

**1** Any patient who fails to turn up for an outpatient appointment at a hospital is described as a 'no-show'. At a particular hospital, on average 15% of patients are no-shows. A random sample of 20 patients who have outpatient appointments is selected.

(i) Find the probability that

(A) there is exactly 1 no-show in the sample, [3]

(B) there are at least 2 no-shows in the sample. [2]

The hospital management introduces a policy of telephoning patients before appointments. It is hoped that this will reduce the proportion of no-shows. In order to check this, a random sample of  $n$  patients is selected. The number of no-shows in the sample is recorded and a hypothesis test is carried out at the 5% level.

(ii) Write down suitable null and alternative hypotheses for the test. Give a reason for your choice of alternative hypothesis. [4]

(iii) In the case that  $n = 20$  and the number of no-shows in the sample is 1, carry out the test. [4]

(iv) In another case, where  $n$  is large, the number of no-shows in the sample is 6 and the critical value for the test is 8. Complete the test. [3]

(v) In the case that  $n \leq 18$ , explain why there is no point in carrying out the test at the 5% level. [2]

**2** Mark is playing solitaire on his computer. The probability that he wins a game is 0.2, independently of all other games that he plays.

(i) Find the expected number of wins in 12 games. [2]

(ii) Find the probability that

(A) he wins exactly 2 out of the next 12 games that he plays, [3]

(B) he wins at least 2 out of the next 12 games that he plays. [3]

(iii) Mark's friend Ali also plays solitaire. Ali claims that he is better at winning games than Mark. In a random sample of 20 games played by Ali, he wins 7 of them. Write down suitable hypotheses for a test at the 5% level to investigate whether Ali is correct. Give a reason for your choice of alternative hypothesis. Carry out the test. [9]

- 3** A manufacturer produces tiles. On average 10% of the tiles produced are faulty. Faulty tiles occur randomly and independently.

A random sample of 18 tiles is selected.

- (i) (A) Find the probability that there are exactly 2 faulty tiles in the sample. [3]  
(B) Find the probability that there are more than 2 faulty tiles in the sample. [3]  
(C) Find the expected number of faulty tiles in the sample. [2]

A cheaper way of producing the tiles is introduced. The manufacturer believes that this may increase the proportion of faulty tiles. In order to check this, a random sample of 18 tiles produced using the cheaper process is selected and a hypothesis test is carried out.

- (ii) (A) Write down suitable null and alternative hypotheses for the test.  
(B) Explain why the alternative hypothesis has the form that it does. [4]
- (iii) Find the critical region for the test at the 5% level, showing all of your calculations. [4]
- (iv) In fact there are 4 faulty tiles in the sample. Complete the test, stating your conclusion clearly. [2]