OCR Maths S1

Topic Questions from Papers

Binomial and Geometric Distributions

1 On average 1 in 20 members of the population of this country has a particular DNA feature. Members of the population are selected at random until one is found who has this feature.

(i) Find the probability that the first person to have this feature is

(a) the sixth person selected, [3]

(b) not among the first 10 people selected. [3]

(ii) Find the expected number of people selected. [2]

(Q5, Jan 2005)

2 It is known that, on average, one match box in 10 contains fewer than 42 matches. Eight boxes are selected, and the number of boxes that contain fewer than 42 matches is denoted by Y.

(i) State two conditions needed to model Y by a binomial distribution. [2]

Assume now that a binomial model is valid.

(ii) Find

(a) $\mathrm{P}(Y = 0)$, [2]

(b) $\mathrm{P}(Y \geqslant 2)$. [2]

(iii) On Wednesday 8 boxes are selected, and on Thursday another 8 boxes are selected. Find the probability that on one of these days the number of boxes containing fewer than 42 matches is 0, and that on the other day the number is 2 or more. [3]

(Q7, Jan 2005)

3 The probability that a certain sample of radioactive material emits an alpha-particle in one unit of time is 0.14. In one unit of time no more than one alpha-particle can be emitted. The number of units of time up to and including the first in which an alpha-particle is emitted is denoted by T.

(i) Find the value of

(a) $\mathrm{P}(T = 5)$, [3]

(b) $\mathrm{P}(T < 8)$. [3]

(ii) State the value of $\mathrm{E}(T)$. [2]

(Q2, June 2005)

4 In a supermarket the proportion of shoppers who buy washing powder is denoted by p. 16 shoppers are selected at random.

(i) Given that $p = 0.35$, use tables to find the probability that the number of shoppers who buy washing powder is

(a) at least 8, [3]

(b) between 4 and 9 inclusive. [2]

(ii) Given instead that $p = 0.38$, find the probability that the number of shoppers who buy washing powder is exactly 6. [3]

(Q3, June 2005)

5 **(i)** The random variable X has the distribution $\mathrm{B}(25, 0.2)$. Using the tables of cumulative binomial probabilities, or otherwise, find $\mathrm{P}(X \geqslant 5)$. [2]

(ii) The random variable Y has the distribution $\mathrm{B}(10, 0.27)$. Find $\mathrm{P}(Y = 3)$. [2]

(iii) The random variable Z has the distribution $\mathrm{B}(n, 0.27)$. Find the smallest value of n such that $\mathrm{P}(Z \geqslant 1) > 0.95$. [3]

(Q4, June 2006)

6 Henry makes repeated attempts to light his gas fire. He makes the modelling assumption that the probability that the fire will light on any attempt is $\frac{1}{3}$.

Let X be the number of attempts at lighting the fire, up to and including the successful attempt.

(i) Name the distribution of X, stating a further modelling assumption needed. [2]

In the rest of this question, you should use the distribution named in part **(i)**.

(ii) Calculate

(a) $\mathrm{P}(X = 4)$, [3]

(b) $\mathrm{P}(X < 4)$. [3]

(iii) State the value of $\mathrm{E}(X)$. [1]

(iv) Henry has to light the fire once a day, starting on March 1st. Calculate the probability that the first day on which fewer than 4 attempts are needed to light the fire is March 3rd. [3]

(Q8, June 2006)

7 A coin is biased so that the probability that it will show heads on any throw is $\frac{2}{3}$. The coin is thrown repeatedly.

The number of throws up to and including the first head is denoted by X. Find

(i) $P(X = 4)$, [3]

(ii) $P(X < 4)$, [3]

(iii) $E(X)$. [2]

(Q6, Jan 2007)

8 A variable X has the distribution $B(11, p)$.

(i) Given that $p = \frac{3}{4}$, find $P(X = 5)$. [2]

(ii) Given that $P(X = 0) = 0.05$, find p. [4]

(iii) Given that $\text{Var}(X) = 1.76$, find the two possible values of p. [5]

(Q9, Jan 2007)

9 On average, 25% of the packets of a certain kind of soup contain a voucher. Kim buys one packet of soup each week for 12 weeks. The number of vouchers she obtains is denoted by X.

(i) State two conditions needed for X to be modelled by the distribution $B(12, 0.25)$. [2]

In the rest of this question you should assume that these conditions are satisfied.

(ii) Find $P(X \leqslant 6)$. [2]

In order to claim a free gift, 7 vouchers are needed.

(iii) Find the probability that Kim will be able to claim a free gift at some time during the 12 weeks. [1]

(iv) Find the probability that Kim will be able to claim a free gift in the 12th week but not before. [4]

(Q7, June 2007)

10 **(i)** A random variable X has the distribution $\text{Geo}\left(\frac{1}{5}\right)$. Find

(a) $E(X)$, [2]

(b) $P(X = 4)$, [2]

(c) $P(X > 4)$. [2]

(ii) A random variable Y has the distribution $\text{Geo}(p)$, and $q = 1 - p$.

(a) Show that $P(Y \text{ is odd}) = p + q^2p + q^4p + \dots$. [1]

(b) Use the formula for the sum to infinity of a geometric progression to show that

$$P(Y \text{ is odd}) = \frac{1}{1+q}.$$ [4]

(Q9, June 2007)

11 A random variable T has the distribution $\text{Geo}\left(\frac{1}{5}\right)$. Find

(i) $P(T = 4)$, [2]

(ii) $P(T > 4)$, [2]

(iii) $E(T)$. [1]

(Q2, Jan 2008)

12 **(i)** 20% of people in the large town of Carnley support the Residents' Party. 12 people from Carnley are selected at random. Out of these 12 people, the number who support the Residents' Party is denoted by U.

Find

(a) $P(U \leqslant 5)$, [2]

(b) $P(U \geqslant 3)$. [3]

(ii) 30% of people in Carnley support the Commerce Party. 15 people from Carnley are selected at random. Out of these 15 people, the number who support the Commerce Party is denoted by V.

Find $P(V = 4)$. [3]

(Q5, Jan 2008)

13 **(i)** Andrew plays 10 tennis matches. In each match he either wins or loses.

(a) State, in this context, two conditions needed for a binomial distribution to arise. [2]

(b) Assuming these conditions are satisfied, define a variable in this context which has a binomial distribution. [1]

(ii) The random variable X has the distribution $\mathrm{B}(21, p)$, where $0 < p < 1$.

Given that $\mathrm{P}(X = 10) = \mathrm{P}(X = 9)$, find the value of p. [5]

(Q7, Jan 2008)

14 Erika is a birdwatcher. The probability that she will see a woodpecker on any given day is $\frac{1}{8}$. It is assumed that this probability is unaffected by whether she has seen a woodpecker on any other day.

(i) Calculate the probability that Erika first sees a woodpecker

(a) on the third day, **[3]**

(b) after the third day. **[3]**

(ii) Find the expectation of the number of days up to and including the first day on which she sees a woodpecker. **[1]**

(iii) Calculate the probability that she sees a woodpecker on exactly 2 days in the first 15 days. **[3]**

(Q3, Jan 2009)

15 At a factory that makes crockery the quality control department has found that 10% of plates have minor faults. These are classed as ‘seconds’. Plates are stored in batches of 12. The number of seconds in a batch is denoted by X.

(i) State an appropriate distribution with which to model X. Give the value(s) of any parameter(s) and state any assumptions required for the model to be valid. **[4]**

Assume now that your model is valid.

(ii) Find

(a) $\mathrm{P}(X = 3)$, **[2]**

(b) $\mathrm{P}(X \geqslant 1)$. **[2]**

(iii) A random sample of 4 batches is selected. Find the probability that the number of these batches that contain at least 1 second is fewer than 3. **[4]**

(Q7, Jan 2009)

16 20% of packets of a certain kind of cereal contain a free gift. Jane buys one packet a week for 8 weeks. The number of free gifts that Jane receives is denoted by X. Assuming that Jane's 8 packets can be regarded as a random sample, find

(i) $\mathrm{P}(X = 3)$, **[3]**

(ii) $\mathrm{P}(X \geqslant 3)$, **[2]**

(iii) $\mathrm{E}(X)$. **[2]**

(Q1, June 2009)

17 30% of people own a Talk-2 phone. People are selected at random, one at a time, and asked whether they own a Talk-2 phone. The number of people questioned, up to and including the first person who owns a Talk-2 phone, is denoted by X. Find

(i) $\mathrm{P}(X = 4)$, **[3]**

(ii) $\mathrm{P}(X > 4)$, **[2]**

(iii) $\mathrm{P}(X < 6)$. **[3]**

(Q4, June 2009)

18 Repeated independent trials of a certain experiment are carried out. On each trial the probability of success is 0.12.

(i) Find the smallest value of n such that the probability of at least one success in n trials is more than 0.95. **[3]**

(ii) Find the probability that the 3rd success occurs on the 7th trial. **[5]**

(Q9, June 2009)

19 Andy makes repeated attempts to thread a needle. The number of attempts up to and including his first success is denoted by X.

(i) State two conditions necessary for X to have a geometric distribution. **[2]**

(ii) Assuming that X has the distribution $\mathrm{Geo}(0.3)$, find

(a) $\mathrm{P}(X = 5)$, **[2]**

(b) $\mathrm{P}(X > 5)$. **[3]**

(iii) Suggest a reason why one of the conditions you have given in part **(i)** might not be satisfied in this context. **[2]**

(Q1, Jan 2010)

20 R and S are independent random variables each having the distribution Geo(p).

(i) Find P($R = 1$ and $S = 1$) in terms of p. **[1]**

(ii) Show that P($R = 3$ and $S = 3$) $= p^2q^4$, where $q = 1 - p$. **[1]**

(iii) Use the formula for the sum to infinity of a geometric series to show that

$$P(R = S) = \frac{p}{2 - p}.$$ **[5]**

(Q9, Jan 2010)

21 **(i)** The random variable W has the distribution $B\left(10, \frac{1}{3}\right)$. Find

(a) $P(W \leqslant 2)$, **[1]**

(b) $P(W = 2)$. **[2]**

(ii) The random variable X has the distribution B(15, 0.22).

(a) Find $P(X = 4)$. **[2]**

(b) Find E(X) and Var(X). **[3]**

(Q4, June 2010)

22 The proportion of people who watch *West Street* on television is 30%. A market researcher interviews people at random in order to contact viewers of *West Street*. Each day she has to contact a certain number of viewers of *West Street*.

(i) Near the end of one day she finds that she needs to contact just one more viewer of *West Street*. Find the probability that the number of further interviews required is

(a) 4, **[3]**

(b) less than 4. **[3]**

(ii) Near the end of another day she finds that she needs to contact just two more viewers of *West Street*. Find the probability that the number of further interviews required is

(a) 5, **[4]**

(b) more than 5. **[2]**

(Q8, June 2010)

23 The random variable X has the distribution Geo(0.2). Find

(i) $P(X = 3)$, **[2]**

(ii) $P(3 \leqslant X \leqslant 5)$, **[3]**

(iii) $P(X > 4)$. **[3]**

Two independent values of X are found.

(iv) Find the probability that the total of these two values is 3. **[3]**

(Q2, Jan 2011)

24 30% of packets of Natural Crunch Crisps contain a free gift. Jan buys 5 packets each week.

(i) The number of free gifts that Jan receives in a week is denoted by X. Name a suitable probability distribution with which to model X, giving the value(s) of any parameter(s). State any assumption(s) necessary for the distribution to be a valid model. **[4]**

Assume now that your model is valid.

(ii) Find

(a) $P(X \leqslant 2)$, **[1]**

(b) $P(X = 2)$. **[2]**

(iii) Find the probability that, in the next 7 weeks, there are exactly 3 weeks in which Jan receives exactly 2 free gifts. **[3]**

(Q5, Jan 2011)

25 **(i)** A random variable, X, has the distribution B(12, 0.85). Find

(a) $P(X > 10)$, **[2]**

(b) $P(X = 10)$, **[2]**

(c) $\text{Var}(X)$. **[2]**

(ii) A random variable, Y, has the distribution $B(2, \frac{1}{4})$. Two independent values of Y are found. Find the probability that the sum of these two values is 1. **[4]**

(Q3, June 2011)

26 A random variable X has the distribution B(13, 0.12).

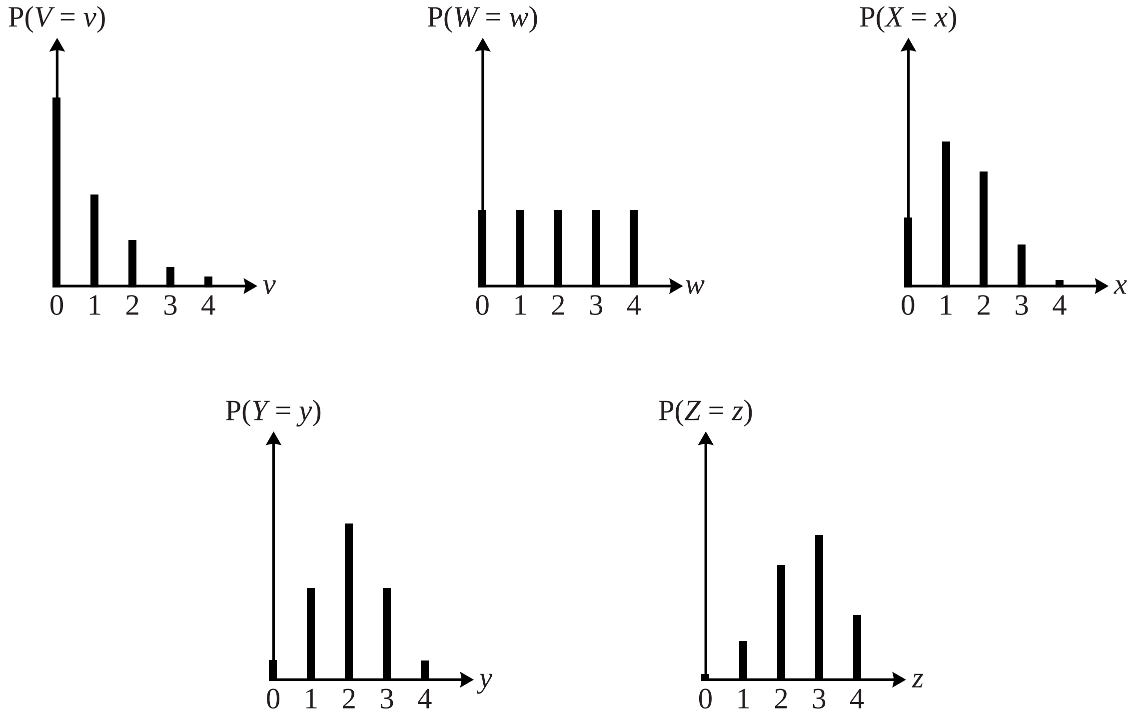
(i) Find $P(X < 2)$. **[3]**

Two independent values of X are found.

(ii) Find the probability that exactly one of these values is equal to 2. **[3]**

(Q3, Jan 2012)

27 The diagrams illustrate all or part of the probability distributions of the discrete random variables V, W, X, Y and Z.



(i) One of these variables has the distribution Geo($\frac{1}{2}$). State, with a reason, which variable this is. **[2]**

(ii) One of these variables has the distribution B(4, $\frac{1}{2}$). State, with reasons, which variable this is. **[3]**

(Q6, Jan 2012)

28 60% of the voters at a certain polling station are women. Voters enter the polling station one at a time. The number of voters who enter, up to and including the first woman, is denoted by X.

(i) State a suitable distribution that can be used as a model for X, giving the value(s) of any parameter(s). State also any necessary condition(s) for this distribution to be a good model. **[4]**

Use the distribution stated in part **(i)** to find

(ii) $P(X = 4)$, **[2]**

(iii) $P(X \geqslant 4)$. **[2]**

(Q7, Jan 2012)

29 On average, half the plants of a particular variety produce red flowers and the rest produce blue flowers.

(i) Ann chooses 8 plants of this variety at random. Find the probability that more than 6 plants produce red flowers. **[3]**

(ii) Karim chooses 22 plants of this variety at random.

(a) Find the probability that the number of these plants that produce blue flowers is equal to the number that produce red flowers. **[2]**

(b) Hence find the probability that the number of these plants that produce blue flowers is greater than the number that produce red flowers. **[3]**

(Q8, Jan 2012)

30 **(i)** The random variable X has the distribution $B(30, 0.6)$. Find $P(X \geqslant 16)$. **[2]**

(ii) The random variable Y has the distribution $B(4, 0.7)$.

(a) Find $P(Y = 2)$. **[2]**

(b) Three values of Y are chosen at random. Find the probability that their total is 10. **[6]**

(Q8, June 2012)

31 **(i)** A clock is designed to chime once each hour, on the hour. The clock has a fault so that each time it is supposed to chime there is a constant probability of $\frac{1}{10}$ that it will not chime. It may be assumed that the clock never stops and that faults occur independently. The clock is started at 5 minutes past midnight on a certain day. Find the probability that the first time it does not chime is

(a) at 0600 on that day, **[3]**

(b) before 0600 on that day. **[3]**

(ii) Another clock is designed to chime twice each hour: on the hour and at 30 minutes past the hour. This clock has a fault so that each time it is supposed to chime there is a constant probability of $\frac{1}{20}$ that it will not chime. It may be assumed that the clock never stops and that faults occur independently. The clock is started at 5 minutes past midnight on a certain day.

(a) Find the probability that the first time it does not chime is at either 0030 or 0130 on that day. **[2]**

(b) Use the formula for the sum to infinity of a geometric progression to find the probability that the first time it does not chime is at 30 minutes past some hour. **[3]**

(Q9, June 2012)

32 A random variable X has the distribution $\mathrm{B}(5, \frac{1}{4})$.

(i) Find

(a) $\mathrm{E}(X)$, **[1]**

(b) $\mathrm{P}(X = 2)$. **[2]**

(ii) Two values of X are chosen at random. Find the probability that their sum is less than 2. **[4]**

(iii) 10 values of X are chosen at random. Use an appropriate formula to find the probability that exactly 3 of these values are 2s. **[3]**

(Q5, Jan 2013)

33 Sandra makes repeated, independent attempts to hit a target. On each attempt, the probability that she succeeds is 0.1.

(i) Find the probability that

(a) the first time she succeeds is on her 5th attempt, **[2]**

(b) the first time she succeeds is after her 5th attempt, **[2]**

(c) the second time she succeeds is before her 4th attempt. **[4]**

Jill also makes repeated attempts to hit the target. Each attempt of either Jill or Sandra is independent. Each time that Jill attempts to hit the target, the probability that she succeeds is 0.2. Sandra and Jill take turns attempting to hit the target, with Sandra going first.

(ii) Find the probability that the first person to hit the target is Sandra, on her

(a) 2nd attempt, **[2]**

(b) 10th attempt. **[3]**

(Q8, Jan 2013)