

1 In a multiple-choice test there are 30 questions. For each question, there is a 60% chance that a randomly selected student answers correctly, independently of all other questions.

(i) Find the probability that a randomly selected student gets a total of exactly 20 questions correct. [3]

(ii) If 100 randomly selected students take the test, find the expected number of students who get exactly 20 questions correct. [2]

2 An environmental health officer monitors the air pollution level in a city street. Each day the level of pollution is classified as low, medium or high. The probabilities of each level of pollution on a randomly chosen day are as given in the table.

Pollution level	Low	Medium	High
Probability	0.5	0.35	0.15

(i) Three days are chosen at random. Find the probability that the pollution level is

(A) low on all 3 days, [2]

(B) low on at least one day, [2]

(C) low on one day, medium on another day, and high on the other day. [3]

(ii) Ten days are chosen at random. Find the probability that

(A) there are no days when the pollution level is high, [2]

(B) there is exactly one day when the pollution level is high. [3]

The environmental health officer believes that pollution levels will be low more frequently in a different street. On 20 randomly selected days she monitors the pollution level in this street and finds that it is low on 15 occasions.

(iii) Carry out a test at the 5% level to determine if there is evidence to suggest that she is correct. Use hypotheses $H_0: p = 0.5$, $H_1: p > 0.5$, where p represents the probability that the pollution level in this street is low. Explain why H_1 has this form. [5]

3 The Department of Health ‘eat five a day’ advice recommends that people should eat at least five portions of fruit and vegetables per day. In a particular school, 20% of pupils eat at least five a day.

(i) 15 children are selected at random.

(A) Find the probability that exactly 3 of them eat at least five a day. [3]

(B) Find the probability that at least 3 of them eat at least five a day. [3]

(C) Find the expected number who eat at least five a day. [2]

A programme is introduced to encourage children to eat more portions of fruit and vegetables per day. At the end of this programme, the diets of a random sample of 15 children are analysed. A hypothesis test is carried out to examine whether the proportion of children in the school who eat at least five a day has increased.

(ii) (A) Write down suitable null and alternative hypotheses for the test.

(B) Give a reason for your choice of the alternative hypothesis. [4]

(iii) Find the critical region for the test at the 10% significance level, showing all of your calculations. Hence complete the test, given that 7 of the 15 children eat at least five a day. [6]