1 Four letters are taken out of their envelopes for signing. Unfortunately they are replaced randomly, one in each envelope.

The probability distribution for the number of letters, X, which are now in the correct envelope is given in the following table.

r	0	1			
$\mathrm{P}(X = r)$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	0	$\frac{1}{24}$

(i) Explain why the case $X = 3$ is impossible. [1]

(ii) Explain why $\mathrm{P}(X = 4) = \frac{1}{24}$. [2]

(iii) Calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [5]

2 A company sells sugar in bags which are labelled as containing 450 grams.

Although the mean weight of sugar in a bag is more than 450 grams, there is concern that too many bags are underweight. The company can adjust the mean or the standard deviation of the weight of sugar in a bag.

(i) State two adjustments the company could make. [2]

The weights, x grams, of a random sample of 25 bags are now recorded.

(ii) Given that $\sum x = 11\,409$ and $\sum x^2 = 5206937$, calculate the sample mean and sample standard deviation of these weights. [3]

3 Jeremy is a computing consultant who sometimes works at home. The number, X, of days that Jeremy works at home in any given week is modelled by the probability distribution

$$P(X = r) = \frac{1}{40}r(r+1) \quad \text{for } r = 1, 2, 3, 4.$$

(i) Verify that $P(X = 4) = \frac{1}{2}$. [1]

(ii) Calculate $E(X)$ and $Var(X)$. [5]

(iii) Jeremy works for 45 weeks each year. Find the expected number of weeks during which he works at home for exactly 2 days. [2]

4 A sprinter runs many 100-metre trials, and the time, x seconds, for each is recorded. A sample of eight of these times is taken, as follows.

10.53 10.61 10.04 10.49 10.63 10.55 10.47 10.63

(i) Calculate the sample mean, $\bar{x}$, and sample standard deviation, s, of these times. [3]

(ii) Show that the time of 10.04 seconds may be regarded as an outlier. [2]

(iii) Discuss briefly whether or not the time of 10.04 seconds should be discarded. [2]

5 The number, X, of children per family in a certain city is modelled by the probability distribution $P(X = r) = k(6 - r)(1 + r)$ for $r = 0, 1, 2, 3, 4$.

(i) Copy and complete the following table and hence show that the value of k is $\frac{1}{50}$. [3]

r	0	1	2	3	4
$P(X = r)$	$6k$	$10k$			

(ii) Calculate $E(X)$. [2]

(iii) Hence write down the probability that a randomly selected family in this city has more than the mean number of children. [1]

6 The weights, w grams, of a random sample of 60 carrots of variety A are summarised in the table below.

Weight	$30 \leqslant w < 50$	$50 \leqslant w < 60$	$60 \leqslant w < 70$	$70 \leqslant w < 80$	$80 \leqslant w < 90$
Frequency	11	10	18	14	7

(i) Draw a histogram to illustrate these data. **[5]**

(ii) Calculate estimates of the mean and standard deviation of w. **[4]**

(iii) Use your answers to part **(ii)** to investigate whether there are any outliers. **[3]**

The weights, x grams, of a random sample of 50 carrots of variety B are summarised as follows.

$$n = 50 \qquad \Sigma x = 3624.5 \qquad \Sigma x^2 = 265\,416$$

(iv) Calculate the mean and standard deviation of x. **[3]**

(v) Compare the central tendency and variation of the weights of varieties A and B. **[2]**

7 A supermarket chain buys a batch of 10 000 scratchcard draw tickets for sale in its stores. 50 of these tickets have a £10 prize, 20 of them have a £100 prize, one of them has a £5000 prize and all of the rest have no prize. This information is summarised in the frequency table below.

Prize money	£0	£10	£100	£5000
Frequency	9929	50	20	1

(i) Find the mean and standard deviation of the prize money per ticket. **[4]**

(ii) I buy two of these tickets at random. Find the probability that I win either two £10 prizes or two £100 prizes. **[3]**