Statistics 1 Estimation Questions

4 The time, x seconds, spent by each of a random sample of 100 customers at an automatic teller machine (ATM) is recorded. The times are summarised in the table.

Time (seconds)	**Number of customers**
$20 < x \leqslant 30$	2
$30 < x \leqslant 40$	7
$40 < x \leqslant 60$	18
$60 < x \leqslant 80$	27
$80 < x \leqslant 100$	23
$100 < x \leqslant 120$	13
$120 < x \leqslant 150$	7
$150 < x \leqslant 180$	3
Total	**100**

(a) Calculate estimates for the mean and standard deviation of the time spent at the ATM by a customer. *(4 marks)*

(b) The mean time spent at the ATM by a random sample of **36** customers is denoted by $\overline{Y}$.

(i) State why the distribution of $\overline{Y}$ is approximately normal. *(1 mark)*

(ii) Write down estimated values for the mean and standard error of $\overline{Y}$. *(2 marks)*

(iii) Hence estimate the probability that $\overline{Y}$ is less than $1\frac{1}{2}$ minutes. *(3 marks)*

4 The weights of packets of sultanas may be assumed to be normally distributed with a standard deviation of 6 grams.

The weights of a random sample of 10 packets were as follows:

498	496	499	511	503	505	510	509	513	508

(a) (i) Construct a 99% confidence interval for the mean weight of packets of sultanas, giving the limits to one decimal place. *(5 marks)*

(ii) State why, in calculating your confidence interval, use of the Central Limit Theorem was **not** necessary. *(1 mark)*

(iii) On each packet it states 'Contents 500 grams'.

Comment on this statement using **both** the given sample **and** your confidence interval. *(3 marks)*

(b) Given that the mean weight of all packets of sultanas is 500 grams, state the probability that a 99% confidence interval for the mean, calculated from a random sample of packets, will **not** contain 500 grams. *(1 mark)*

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4 A very popular play has been performed at a London theatre on each of 6 evenings per week for about a year. Over the past 13 weeks (78 performances), records have been kept of the proceeds from the sales of programmes at each performance. An analysis of these records has found that the mean was £184 and the standard deviation was £32.

(a) Assuming that the 78 performances may be considered to be a random sample, construct a 90% confidence interval for the mean proceeds from the sales of programmes at an evening performance of this play. *(4 marks)*

(b) Comment on the likely validity of the assumption in part (a) when constructing a confidence interval for the mean proceeds from the sales of programmes at an evening performance of:

(i) this particular play;

(ii) any play. *(3 marks)*

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3 (a) A sample of 50 washed baking potatoes was selected at random from a large batch. The weights of the 50 potatoes were found to have a mean of 234 grams and a standard deviation of 25.1 grams.

Construct a 95% confidence interval for the mean weight of potatoes in the batch. *(4 marks)*

(b) The batch of potatoes is purchased by a market stallholder. He sells them to his customers by allowing them to choose any 5 potatoes for £1.

Give a reason why such chosen potatoes are unlikely to represent a random sample from the batch. *(1 mark)*

Statistics 1 Estimation Answers

4(a)	$\Sigma fx = 8025$ $\Sigma fx^2 = 739975$			
	Mean ($\bar{x}$) = 80.2 to 80.3	B2		AWFW 80.25
	Standard Deviation (s_n, s_{n-1}) = 30.9 to 31.2	B2		AWFW 30.97882 or 31.13489
	MPs (x): 25, 35, 50, 70, 90, 110, 135, 165	(B1)		At least 4 correct
	Mean $(\bar{x}) = \frac{\Sigma fx}{100}$	(M1)	4	Use of
(b)(i)	**Large** (n > 30) **sample** **or** **Central Limit Theorem**	B1	1	OE
(ii)	Mean $(\bar{Y})$ = 80.2 to 80.3	B1√		√ on (a)
	Standard error $(\bar{Y}) = \frac{30.9 \text{ to } 31.2}{\sqrt{36}}$			
	= 5.1 to 5.25	M1	2	$\sqrt{s^2} > 0$ in (a) ÷ $\sqrt{36}$ or 6
(iii)	$P(\bar{Y} < 90) = P\left(Z < \frac{90-(80.2 \text{ to } 80.3)}{(5.1 \text{ to } 5.25)}\right)$	M1 M1		Standardising 90 Using values from (b)(ii) with $\sqrt{s^2/36} > 0$ or $\sqrt{s^2/100} > 0$
	$= P(Z < 1.84 \text{ to } 1.93)$ = 0.967 to 0.974	A1	3	AWFW
	Total		**10**	

4(a)(i)	Mean, $\overline{x}$ = 505.2	B1		CAO; stated or implied
	99% $\Rightarrow$ $z = 2.57$ to 2.58 or	B1		AWFW (2.5758)
	99% $\Rightarrow$ $t = 3.25$ (Knowledge of the *t*-distribution is not required in this unit)	B1		AWRT (3.250)
	CI for μ is $\overline{x} \pm (z \text{ or } t) \times \dfrac{(\sigma \text{ or } s)}{\sqrt{n}}$	M1		use of; must have $\left(\div\sqrt{n}\right)$ with $n > 1$
	Thus $505.2 \pm 2.5758 \times \dfrac{6}{\sqrt{10}}$	A1√		√ on $\overline{x}$ and z only
	or $505.2 \pm 3.25 \times \left(\dfrac{5.96}{\sqrt{10}} \text{ or } \dfrac{5.65}{\sqrt{9}}\right)$	A1√		√ on $\overline{x}$ only
	Hence 505.2 ± 4.9			use of t $\Rightarrow$ 505.2 ± 6.1
	or (500.3, 510.1)	A1	5	AWRT
(ii)	Weights of packets can be assumed to be normally distributed	B1	1	accept 'population of weights'; not 'sample of weights' or 'it'
(iii)	Given sample: 3 in 10/ some of packets have weights below 500 grams	B1		or equivalent
	Confidence interval: CI > 500	B1√		√ on CI in (a)(i)
	Conclusion: Statement does not appear justified	B1 dep	3	or equivalent dependent on both B1 and B1√
(b)	0.01 or 1%	B1	1	CAO; or equivalent
	Total		**10**	

4(a)	90% $\Rightarrow z = 1.64$ to 1.65 **or** 90% $\Rightarrow t = 1.66$ to 1.67 (Knowledge of the *t*–distribution is **not** required in this unit)	B1 (B1)		AWFW (1.6449) AWFW (1.6649)
	CI for μ is $\bar{x} \pm (z \text{ or } t) \times \frac{(s_{n-1} \text{ or } s_n)}{\sqrt{n}}$	M1		Used; must have $\sqrt{n}$ with $n > 1$
	Thus $184 \pm (1.6449 \text{ or } 1.6649) \times \frac{(32 \text{ or } 32.2)}{(\sqrt{78} \text{ or } \sqrt{77})}$	A1√		√ on *z* or *t* only
	Hence $184 \pm (5.94$ to $6.13)$ **or** £184 ± £6 **or** (£178, £190)	A1	4	AWRT; ignore units
(b)(i)	**Likely** to be **valid**	B1		Accept 'valid' or equivalent
(ii)	Different plays have different: programme prices, sales, marketing, etc theatre or audience sizes, etc popularity, artists, etc so **Unlikely** to be **valid**	B1 ↑Dep↑ B1	3	Accept 'not valid' or equivalent
	Total		**7**	

3(a)	95% $\Rightarrow z = 1.96$ or 95% $\Rightarrow t = 2.0$ to 2.01 (Knowledge of the *t*–distribution is not required in this unit)	B1 (B1)		CAO AWFW (2.009)
	CI for μ is $\bar{x} \pm (z \text{ or } t) \times \frac{(s_{n-1} \text{ or } s_n)}{\sqrt{n}}$	M1		Used; must have $\sqrt{n}$ with $n > 1$
	Note that $25.1 \times \sqrt{\frac{50}{49}} = 25.35483$			$25.1 \times \frac{50}{49} = 25.61224$ Max of B1 M1 A0√ A1
	Thus $234 \pm (1.96 \text{ or } 2.009) \times \frac{(25.1 \text{ or } 25.3 \text{ to } 25.4)}{(\sqrt{50} \text{ or } \sqrt{49})}$	A1√		√ on *z* or *t* only
	Hence $234 \pm (6.95$ to $7.30)$ ie 234 ± 7 or (227, 241)	A1	4	AWRT
(b)	Customers are likely to choose large / similar sized potatoes	B1	1	OE; accept any sensible alternative
	Total		**5**	