \times 0.973^2 \times 0.027$ $= 0.921167 + 0.076685$ $p = 1 - 0.99785$ $= 0.0021$	(M1) (M1) (A1)		for either term correct for 1 – [sum of two correct terms] 0.0021 or 0.0022 [iff M1M1 (+ 4dp)]
	Total		**13**	

MS2B

Q	Solution	Marks	Total	Comments
6(a)	H_0: No association between A level grade and class of degree H_1: Association between A level grade and class of degree	B1		At least H_0 correct
	O_i / E_i: 20/11.6; 9/17.4; 36/36.4; 55/54.6; 22/28; 48/42; 2/4; 8/6; 200/200	M1		For E_i's attempted
	Combine Class 2(ii) and 3	M1		For combining attempted
	20 | 11.6 | 8.4 | 6.0827 9 | 17.4 | −8.4 | 4.0552 36 | 36.4 | −0.4 | 0.0044 55 | 54.6 | 0.4 | 0.0029 24 | 32 | −8 | 2.0 56 | 48 | 8 | 1.3333 200 | 200 | 0 | 13.47	M1 A1		For final column attempted (awrt 13.5)
	$\nu = 2$ $\chi^2_{1\%}(2) = 9.210$	B1 B1		[$\nu = 3$ with $\chi^2 = 11.345$ (B0B1ft)]
	Reject H_0	A1		Dep on B1 M1M1M1 B1B1, not A1
	Fiona's belief justified	E1	9	Dep on B1 M1M1M1 B1B1, not A1
(b)	**Fewer than expected gained a Class 1** degree having gained grade B in A-level Mathematics.	E1		
	More than expected gained a Class 2(ii) degree having gained grade B in A-level Mathematics.	E1	2	correct comments (see below)
	1 / 2(i) / 2(ii) / 3 / comb **9 / 55 / 48 / 8 / 56** **17.6 / 54.6 / 42 / 6 / 48** **A / B / C / D / E**			A: fewer than expected B: as expected C: more than expected D: more or similar than expected E: more than expected
	Total		**11**	

MS2B

Q	Solution	Marks	Total	Comments
7(a)		B2,1	2	Straight line from (1 , 0.5) to (3 , ⅙). Horizontal straight line from (3 , ⅙) to (5 , ⅙).
(b)	$\mathrm{E}(X)=\frac{1}{6}\int_1^3 x(4-x)\,\mathrm{d}x+\frac{1}{6}\int_3^5 x\mathrm{d}x$	M1		ignore limits (**both** parts attempted)
	$=\frac{1}{6}\left[2x^2-\frac{x^3}{3}\right]_1^3+\frac{1}{6}\left[\frac{x^2}{2}\right]_3^5$	A1		ignore limits (**both** correct)
	$=\frac{1}{6}\left[(18-9)-\left(2-\frac{1}{3}\right)\right]+\frac{1}{6}\left[\frac{25}{2}-\frac{9}{2}\right]$			
	$=\frac{1}{6}\left[7\frac{1}{3}+8\right]$	m1		use of correct limits. dep on M1A1
	$=2\frac{5}{9}$	A1	4	(AG)
(c)(i)	$\mathrm{P}(X>2.5)=\frac{1}{3}+\frac{1}{2}\times\left(0.25+\frac{1}{6}\right)\times\frac{1}{2}$	M1		Or $1-\int_1^{2.5}\frac{1}{6}(4-x)\mathrm{d}x=1-\left[\frac{1}{6}\left(4x-\frac{x^2}{2}\right)\right]_1^{2.5}$
	$=\frac{7}{16}$	A1	2	cao (0.4375)
(ii)	$\mathrm{P}(1.5<X<4.5)=\frac{1}{2}\times\left(\frac{5}{12}+\frac{1}{6}\right)\times1.5+(4.5-3)\times\frac{1}{6}$	M1		Or $\int_{1.5}^{3}\frac{1}{6}(4-x)\mathrm{d}x+\int_3^{4.5}\frac{1}{6}\mathrm{d}x$
	$=\frac{7}{16}+\frac{1}{4}$	A1		
	$=\frac{11}{16}$	A1	3	cao ($=\frac{11}{16}$ or 0.6875)
(iii)	$\mathrm{P}(X>2.5\ \textbf{and}\ 1.5<X<4.5)$ $=\mathrm{P}(2.5<X<4.5)$			$\int_{2.5}^{3}\frac{1}{6}(4-x)\mathrm{d}x=\left[\frac{1}{6}\left(4x-\frac{x^2}{2}\right)\right]_{2.5}^{3}=\frac{5}{48}$
	$=\frac{1}{2}\times\left(0.25+\frac{1}{6}\right)\times0.5+\frac{1}{4}$	M1		
	$=\frac{5}{48}+\frac{1}{4}$			
	$=\frac{17}{48}$	A1	2	cao $(0.35416\dot{})$
(iv)	$\mathrm{P}(X>2.5\|1.5<X<4.5)=\frac{17/48}{11/16}$	M1		their $\frac{\text{(iii)}}{\text{(ii)}}$ iff 0 < p's < 1
	$=\frac{17}{33}$	A1	2	cao (allow $0.5\dot{1}$)

MS2B

Q	Solution	Marks	Total	Comments
7(c)	**Alternative Solution**			
	$F(x)=\begin{cases}0 & x<1\\ \frac{1}{12}(x-1)(7-x) & 1\le x<3\\ \frac{1}{6}(x+1) & 3\le x<5\\ 1 & x\ge 5\end{cases}$			
(i)	$P(X>2.5)=1-F(2.5)$			
	$=1-\frac{1}{12}(2.5-1)(7-2.5)$	(M1)		
	$=1-\frac{1}{12}\times 1.5\times 4.5$			
	$=1-0.5625$			
	$=0.4375$ **or** $\frac{7}{16}$	(A1)		cao
(ii)	$P(1.5<X<4.5)=F(4.5)-F(1.5)$			
	$=\frac{1}{6}(4.5+1)-\frac{1}{12}(1.5-1)(7-1.5)$	(M1)		
	$=\frac{11}{12}-\frac{11}{48}$	(A1)		
	$=\frac{11}{16}$ **or** 0.6875	(A1)		cao
(iii)	$P(X>2.5$ **and** $1.5<X<4.5)$			
	$=P(2.5<X<4.5)$			
	$=F(4.5)-F(2.5)$			
	$=\frac{11}{12}-\frac{9}{16}$	(M1)		
	$=\frac{17}{48}$	(A1)		cao
(iv)	$P(X>2.5\mid 1.5<X<4.5)$			
	$=\frac{F(4.5)-F(2.5)}{F(4.5)-F(1.5)}$ **or** $\frac{\text{their (iii)}}{\text{their (ii)}}$	(M1)		
	$=\frac{17/48}{11/16}$			
	$=\frac{17}{33}$ **or** $(\text{allow } 0.\dot{5}\dot{1})$	(A1)		cao
	Total		**15**	
	TOTAL		**75**	

Version

General Certificate of Education (A-level)
January 2013

Mathematics MS2B

(Specification 6360)

Statistics 2B

Final

Mark Scheme

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Registered address: AQA, Devas Street, Manchester M15 6EX.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Q	Solution	Marks	Total	Comments
1(a)	Sample mean = 53.06, s = 1.140	B1		Both. For s AWRT 1.14
	t_5 = 2.571	B1		AWRT 2.57
	Sample mean $\pm$ t $\times$ s/$\sqrt{6}$	M1		For $\sqrt{6}$
		m1		Rest of formula. Allow t_5 = 2.01 to 2.02, or t_6 = 2.45
	(53.06 $\pm$ 1.20) = (51.86, 54.26)	A1		Either form $\pm$ 0.01 in total.
			5	
(b)	Sample mean is lower than last year's mean so claim **may** be true.			E1 each for sensible comments either supporting or against statement up to a maximum of 2. Comment must be **uncertain.**
	53.41 lies within c.i. so **not certain** that mean time is better.	E2		
	Performance in competition does not depend on mean time.			
	Times seem to be improving.			
			2	
		Total	**7**	

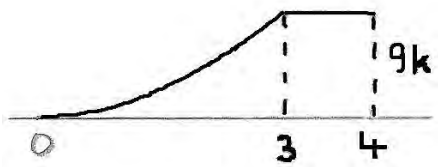
Q	Solution	Marks	Total	Comments
2 (a)		M1		Any two correct to 2 d.p.
		A1		All correct, here or below, to 2 d.p.
		E1		Must be **expected** value, applied to this case, not just general statement.
		B1		For combining first two E columns, at least 1 correct.
	H_0: No association between property type and time to sell.			H_0 seen somewhere in solution
	H_1: Association between property type and time to sell.	B1		If "independent" used then must be correct way round
		M1		For attempt at $(O_i-E_i)^2/E_i$
		A1		$4.7 < X^2 < 4.8$
	2 degrees of freedom	B1		PI by 4.605, 5.991, 7.378, 9.210 or 10.597 seen
	C.V. of χ^2 for 2 d.f. = 4.605	B1		AWFW 4.60 to 4.61
	4.74 > 4.605 so reject H_0 significant evidence of an association between property type and time to sell.	A1		Context conclusion. Dep. on B1 for H_0 , A1 for χ^2 and B1 for c.v.
			10	
(b)(i)	More in total than any other type so likely to have biggest effect	E1		Or similar referring to large number
(ii)	Far away from expected values	E1		Or opposite pattern to other three
			2	
		Total	**12**	

	If Flats and Detached combined:			.
		B1		For combined F and D
		M1		For attempt at $(O_i-E_i)^2/E_i$
		A0		
	Accept H_0	A0		**Max of 8 marks**

MS2B (cont)

Q	Solution	Marks	Total	Comments
3 (a)(i)	$e^{-1.5} \times 1.5^3/3!$	M1		
	= 0.126	A1		0.125 to 0.126
			2	
(ii)	Using Po(1), $P(X > 1) = 1 - P(X \leq 1)$	M1		
	= 1 – 0.7358 = 0.264	A1		SC Award M1 only if obtain 0.0902 using Po(0.5)
			2	
(iii)	Weekdays Po(7.5) weekend Po(1)	M1		Weekdays = 7.5
	Total Po(8.5)	A1		
	P(Total < 10) = P(Total ≤ 9)	m1		Applied (0.7764, 0.7166, 0.6530 are evidence)
	= 0.653	A1		
			4	
(b)	Using Total Po from (a)(iii)			M1 using their total providing supporting probabilities seen
	P(>15) = 0.0138, P(>16) = 0.0066	M1		OE use of P(Total ≤ 15 & 16)
	So needs 16 tubes	A1		CAO Answer alone scores B2
			2	
(c)	Average rate of failure unlikely to be constant over the course of a day. Very little use of lights over this period.	E1		One mark for any sensible comment
			1	
		Total	**11**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
4(a)		B1 B1 B1	**3**	Curve + rectangle Some indication of x values $9k$ or 0.5 indicated for vertical height
(b)	Attempt to integrate kx^2 between 0 & 3 Obtain $9k$ Area under rectangle = $9k$ $9k + 9k = 1$ so $k = {}^1/_{18}$	M1 A1 B1 B1	**4**	Show $\frac{kx^3}{3}$ For $9k + 9k = 1$. AG
(c)(i)	3	B1		
(ii)	Attempt to integrate kx^2 between 0 & Q_1 put = 0.25 $\frac{(Q_1)^3}{54} = 0.25$ $Q_1 = 2.38$	M1 A1 A1	**4**	 AWRT or any equivalent exact form $\sqrt[3]{13.5} = {}^3/(\sqrt[3]{2}) = {}^3/_2(\sqrt[3]{4})$
		Total	**11**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
5(a)	Mean = $0 \times 0.1 + 1 \times 0.35$ ……	M1		AG
	= 1.85			
	$E(X^2) = 0^2 \times 0.1 + 1^2 \times 0.35$ …	M1		Full method including -1.85^2
	= 4.75 $\quad$ $Var(X) = 4.75 - 1.85^2$	A1		For $E(X^2) = 4.75$
	= 1.3275	A1		For final answer AWRT 1.33
			4	
(b)(i)	$T = c + nX$	B1		
			1	
(ii)	$E(c + nX) = c + nE(X)$	M1		Getting at least as far as $c + E(nX)$
	$= c + 1.85n$	A1		CAO
	$Var(c + nX) = Var(c) + Var(nX)$	M1		Getting at least as far as $(0 +) Var(nx)$
	$= 0 + n^2 var(X) = 1.3275n^2$	A1F		FT their $Var(X)$ if $0 < Var(X) < 4$
			4	
		Total	**9**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
6(a)	Putting $\frac{t^3}{216} = 0.9$	M1		
	$t = 5.793$	A1		5.79 to 5.80
	41 days.	A1		Accept 40 days in this context
			3	
(b)	Attempt to differentiate F(t)	M1		ct^2 seen
	$f(t) = \frac{1}{72}t^2 \quad 0 \le t \le 6$	A1		Condone domain missing here
	$= 0$ otherwise	A1		For **complete** function
			3	
(c)	Attempt to integrate $tf(t)$ from 0 to 6	M1		Using their f(t) from **(b)** ct^4 seen
	$E(T) = 4.5$	A1		
	Attempt to integrate $t^2f(t)$ from 0 to 6	M1		Using their f(t) from **(b)** ct^5 seen
	$E(T^2) = 21.6$	A1		
	$Var(T) = E(T^2) - E(T)^2$	m1		**Applied** in this case. Dependent on both M1
	$= 21.6 - 4.5^2 = 1.35$	A1		
			6	
(d)	S.d. $= \sqrt{1.35} = 1.162$	M1		For $\sqrt{}$ (their Var) $0 < Var(T) < 9$
	Use of F(5.662)	m1		For F(their s.d. + their E(T)) provided $0 <$ Total < 6
	$1 - \frac{5.662^3}{216}$	m1		
	$= 0.160$	A1		AWFW 0.159 to 0.161
			4	
		Total	**16**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
7(a)	H_0: $\mu_y = 20$ $\mu_x = 3020$ H_1: $\mu_y \neq 20$ $\mu_x \neq 3020$	B1		Both
	$\bar{y} = 18.47$ $\bar{x} = 3018.47$ $s = 8.00$	B1 B1		x or y.
	$z = (18.47 - 20)/(\frac{8}{\sqrt{100}})$ o.e. for x	M1 m1		$\sqrt{100}$ rest of formula for z (either way round in numerator)
	$= -1.9125$	A1		AWFW –1.93 to –1.91.
	c.v. = ±1.96 (or ±1.98 from t_{99})	B1		Sign consistent with z value.
	So test statistic not in critical region. Accept H_0, no significant evidence that mean has changed.	A1	**8**	Comparison not just with opposite C.V. Mention of water not necessary. Dep on B1 for H_0 & H_1, A1 and B1 for c.v.
(b)	No error if (a) is "Accept H_0" Type I if (a) is "Reject H_0"	B1F	**1**	MUST FOLLOW THROUGH
	Alternatives: $(3018.47 - 3020)/(\frac{8}{\sqrt{100}})$ for z p value is 0.0558 and must be compared with 0.05 Critical values for $\bar{y}$ are 18.43 or 18.44 and 21.56 or 21.57			
		Total	**9**	
	TOTAL		**75**	

Version 1.0

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Otherwise we require evidence of a correct method for any marks to be awarded.

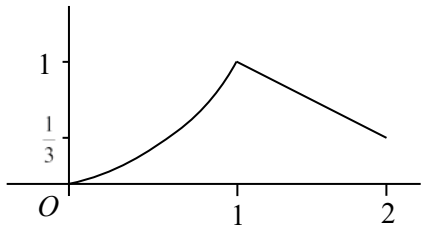
Q	Solution	Marks	Total	Comments
1(a)	$\bar{x} = 948$ and $s^2 = 4817.25$	B1		Both; AWRT 4820 ($s = 69.406$)
	$t_8 = 2.896$	B1		AWRT 2.90
	C.I. $= 948 \pm 2.896 \times \sqrt{\frac{4817.25}{9}}$	M1		For division by $\sqrt{9}$
		m1		For rest of expression, must be t_8 or t_9 (= 2.821)
	$= 948 \pm 67.0 = (881, 1015)$	A1	5	Either form AWRT ± 67 Accept 1010 or 1020 as upper limit
(b)(i)	$(927 + 1063) \div 2 = 995$	B1	1	CAO
(ii)	**Dependent on partial overlap**			
	Because of the overlap by the confidence intervals ...	E1		
	… no definite conclusion is possible	Edep1	2	Accept "No evidence"
SC	Reference to evidence provided by the mean or the limits being lower 'suggesting' or 'providing evidence' or 'supporting' weight reduction scores 1	(E1)		The statement must be not definite. Anything definite, eg. 'proves that' or 'shows that' scores 0
	Total		**8**	

Q	Solution	Marks	Total	Comments
2(a)				
	O_i | E_i | $(\lvert O_i - E_i\rvert - 0.5)^2/E_i$ 30 | 35.2 | 0.6276 14 | 8.8 | 2.5102 130 | 124.8 | 0.1770 26 | 31.2 | 0.7080 | χ^2 | **4.0228**	M1		E attempted (at least two correct to 1 d.p.)
		M1		Yates' correction attempted; at least one correct value in final column
		M1		χ^2 attempted
		A1		AWFW 4.02 to 4.03
	H_0: No association between method of receiving information and outcome	B1		At least one correct
	H_1: Association between method of receiving information and outcome			If "independent" used, it must be the right way round
	CV of χ^2 for 1 df = 3.84(1)	B1		
	4.02 > 3.841 so reject H_0 There is significant evidence of an association between method of receiving information and outcome	A1		Dep on A1 and B1 for CV
	Applications higher than expected for telephone calls, so council's belief seems to be true	Adep1	8	Dep on previous A1 Context conclusion about council's belief, referring to higher than expected for telephone
	Alternative if Yates' not used			
	O_i | E_i | $(O_i - E_i)^2/E_i$ 30 | 35.2 | 0.7682 14 | 8.8 | 3.0727 130 | 124.8 | 0.2167 26 | 31.2 | 0.8667 | χ^2 | **4.9243**			Loses M1 for Yates' and A1 for final χ^2 value but can score all the other 6 marks Final 2 A1 marks dep on 4.92 to 4.93 and B1 for CV
(b)	Type I error was made because H_0 has been rejected (when it was true)	E1 Edep		Dep on previous E1
SC	**If 'H_0 accepted' when their χ^2 less than their CV** No error was made because H_0 has been accepted (when it was true)	(E1) (Edep1)	2	Dep on previous (E1)
	Total		**10**	

Q	Solution	Marks	Total	Comments
3(a)(i)	Just catches a tram	E1		Must refer to the 0 in some way to score the E1
	= 2 (+ 0) + 20 + 5 = 27	B1		but can score B1 for 2 + 20 + 5 = 27
(ii)	$b = 37$	B1	3	
(b)	E(T) = 32	B1		
	Var(T) = $10^2/12$			
	= 100/12 = 25/3 = $8^1/_3$ = 8.33	B1	2	Any form
(c)	(35 – 27) = 8	M1		Or by integration from 27 to 35
	× 0.1 = 0.8	A1	2	
	Total		**7**	
4(a)(i)	$\frac{e^{-3.5} \times 3.5^4}{4!}$	M1		
	= 0.189	A1	2	AWRT 0.189 Answer only gets B2
(ii)	Using or stating Po(0.5)	B1		An answer of 0.0144, 0.3935, 0.6065, 0.9098 or 0.9856 implies award of B1 but no further marks
	P(≥ 2) = 1 – P(≤ 1)	M1		
	or = 1 – 0.9098			
	= 0.0902	A1	3	Accept 0.09
(iii)	Using Po(14)	B1		Sight of 0.1094, 0.1757, 0.9235, 0.9521
	P(≤ 19) – P(≤ 10) = 0.9235 – 0.1757	M1		Allow 0.8752 – 0.1185
				or 0.9573 – 0.2517 for M1
	= 0.7478	A1	3	AWFW 0.747 to 0.748
(b)	**GRBs/explosions/events/etc** will be random and/or independent			
		E1	1	For any valid point
	GRBs/etc short in comparison to observation period (non-overlapping)			
	Total		**9**	

Q	Solution	Marks	Total	Comments
5(a)(i)	$1 - ({}^1/_3 + {}^1/_4 + {}^1/_5 + {}^1/_6)$	M1		OE
	$= {}^1/_{20} = 0.05$	A1	2	AG
(ii)	$E(X) =$			
	$1\times{}^1/_3 + 2\times{}^1/_4 + 3\times{}^1/_5 + 4\times{}^1/_6 + 5\times{}^1/_{20}$	M1		At least 2 terms
	$= 2.35$	A1	2	OE: give B2 for only 2.35 seen
(iii)	$E(X^2) =$			
	$1\times{}^1/_3 + 4\times{}^1/_4 + 9\times{}^1/_5 + 16\times{}^1/_6 + 25\times{}^1/_{20}$ $(= 7.05)$	M1		All 5 terms $E(X^2) = 7.05$ with no working scores M0 Correct working but labelled $\text{Var}(X)$ and then no more done also scores M0
	$\text{Var}(X) = E(X^2) - E(X)^2$	m1		Applied to this problem
	$= 1.5275$	A1	3	AG
(iv)	$1 - ({}^1/_3 + {}^1/_4)$ **or** $({}^1/_5 + {}^1/_6 + {}^1/_{20})$	M1		
	$= {}^5/_{12}$ or 0.417	A1	2	AWRT Accept answer only for B2
(b)	'2.35' × 100 – 50	M1		Their value of mean
	$= 185$	A1F		FT from **(a)(ii)** Give B2 for only 185 seen
	$100^2 \times 1.5275$ **or** $100 \times \sqrt{1.5275}$	M1		
	$SD = \sqrt{15275} = 5\sqrt{611} = 124$	A1	4	AWFW 123.5 to 124 or $5\sqrt{611}$ Give B2 for only 123.5 to 124 or $5\sqrt{611}$ seen
	Total		**13**	

Q	Solution	Marks	Total	Comments
6(a)	H_0: $\mu = 175$ H_1: $\mu < 175$	B1		Both; accept H_0: $\mu \geq 175$ Do not accept mean or $\bar{x}$ but accept population mean
	$\bar{x} = 168.1$	B1		
	$z = \dfrac{\text{'168.1'} - 175}{9.4/\sqrt{6}}$	M1		For use of $9.4/\sqrt{6}$
		m1		For rest of formula (ignore sign)
	$= -1.798$	A1		Must be negative AWRT -1.80
	CV = -1.6449	B1		AWFW -1.64 to -1.65
	$-1.6449 > -1.798$ so test statistic in critical region Reject H_0, significant evidence that batch **mean** is less than 175grams	A1	7	Comparison of correct test statistic with correct CV **must be seen** (diagram or words) OE; suspicion supported Must be in context AG
(b)	H_0: $\mu = 175$ H_1: $\mu < 175$			Award B1 for both correct if not scored in **(a)**
	$t = \dfrac{169.4 - 175}{11.2/\sqrt{20}}$	M1		For use of $11.2/\sqrt{20}$
		m1		For rest of formula (ignore sign)
	$= -2.236$	A1		Must be negative AWRT -2.24
	CV(t_{19}) = -2.539	B1		AWRT -2.54
	$-2.236 > -2.539$ so test statistic not in critical region			Comparison of correct test statistic with correct CV (need not be seen)
	Accept H_0, no significant evidence that batch mean/weight is less than 175grams	A1	5	OE; suspicion not supported
(c)	Because the significance level is 1% instead of 5%	E1	1	OE; eg SL is different Reference to sample size $\Rightarrow$ E0
	Total		**13**	

Q	Solution	Marks	Total	Comments
7(a)		B1 B1 B1	 3	Curve concave upwards between (0, 0) and $(1, y_1)$ Negative gradient line between $(1, y_1)$ and $(2, y_2)$ with $y_2 > 0$ (and not beyond 2) $y_1 = 1$ and $y_2 = \frac{1}{3}$ shown
(b)(i)	Attempt to integrate t^2 **between 0 and x** $F(x) = \frac{1}{3}x^3$	M1 A1	 2	Accept integral of x^2
(ii)	Their $F(x) = 0.25$ $x = 0.909$	M1 A1	 2	AWRT; accept $\sqrt[3]{0.75}$ OE
(c)(i)	$F(1) = \frac{1}{3}$ $\int_1^x \frac{1}{3}(5-2t)\,dt = \left[\frac{1}{3}(5t-t^2)\right]_1^x$ $= \frac{1}{3}(5x-x^2) - \frac{4}{3}$ $F(x) = \frac{1}{3}(5x-x^2) - \frac{4}{3} + \frac{1}{3}$ $= \frac{1}{3}(5x-x^2-3)$	B1 M1 A1 A1	 4	 For integral attempted with correct limits For limits substituted in correct expression F(1) added to give complete $F(x)$ AG
(ii)	$\frac{1}{3}(5q-q^2-3) = 0.75$ **or** integral of $f(x)$ from q to $2 = 0.25$ $4q^2 - 20q + 21 = 0$ **or** $q^2 - 5q + 5.25 = 0$ $(2q-3)(2q-7) = 0$ **or** $q = 2.5 \pm 1$ $q = 1.5$	M1 A1 m1 A1	 4	Setting up equation Reaching correct simplified quadratic Factorising for two solutions or using formula or calculator Selecting only this one
	Total		**15**	
	TOTAL		**75**	

A-LEVEL
Mathematics

Statistics 2B – MS2B
Mark scheme

6360
June 2014

Version/Stage: Final

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A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

A-level Mathematics June 2014 MS2B

Q1	Solution	Marks	Total	Comments
(a)	Sample mean = $1904 \div 5 = 380.8$	B1		CAO
	$s = 4.38$ or $s^2 = 19.2$	B1		AWRT
	$t_4 = 2.132$	B1		AWRT 2.13
	C.I. = $380.8 \pm 2.132 \times$'$\frac{4.38}{\sqrt{5}}$' or $\sqrt{('19.2')/5}$	M1		Use of their $4.38/\sqrt{5}$ or $\sqrt{('19.2')/5}$
		m1		Rest of formula (using t_4 or t_5 (2.015))
	= (377, 385)	A1		AWRT
			6	
(b)	3	B1		CAO
			1	
			7	

Q2	Solution	Marks	Total	Comments
(a)	<table><tr><th></th><th>E</th><th>S</th><th>W</th><th>NI</th><th>Total</th></tr><tr><td>Male</td><td>57</td><td>44</td><td>27</td><td>17</td><td>145</td></tr><tr><td>Female</td><td>39</td><td>43</td><td>19</td><td>4</td><td>105</td></tr><tr><td>Total</td><td>96</td><td>87</td><td>46</td><td>21</td><td>250</td></tr></table>	B2,1	**2**	B2 all correct, B1 one slip.
(b)	<table><tr><td>Expected</td><th>E</th><th>S</th><th>W</th><th>NI</th></tr><tr><td>Male</td><td>55.68</td><td>50.46</td><td>26.68</td><td>12.18</td></tr><tr><td>Female</td><td>40.32</td><td>36.54</td><td>19.32</td><td>8.82</td></tr></table> <table><tr><td>0.03129..</td><td>0.82702..</td><td>0.00383..</td><td>1.90742..</td></tr><tr><td>0.04321..</td><td>1.14207..</td><td>0.00530..</td><td>2.63405..</td></tr></table> Sum = 6.59 $\nu = (4 - 1)(2 - 1) = 3$ Critical value = 6.251 H_o:No association between country & gender H_1:Association between country & gender Test statistic in critical region, reject H_o There is significant evidence of association between country and gender.	M1 M1 A1 B1 B1 B1 A1 E1	**8**	Expected attempted, at least 2 correct to 3 s.f. $(O - E)^2/E$ attempted, at least 1 correct to 3 s.f. AWFW 6.58 – 6.60 CAO AWRT 6.25 B2 for just 6.25 seen At least 1 correct – must be in context. Comparison of 6.59 with 6.251 Dep on 6.59 A1 and 6.251 B1 and on hypotheses B1 Conclusion in context Dep on previous A1 and B1
(c)	More females than expected from Scotland Fewer females than expected from N.I. About the right number of females from England and/or Wales	B1	**1**	For any one of these
			11	

If they combine Wales and Northern Ireland

Q2	Solution	Marks	Total	Comments
(b)	<table><tr><td>Expected</td><th>E</th><th>S</th><th>W + NI</th></tr><tr><td>Male</td><td>55.68</td><td>50.46</td><td>38.86</td></tr><tr><td>Female</td><td>40.32</td><td>36.54</td><td>28.14</td></tr></table> <table><tr><td>0.03129..</td><td>0.82702..</td><td>0.67986..</td></tr><tr><td>0.04321..</td><td>1.14207..</td><td>0.93886</td></tr></table> Sum = 3.66 $\nu = (3 - 1)(2 - 1) = 2$ Critical value = 4.605 H_o:No association between country & gender H_1:Association between country & gender Test statistic not in critical region, accept H_o There is no significant evidence of association between country and gender.	M1 M1 A0 B1F B1F B1 A0 E0	A maximum of 5 out of 8	Expected attempted, at least 2 correct to 3 s.f. $(O - E)^2/E$ attempted, at least 1 correct to 3 s.f. AWFW 4.60 to 4.61 B2F for just 4.60 or 4.61 seen At least 1 correct – must be in context A maximum of 5 out of 8

Q3	Solution	Marks	Total	Comments
(a)	$P(X \leq 4) = 0.3$ So $P(\text{Both} \leq 4) = 0.3^2 = 0.09$	M1 A1	**2**	CAO
(b)(i)	$0.1 + 0.2 + a + 0.3 + b = 1$ so $a + b = 0.4$ $3 \times 0.1 + 4 \times 0.2 + 5a + 6 \times 0.3 + 7b = 5.1$ $5a + 7b = 2.2$ and $5a + 5b = 2.0$ or substitution of $b = 0.4 - a$ or $a = 0.4 - b$ leading to $a = 0.3$, $b = 0.1$	B1 M1 m1 A1	**4**	Correct treatment of simultaneous equations, starting with correctly simplified $5a + 7b = 2.2$ CAO
(ii)	$E(X^2) = 0.1\times3^2+0.2\times4^2+0.3\times5^2+0.3\times6^2+0.1\times7^2$ ($= 27.3$) $\text{Var}(X) = E(X^2) - E(X)^2 = 27.3 - 5.1^2 = 1.29$	M1 A1	**2**	Not simply $E(X^2) = 27.3$ AG
(iii)	Using $N = 2X - 5$ $E(N) = 2E(X) - 5 = 5.2$ $\text{Var}(N) = 2^2\text{Var}(X) = 5.16$ so $\sigma_N = 2.27$	M1 A1 A1	**3**	Or by use of 1, 3, 5, 7, 9 AWRT Or $2 \times \sqrt{1.29}$
			11	

Q4	Solution	Marks	Total	Comments
(a)(i)	Area of rectangle = 1 (or total probability) $= {}^1/_k \times (b - a) \rightarrow (b - a) = k$	M1 A1	**2**	AG
(ii)	$E(X) = {}^1/_2(a + b)$ (or $a + {}^1/_2k$)	B1	**1**	
(iii)	$E(X^2) = \int_a^b \frac{x^2}{k}\,dx$ $= [{}^1/_{3k}x^3]_a^b$ $= \frac{(b^3 - a^3)}{3(b - a)} = {}^1/_3(b^2 + ab + a^2)$	M1 m1 A1	**3**	k or $(b - a)$ For integration. Ignore limits Use of correct limits AG
(iv)	$\text{Var}(X) = E(X^2) - [E(X)]^2$ $= {}^4/_{12}(b^2 + ab + a^2) - {}^3/_{12}(a + b)^2$ $= {}^1/_{12}(b^2 - 2ab + a^2) = {}^1/_{12}(b - a)^2$.	M1 A1	**2**	Applied to this case (their mean) Either form or continued to ${}^1/_{12}k^2$
(b)	${}^1/_{12}(b - a)^2 = 3 \rightarrow (b - a) = 6$ b = 10 $E(X) = {}^1/_2(a + b) = 7$	M1 A1 A1	**3**	
			11	

Q5	Solution	Marks	Total	Comments
(a)	$\mu = 128 \div 40 = 3.2$ as required for λ $s^2 = 3.2410\ldots$ (Condone $\sigma^2 = 3.16$) which is close to λ, as required for Poisson	B1 B1 E1	**3**	 AWRT 3.24 or 3.16 Clearly stated (for either s^2 or σ^2)
(b)(i)	$1 - P(X \leq 5) = 1 - 0.8946$ $= 0.105(4)$	M1 A1	**2**	For attempt to subtract $P(X \leq 5)$ AWRT
(ii)	$P(X \leq 7) - P(X \leq 2)$ $0.9832 - 0.3799$ $= 0.603(3)$	M1 B1 A1	**3**	Attempt to use these two For either. AWFW 0.603 to 0.604
(iii)	$P(X = 0) = 0.0408$ or $e^{-3.2}$ or $P(X \geq 0) = 0.9592$ $1 - 0.9592^2$ (or $0.0408^2 + 2\times 0.0408 \times 0.9592$) $= 0.0799$	 B1 M1 A1	**3**	 For any of these seen to 3 d.p. AWFW 0.079 to 0.081
(c)	Using Po(8.2) $e^{-8.2} \times 8.2^9 \div 9! + e^{-8.2} \times 8.2^{10} \div 10!$ $= 0.231$	M1 m1 A1	**3**	Stated or use in formula or either of figures below seen Or Calc $P(\leq 10) - P(\leq 8)$ $= 0.79555 - 0.56465$ AWRT
			14	

Q6	Solution	Marks	Total	Comments
(a)	H_0: $\mu = 20$, H_1: $\mu \neq 20$	B1		Both
	$\bar{x} = 22.625$	B1		CAO
	$s = 4.5650066$ (or $\sigma = 4.27$)	B1		AWFW 4.56 – 4.57 (or AWRT 4.27)
	test stat = $\dfrac{22.625 - 20}{(4.5650066 \div \sqrt{8})}$	M1		Or $\sqrt{7}$ if $\sigma = 4.27$ used
	$= 1.626$	A1		AWRT 1.63
	$t_7 = \pm 1.895$	B1		
	Test statistic not in critical region, accept H_o	A1		Comparison of test stat with t_7
	There is insufficient evidence that Gary does not take a mean time of 20 minutes for an annual service.	E1		In context. These last two marks dep on both A1s and hypotheses B1. E1 also dep on previous A1.
	Alternative: If the boundaries of the critical region are calculated, marks as above except $20 \pm 1.895 \times (4.5650066 \div \sqrt{8})$ M1 ((16.94), 23.06) A1 (AWRT)		**8**	
(b)	5% sig gives $z = 1.64$ to 1.65	B1		AWFW
	$20 + 1.6449 \times (4.6 \div \sqrt{100})$	M1		OE
	$= 20 + 0.754$ to 0.759	A1		AWFW
	So to not support suspicion need $\bar{x} \leq 20.75$	A1		
	SC 20.76 using this method scores B1, M1, A1, A0		**4**	
			12	

Q7	Solution	Marks	Total	Comments
(a)	$P(X < 1) = \int_0^1 \frac{4x}{5}\,dx$ or $\frac{1}{2} \times 1 \times \frac{4}{5}$ $= \left[\frac{2}{5}x^2\right]_0^1 = \frac{2}{5}$	M1 A1	**2**	Including limits
(b)(i)	$\int_1^x \frac{1}{20}(3t^2 - 20t + 33)\,dt$ $= \left[\frac{1}{20}(t^3 - 10t^2 + 33t)\right]_1^x$ $= \frac{1}{20}(x^3 - 10x^2 + 33x) - \frac{1}{20}(1 - 10 + 33)$ $F(x) = \frac{2}{5} + \frac{1}{20}(x^3 - 10x^2 + 33x) - \frac{24}{20}$ $= \frac{1}{20}(x^3 - 10x^2 + 33x - 16)$	M1 A1 m1 A1	**4**	Accept x integral Correct integration with limits Use of limits With $\frac{2}{5}$ included AG
(ii)	$F(1.13) = 0.49819\ldots$ $F(1.14) = 0.50527\ldots$ Median requires $F(x) = 0.5$ $0.49819\ldots < 0.5 < 0.50527\ldots$ So $1.13 <$ median < 1.14 Alternative scheme for **(b)(ii)** If a calculator, or trial and improvement, has been used to solve the cubic equation directly: $\frac{1}{20}(x^3 - 10x^2 + 33x - 16) = 0.5$ median = AWFW 1.132 to 1.133 which lies between 1.13 and 1.14	B1 B1 E1 M1 A1 E1	**3**	At least 3 s.f. At least 3 s.f. Must clearly indicate that median requires $F(x) = 0.5$
			9	

A-LEVEL

Mathematics

Statistics MS2 – MS2B
Mark scheme

6360
June 2015

Version/Stage 1.0 Final

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Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Marks	Total	Comments
(a)	Use of Po(2.8)	M1		Stated or table value (0.8477, 0.9349, 0.9756 or 3sf equivalents) seen
	P(≤ 5) = **0.935**	A1	**2**	AWRT
(b)	Use of Po(4.4)	B1		Stated or attempt at method seen
	$e^{-4.4} \times 4.4^2 \div 2$	M1		Correct formula or by calculator
	= **0.119**	A1	**3**	AWRT
(c)	Use of Po(15)	B1		Stated or any 1 of 4 relevant values seen 0.1848, 0.2676, 0.7489, 0.8195
	We require P(≤ 17)	M1		Stated **or** use of 0.7489
	– P(≤ 11)	M1		Indep. Stated **or** use of 0.1848
	= 0.7489 – 0.1848 = **0.564**(1)	A1	**4**	AWRT 0.564
		Total	**9**	

Note: (a) The mark is not awarded for simply 2.8. Some indication of Poisson is needed. Eg. Po(2.8) or $\lambda = 2.8$

(b) As for part (a), not simply 4.4.

(c) If Po(15) and P(≤ 17) – P(≤ 11) are seen, 3 marks have been earned irrespective of later numbers.

Q2	Solution	Marks	Total	Comments
(a)	$k = (b - a)$	B1	**1**	CAO
(b)(i)	$\frac{1}{2}(a + b) = 1$ and $\frac{1}{12}(b - a)^2 = 3$	B1		For both equations (not including k)
	$(b - a)^2 = 36 \rightarrow (b - a) = \pm 6$	M1		6 or ±6 required for this mark
	$b > a$ stated giving $b - a = 6$ only **or** both $b - a = 6$ and $b - a = -6$ used	m1		Consideration of two solutions
	$\boldsymbol{b = 4}$ and $\boldsymbol{a = -2}$	A1	**4**	CAO not dependent on m1
(ii)	$P(X < 0) = \frac{1}{3}$	B1		Stated or used (accept 0.333)
	$4 \times p \times (1 - p)^3$ where p = candidate's stated $P(X < 0)$	M1		$0 < p < 1$
	$= \frac{32}{81}$ **(= 0.395)**	A1	**3**	CAO or AWRT 0.395
		Total	**8**	

(b)(i)	**Alternative solution** $\frac{1}{2}(a + b) = 1$ and $\frac{1}{12}(b - a)^2 = 3$	B1		For both equations (not including k)
	$b = 2 - a \rightarrow 4a^2 - 8a - 32 = 0$ **or** $a = 2 - b \rightarrow 4b^2 - 8b - 32 = 0$	M1		For obtaining one of these quadratics or equivalent
	$a = -2$, $b = 4$ and $a = 4$, $b = -2$	A1		For both correct pairs of solutions or one pair with any justification
	Selection of correct solution $\boldsymbol{b = 4}$ and $\boldsymbol{a = -2}$	A1	**4**	CAO not dependent on previous A1

Note: (b) Integration may be used but must reach the two correct equations to earn any marks.

Many will use $b - a = 6$, ignoring the ±, and obtain the correct values for a and b.
This scores B1 M1 m0 A1.

Q3	Solution	Marks	Total	Comments
(a)(i)	Mean of sample is **909.2**	B1		If wrong here, the B1 here may be earned for a correct value seen in (ii)
	Use of **1.96**	B1		AWRT
	$909.2 \pm 1.96 \times \frac{2.2}{\sqrt{8}}$	M1		Allow for M1 if AWFW 1.64 to 1.65 used instead of 1.96
	907.7, 910.7	A1	**4**	For both. AWRT
	Notes: **1** Seen use of s ⇒ B1 B1 M0 A0 max **2** Seen use of t ⇒ B1 B0 M0 A0 max **3** Seen use of t and s ⇒ B1 B0 M0 A0 max			
(ii)	$t_7 = 2.365$	B1		AWFW 2.36 to 2.37
	$s = 2.39$ or 2.24 (or $s^2 = 5.72$ or $5.00(5)$)	B1		AWRT
	$909.2 \pm (2.36 \text{ to } 2.37) \times se$	M1		Allow for M1 if AWFW 1.89 to 1.90 used instead of (2.36 to 2.37)
	where $se = 2.39/\sqrt{8}$ or $2.24/\sqrt{7}$			OE in terms of s^2
	907.2, 911.2	A1	**4**	For both. AWRT
	Notes: **1** Seen use of 2.2 ⇒ B1 B0 M0 A0 max **2** Seen use of z ⇒ B0 B1 M0 A0 max **3** Seen use of z and 2.2 ⇒ B0 B0 M0 A0			
(b)	**Both** confidence intervals are **above 907** so **mean/average** weight is probably acceptable	Edep1		OE Dependent on A1 in (i) and A1 in (ii). Must specify **both, 907** and **mean/average.**
	One of data values (or 905.6) is below 907 (or underweight)	E1	**2**	
		Total	**10**	

Note: In both (a)(i) and (ii), **where working is shown**, condone accuracy to more than 4 s.f. **Where working is not shown**, if accurate to 4 s.f. allow B4. If not accurate to 4 s.f., award B1 for AWRT 908 – 911 in (i) and another B1 for AWRT 907 – 911 in (ii).

Q4	Solution	Marks	Total	Comments
(a)	(The 100 vehicles can be regarded as a) **random (**sample).	B1	**1**	Must say random and be about the sample. Do not penalise "and independent", but any mention of "normal" anywhere in (a) scores B0
(b)	H_0: $\mu_x = 44.1$ H_1: $\mu_x < 44.1$	B1		Both. Must be "Population mean", μ_x or μ.
	($\bar{x}$ =) **43.27**	B1		CAO
	sd = 3.0579 (var = 9.35 AWRT) or sd = 3.0425 (var = 9.26 AWRT)	B1		AWFW 3.055 to 3.060. AWFW 3.040 to 3.045
	$z / t = \dfrac{(43.27 - 44.1)}{\dfrac{(3.055\text{ to }3.060)}{\sqrt{100}}}$ or $\dfrac{(43.27 - 44.1)}{\dfrac{(3.040\text{ to }3.045)}{\sqrt{99}}}$	M1 m1		Denominator is division of candidate's sd by √100 or √99 Numerator is $\pm(\bar{x} - 44.1 \text{ or } 40)$
	= **–**2.71…	A1		AWFW –2.695 to –2.735
	CV: z = **–2.32(63)** or t = **–2.36(46)**	B1		AWFW –2.32 to –2.33 AWFW –2.36 to –2.37
	So test statistic in critical region. (Reject H_0), significant evidence that **mean speed has reduced.**	Adep1	**8**	Dep on preceding A1 and B1, but not on B1 for hypotheses. Must have context and mean (or average).
(c) (i)	Concluding that the mean speed has reduced (or changed) when in fact it has not	E1		Must be in context. Must refer to **mean** speed (μ)
(ii)	Concluding that the mean speed is still 44.1 when in fact it has reduced (or changed)	E1	**2**	Must be in context. Must refer to **mean** speed (μ)
		Total	**11**	

Note: (a) "It is random" is sufficient for B1.
"It is random and normally distributed" scores B0.
"The vehicles arrive at random" scores B0

The final A mark is not awarded for the negative statement "There is no significant evidence that the mean speed is 44.1" or equivalent. There **is** significant evidence of a reduction in the mean. A definite statement "the mean speed has reduced" is accepted for A1.

Alternative method for (b) using critical value for $\overline{x}$

Q4	Solution	Marks	Total	Comments
(b)	H_0: $\mu_x = 44.1$ H_1: $\mu_x < 44.1$	B1		Both. Must be "Population mean", μ_x or μ.
	($\overline{x}$ =) **43.27**	B1		CAO
	sd = 3.0579 (var = 9.35 AWRT) or sd = 3.0425 (var = 9.26 AWRT)	B1		AWFW 3.055 to 3.060. AWFW 3.040 to 3.045
	CV: z = **–2.32(63)** **or** t = **–2.36(46)**	B1		AWFW –2.32 to –2.33 AWFW –2.36 to –2.37
	$\overline{x}_{cv} = 44.1 - CV \times \frac{3.0579}{\sqrt{100}}$ or $\times \frac{3.0425}{\sqrt{99}}$	M1		Division of candidate's sd by $\sqrt{100}$ or $\sqrt{99}$
		m1		Rest of formula
	= 43.37 to 43.395	A1		AWFW 43.37 to 43.395
	43.27 < 43.37 or 43.395 So test statistic in critical region. (Reject H_0), significant evidence that **mean speed has reduced.**	Adep1		Dep on preceding A1 and B1, but not on B1 for hypotheses. Must have context and mean (or average).
			8	

Alternative method for (b) using confidence interval for $\overline{x}$

Q4	Solution	Marks	Total	Comments
(b)	H_0: $\mu_x = 44.1$ H_1: $\mu_x < 44.1$	B1		Both. Must be "Population mean", μ_x or μ.
	($\overline{x}$ =) **43.27**	B1		CAO
	sd = 3.0579 (var = 9.35 AWRT) or sd = 3.0425 (var = 9.26 AWRT)	B1		AWFW 3.055 to 3.060. AWFW 3.040 to 3.045
	CV: z = **–2.32(63)** or t = **–2.36(46)**	B1		AWFW –2.32 to –2.33 AWFW –2.36 to –2.37
	Upper limit of confidence interval $= 43.27 + CV \times \frac{3.0579}{\sqrt{100}}$ or $\times \frac{3.0425}{\sqrt{99}}$	M1		Division of candidate's sd by $\sqrt{100}$ or $\sqrt{99}$
		m1		Rest of formula
	= 43.975 to 43.999	A1		AWFW 43.975 to 43.999
	44.1 > 43.975 to 43.999 So previous mean above confidence interval. (Reject H_0), significant evidence that **mean speed has reduced.**	Adep1		Dep on preceding A1 and B1, but not on B1 for hypotheses. Must have context and mean (or average)
			8	

Q5	Solution	Marks	Total	Comments
(a)	H_o:**No association** (between the age at which they had left education and the rate of income tax that they were paying) (H_1:Association …..)			Allow "rate of tax **independent** of age of leaving" but no other words.
		B1		For at least H_o stated correctly.
	29.445 | 3.9 | 5.655 98.905 | 13.1 | 18.995 22.65 | 3 | 4.35	M1		Expected values attempted, seen here or after combining (at least 4 correct (at least 2dp in 1st and 3rd columns))
	Combine last two columns **Observed**: ≤ 16 | > 16 32 | **7** 102 | **29** 17 | **13** **Expected**: ≤ 16 | > 16 29.445 | **9.555** 98.905 | **32.095** 22.65 | **7.35**	M1		Attempt at combining columns 2 & 3 (not just individual cells)
		A1		Combined columns numerically correct (six values)
	$\Sigma (O_i - E_i)^2/E_i = 0.2217.. + 0.0968.. + 1.4093.. + 0.6832.. + 0.2984.. + 4.3431..$	m1		Attempt at $\Sigma(O_i - E_i)^2/E_i$ dep on first M1 (at least 2 values correct to 3sf) Can be implied by correct answer.
	= **7.05**	A1		AWFW 7.0 to 7.1
	$v = (3 - 1)(2 - 1) = 2$	B1		Can be implied by correct answer. Correct v or $v = 4$ from no combining
	Crit val = **5.99**(1)	B1		AWRT 5.99 or 9.488 from no combining
	(Reject H_o) Significant evidence that there is an **association** between age at leaving education and rate of income tax paid.	Adep1	**9**	Dep on A1 for 7.05, B1 for 5.99. For conclusion in context.
(b)	Belief supported (or equivalent).	E1	**1**	Must be supported by reference to stated O and E values, comparing 8 with 4.35 or 13 with 7.35, or other numerical justification, comparing $^8/_{29}$ (27.6%) with $^{17}/_{151}$ (11.3%) or $^{13}/_{49}$ (26.5%) with $^{17}/_{151}$ (11.3%).
		Total	**10**	
	No combining can score B1 M1 M0 A0 m1 A0 B1 B1 Adep0 = max of 5 out of 9 (gives 7.118) Combining first and third rows can also score B1 M1 M0 A0 m1 A0 B1 B1 Adep0 = max of 5 out of 9 (gives 1.156). Use of Yates automatically loses m1 A1 and Adep1.			

Note: (a) Final A mark is not awarded for the double negative statement "No significant evidence that there is no association ….". There **is** significant evidence of an association. A definite conclusion "there is an association between age at leaving education and rate …" is accepted for A1 "**Association**" is the expected word. Use of **independent** must say "tax rate is not independent of age …". No other words are accepted.

Q6	Solution	Marks	Total	Comments
(a)	$F(0.4) = \frac{0.4}{2} - \frac{0.16}{16} = 0.2 - 0.01 = \mathbf{0.19}$ $F(0.8) = \frac{0.8}{2} - \frac{0.64}{16} = 0.4 - 0.04 = \mathbf{0.36}$ $P(0.4 < X < 0.8) = 0.36 - 0.19 = \mathbf{0.17}$	M1 A1	**2**	For either, can be implied by correct answer. CAO
(b)	Clear correct use of differentiation of $F(x)$.	B1	1	AG Sight of $F'(x)$, $\frac{d}{dx}$, $\frac{dy}{dx}$ etc. = correct answer
(c)(i)	$E(X) = \int_0^4 (\frac{1}{2}x - \frac{1}{8}x^2)\,dx$ $= [\frac{1}{4}x^2 - \frac{1}{24}x^3]_0^4$ $= 4 - \frac{8}{3} = \mathbf{\frac{4}{3}}$	M1 A1 A1	**3**	Attempt at integrating $xf(x)$ (condone omission of limits and dx) Integration completed **correctly with limits** OE exact form
(ii)	$E(X^2) = \int_0^4 (\frac{1}{2}x^2 - \frac{1}{8}x^3)\,dx$ $= [\frac{1}{6}x^3 - \frac{1}{32}x^4]_0^4$ $= \frac{32}{3} - 8 = \mathbf{\frac{8}{3}}$ $Var(X) = E(X^2) - E(X)^2 = \frac{8}{3} - (\frac{4}{3})^2 \quad (= \mathbf{\frac{8}{9}})$	M1 A1 A1 A1	**4**	Attempt at integrating $x^2f(x)$ (condone omission of limits and dx) Integration completed **correctly with limits** OE exact form AG
(d)	$E(Y) = 3E(X) - 2 = 3 \times \frac{4}{3} - 2 = \mathbf{2}$ $Var(Y) = 3^2 \times Var(X) = 9 \times \frac{8}{9} = \mathbf{8}$	B1F B1	**2**	FT their (c)(i) provided $0 < E(X) < 4$ CAO
		Total	**12**	

Q7	Solution	Marks	Total	Comments
(a)	(I) a requires the "= 3" value using Po(2) $= (e^{-2} \times 2^3) \div 3!$ or 0.8571 – 0.6767 or 0.1804… from calculator = **0.180** (II) $b = 1 - P(\text{demand} \leq 3) = 1 - 0.8571 =$ **0.143** (III) $b = 1 - (0.135+0.271+0.271+ 0.180) =$ **0.143** (IV) $a = 1 - (0.135+ 0.271+ 0.271+ 0.143) =$ **0.180** (I) & (II) or (I) & (III) or (II) & (IV) SC If M0 can award B1 for $a + b = 0.323$ derived from sum of probabilities = 1	M1 m1 A1	**3**	One M1 for correct use of correct Poisson for either a or b. A dependent m1 for use of Poisson again for b or a or for subsequent use of probability sum = 1 A1 for both correct calculations AG
(b)	$E(X) =$ $1\times0.135+2\times0.271+3\times0.271+4\times0.180+5\times0.143$ $(= 0.135 + 0.542 + 0.813 + 0.72 + 0.715)$ $=$ **2.925** $E(X^2) = 1^2\times0.135 + 2^2\times0.271 + 3^2\times0.271 +$ $4^2\times0.180 + 5^2\times0.143$ $(= 0.135 + 1.084 + 2.439 + 2.88 + 3.575)$ $=$ **10.113** S.D. $= \sqrt{(10.113 - 2.925^2)} =$ **1.25**	M1 A1 M1 A1 B1	**5**	Evidence of at least two of the five products added OE AWFW 2.92 to 2.93 Evidence of at least two of the five products added AWRT 10.1 AWRT
(c)	$1 \times E(X) - 0.5 \times (5 - E(X))$ = £**1.89** **or profit/loss table** E(Profit) = –0.135 + 0.135 + 0.542 + 0.630 + 0.715 = £**1.89**	M1 A1 (M1) (A1)	**2**	Candidate's $E(X)$ AWRT Condone omission of '£' AWRT Condone omission of '£'

Profit	-1	0.5	2	3.5	5
$P(X = x)$	0.135	0.271	0.271	0.180	0.143

Note: (a) One of the three methods of getting 0.180 – formula, subtraction of two figures from tables, or direct calculation showing fourth decimal place (4) – must be seen before the M1 for use of Poisson is awarded. Similarly for 0.143 (0.1429) done using Poisson.

If value of $E(X)$ calculated in part (b) is used retrospectively in part (a) to calculate a and b, then only the SC B1 can be earned.

Q7	Solution	Marks	Total	Comments
(d)	New distribution x | 1 | 2 | 3 | 4 $P(X = x)$ | 0.135 | 0.271 | 0.271 | 0.323	M1 A1		P(demand ≥ 3)= P($X = 4$) Complete distribution (not necessarily in a table)
	$E(X) = 1\times0.135+2\times0.271+3\times0.271+4\times0.323$ $= 2.782$	A1		E(X) = 2.78(2) without shown working scores B3
	E(Profit) = $1 \times E(X) - 0.5 \times (4 - E(X))$ = £**2.17** (which is more than £1.89)	M1 A1		AWRT Condone omission of '£'
	or profit/loss table Profit | -0.5 | 1 | 2.5 | 4 $P(X = x)$ | 0.135 | 0.271 | 0.271 | 0.323	(M1) (A1) (A1)		Any two profit values correct P(demand ≥ 3)= P($X = 4$) Complete table
	E(Profit) = $-0.5\times0.135 + 1\times0.271 + 2.5\times0.271 + 4\times0.323$ = £**2.17** (which is more than £1.89)	(M1) (A1)	**5**	AWRT Condone omission of '£'
		Total	**15**	