1 A coffee shop provides free internet access for its customers. It is known that the probability that a randomly selected customer is accessing the internet is 0.35, independently of all other customers.

(i) 10 customers are selected at random.

(A) Find the probability that exactly 5 of them are accessing the internet. **[3]**

(B) Find the probability that at least 5 of them are accessing the internet. **[2]**

(C) Find the expected number of these customers who are accessing the internet. **[2]**

Another coffee shop also provides free internet access. It is suspected that the probability that a randomly selected customer at this coffee shop is accessing the internet may be different from 0.35. A random sample of 20 customers at this coffee shop is selected. Of these, 10 are accessing the internet.

(ii) Carry out a hypothesis test at the 5% significance level to investigate whether the probability for this coffee shop is different from 0.35. Give a reason for your choice of alternative hypothesis. **[9]**

(iii) To get a more reliable result, a much larger random sample of 200 customers is selected over a period of time, and another hypothesis test is carried out. You are given that 90 of the 200 customers were accessing the internet. You are also given that, if X has the binomial distribution with parameters $n = 200$ and $p = 0.35$, then $\mathrm{P}(X \geqslant 90) = 0.0022$. Using the same hypotheses and significance level which you used in part **(ii)**, complete this test. **[2]**

2 A manufacturer produces titanium bicycle frames. The bicycle frames are tested before use and on average 5% of them are found to be faulty. A cheaper manufacturing process is introduced and the manufacturer wishes to check whether the proportion of faulty bicycle frames has increased. A random sample of 18 bicycle frames is selected and it is found that 4 of them are faulty. Carry out a hypothesis test at the 5% significance level to investigate whether the proportion of faulty bicycle frames has increased. **[8]**

3 It is known that 25% of students in a particular city are smokers. A random sample of 20 of the students is selected.

(i) (*A*) Find the probability that there are exactly 4 smokers in the sample. **[3]**

(*B*) Find the probability that there are at least 3 but no more than 6 smokers in the sample **[3]**

(*C*) Write down the expected number of smokers in the sample. **[1]**

A new health education programme is introduced. This programme aims to reduce the percentage of students in this city who are smokers. After the programme has been running for a year, it is decided to carry out a hypothesis test to assess the effectiveness of the programme. A random sample of 20 students is selected.

(ii) (*A*) Write down suitable null and alternative hypotheses for the test. **[3]**

(*B*) Explain why the alternative hypothesis has the form that it does **[1]**

(iii) Find the critical region for the test at the 5% level, showing all of your calculations. **[4]**

(iv) In fact there are 3 smokers in the sample. Complete the test, stating your conclusion clearly. **[2]**