1 A multinational accountancy firm receives a large number of job applications from graduates each year. On average 20% of applicants are successful.

A researcher in the human resources department of the firm selects a random sample of 17 graduate applicants.

(i) Find the probability that at least 4 of the 17 applicants are successful. [3]

(ii) Find the expected number of successful applicants in the sample. [2]

(iii) Find the most likely number of successful applicants in the sample, justifying your answer. [3]

It is suggested that mathematics graduates are more likely to be successful than those from other fields. In order to test this suggestion, the researcher decides to select a new random sample of 17 mathematics graduate applicants. The researcher then carries out a hypothesis test at the 5% significance level.

(iv) (*A*) Write down suitable null and alternative hypotheses for the test.

(*B*) Give a reason for your choice of the alternative hypothesis. [4]

(v) Find the critical region for the test at the 5% level, showing all of your calculations. [4]

(vi) Explain why the critical region found in part **(v)** would be unaltered if a 10% significance level were used. [2]

2 When onion seeds are sown outdoors, on average two-thirds of them germinate. A gardener sows seeds in pairs, in the hope that at least one will germinate.

(i) Assuming that germination of one of the seeds in a pair is independent of germination of the other seed, find the probability that, if a pair of seeds is selected at random,

(A) both seeds germinate,

(B) just one seed germinates,

(C) neither seed germinates. [3]

(ii) Explain why the assumption of independence is necessary in order to calculate the above probabilities. Comment on whether the assumption is likely to be valid. [2]

(iii) A pair of seeds is sown. Find the expectation and variance of the number of seeds in the pair which germinate. [3]

(iv) The gardener plants 200 pairs of seeds. If both seeds in a pair germinate, the gardener destroys one of the two plants so that only one is left to grow. Of the plants that remain after this, only 85% successfully grow to form an onion. Find the expected number of onions grown from the 200 pairs of seeds. [3]

If the seeds are sown in a greenhouse, the germination rate is higher. The seed manufacturing company claims that the germination rate is 90%. The gardener suspects that the rate will not be as high as this, and carries out a trial to investigate. 18 randomly selected seeds are sown in the greenhouse and it is found that 14 germinate.

(v) Write down suitable hypotheses and carry out a test at the 5% level to determine whether there is any evidence to support the gardener's suspicions. [7]

3 Douglas plays darts, and the probability that he hits the number he is aiming at is 0.87 for any particular dart.

Douglas aims a set of three darts at the number 20; the number of times he is successful can be modelled by $\mathrm{B}(3, 0.87)$.

(i) Calculate the probability that Douglas hits 20 twice. [3]

(ii) Douglas aims fifty sets of 3 darts at the number 20. Find the expected number of sets for which Douglas hits 20 twice. [1]

(iii) Douglas aims four sets of 3 darts at the number 20. Calculate the probability that he hits 20 twice for two sets out of the four. [2]

4 A geologist splits rocks to look for fossils. On average 10% of the rocks selected from a particular area do in fact contain fossils.

The geologist selects a random sample of 20 rocks from this area.

(i) Find the probability that

(*A*) exactly one of the rocks contains fossils, [3]

(*B*) at least one of the rocks contains fossils. [3]

(ii) A random sample of n rocks is selected from this area. The geologist wants to have a probability of 0.8 or greater of finding fossils in at least one of the n rocks. Find the least possible value of n. [3]

(iii) The geologist explores a new area in which it is claimed that less than 10% of rocks contain fossils. In order to investigate the claim, a random sample of 30 rocks from this area is selected, and the number which contain fossils is recorded. A hypothesis test is carried out at the 5% level.

(*A*) Write down suitable hypotheses for the test. [3]

(*B*) Show that the critical region consists only of the value 0. [4]

(*C*) In fact, 2 of the 30 rocks in the sample contain fossils. Complete the test, stating your conclusions clearly. [2]