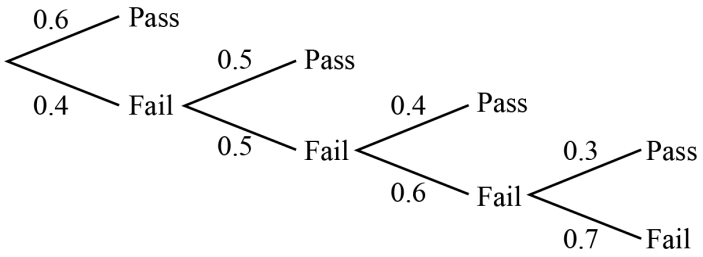
Practice paper

1 a



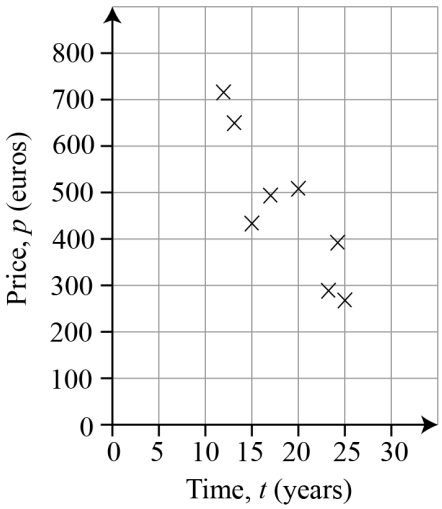
b Let X be the probability that Sudeshna passes on the first or second attempt.

$$P(X) = 0.6 + 0.4 \times 0.5$$
$$= 0.8$$

Let Y be the probability that Sudeshna gets a certificate.

$$P(X \mid Y) = \frac{0.8}{0.916}$$
$$= 0.873 \text{ (3 s.f.)}$$

2 a



b Negative correlation

2 c $\bar{t} = 18.63,\ \bar{p} = 468.75,\ S_{tp} = -5023.75,\ S_{tt} = 181.88,\ S_{pp} = 179\,337.50$

$p = a + bt$ where $b = \dfrac{S_{tp}}{S_{tt}}$ and $a = \bar{p} - b\bar{t}$

$$b = \frac{S_{tp}}{S_{tt}}$$
$$= \frac{-5023.75}{181.88}$$
$$= -27.6...$$
$$a = 468.75 + 27.6... \times 18.63$$
$$= 983.33...$$

Therefore:

$p = 983 - 27.6t$ (3 s.f.)

d when $t = 22$

$$p = 983.33... - 27.62...(22)$$
$$= 375.66...$$

Therefore, it should cost £376

e Not a good decision as 39 years is outside the range of the data.

3 a $\sum go = 29 \times 130 + 31 \times 128 + 36 \times 126 + 41 \times 123 + 43 \times 121 + 46 \times 91 + 48 \times 83 + 50 \times 75$

$= 34\,440$

b $S_{go} = \sum go - \dfrac{\sum g \sum o}{n}$

$$= 34\,440 - \frac{324 \times 877}{8}$$
$$= -1078.5$$

c $S_{oo} = 3583.88$ and $S_{gg} = 426$

$$r = \frac{S_{go}}{\sqrt{S_{gg} S_{oo}}}$$
$$= \frac{-1078.5}{\sqrt{426 \times 3583.33}}$$
$$= -0.873 \text{ (3 s.f.)}$$

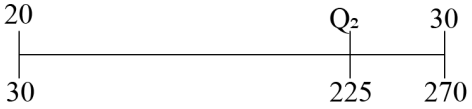
d As the price of oil decreases, the price of gold increases.

e The ninth year goes against the correlation. As a result, the PMMC will decrease (get weaker).

4 a Total area of shaded bars is:
$5\times7.5+10\times30+5\times22.5+5\times7.5+5\times15=562.5$ small squares, which represents 450 employees. Therefore 1.25 small squares represents 1 employee.
The number of employees claiming more than 35 hours overtime is:
$$\frac{5\times7.5+5\times15}{1.25}=90$$

b $$\bar{x}=\frac{12.5\times30+25\times240+32.5\times90+37.5\times30+42.5\times60}{450}$$
$= 28.8$ hours (3 s.f.)

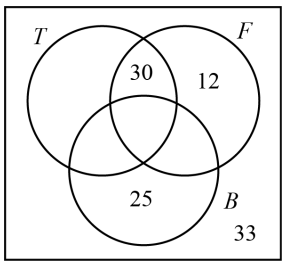
c By linear interpolation:



$$Q_2=20+\frac{195}{240}\times10$$
$$Q_2=28.1 \text{ (3 s.f.)}$$

d The median, as the data is skewed.

5 a



b Tennis and basketball (or football and basketball).

c $$P(F'\cup T'\cup B')=\frac{33}{100}$$

d $$P(F)=\frac{42}{100}=\frac{21}{50}$$

e $$P(T\cup B)=\frac{55}{100}=\frac{11}{20}$$

5 f $$P(T\mid F)=\frac{\frac{30}{100}}{\frac{42}{100}}$$
$$=\frac{5}{7}$$

6 a i $E(X) = 0.6$ and $E(X^2) = 4$

$E(X) = -2 \times a + 0 \times b + 2 \times a + 4 \times c$

$4c = 0.6$

$c = 0.15$

ii $E(X^2) = (-2)^2 \times a + 0 \times b + 2^2 \times a + 4^2 \times 0.15$

$8a + 2.4 = 4$

$a = 0.2$

Since $2a + b + c = 1$

$b = 0.45$

b $\text{Var}(X) = E(X^2) - (E(X))^2$

$= 4 - 0.6^2$

$= 3.64$

c $Y = 7 - 4X$

$E(Y) = E(7 - 4X)$

$= 7 - 4E(X)$

$= 7 - 4(0.6)$

$= 4.6$

d $Y = 7 - 4X$

$\text{Var}(Y) = \text{Var}(7 - 4X)$

$= 4^2\text{Var}(X)$

$= 16(3.64)$

$= 58.24$

e

x	-2	0	2	4
y	15	7	-1	-9
$\mathbf{P(Y = y)}$	0.2	0.45	0.2	0.15

$P(Y \geq 0) = 0.8$

7 a Let X be the volume of coffee, in ml, delivered by the machine to a cup.

$X \sim \mathrm{N}(505, 1.8^2)$

i $\mathrm{P}(X > 507) = 1 - \mathrm{P}(X < 507)$

$$= 1 - \mathrm{P}\left(Z < \frac{507 - 505}{1.8}\right)$$
$$= 1 - \mathrm{P}(Z < 1.1111)$$
$$= 1 - 0.8667$$
$$= 0.133 \text{ (3 s.f)}$$

ii $\mathrm{P}(501 < X < 505) = \mathrm{P}(X < 505) - \mathrm{P}(X < 501)$

$$= \mathrm{P}(X < 509) - 0.5$$
$$= \mathrm{P}\left(Z < \frac{509 - 505}{1.8}\right) - 0.5$$
$$= \mathrm{P}(Z < 2.2222) - 0.5$$
$$= 0.9868 - 0.5$$
$$= 0.487 (3\text{s.f})$$

b $X \sim \mathrm{N}(503, 1.6^2)$

$\mathrm{P}(1006 - w < X < w) = \mathrm{P}(X < w) - \mathrm{P}(X < 1006 - w)$

$\mathrm{P}(X < w) - \mathrm{P}(X < 1006 - w) = 0.9246$

$$\mathrm{P}\left(Z < \frac{w - 503}{1.6}\right) - \mathrm{P}\left(Z < \frac{(1006 - w) - 503}{1.6}\right) = 0.9246$$

$$\frac{w - 503}{1.6} - \frac{(1006 - w) - 503}{1.6} = 1.4367...$$

$w - 503 - 1006 + w + 503 = 2.2987...$

$2w - 1006 = 2.2987...$

$w = 504 \text{ (3s.f)}$

7 c $X \sim \mathrm{N}(r, q^2)$

$\mathrm{P}(Z < z_1) = 0.01 \Rightarrow z_1 = -2.3263$

$\frac{499 - r}{q} = -2.3263 \Rightarrow q = \frac{r - 499}{2.3263}$ **(1)**

$\mathrm{P}(Z > z_2) = 0.05 \Rightarrow z_2 = 1.6449$

$\frac{505 - r}{q} = 1.6449 \Rightarrow q = \frac{505 - r}{1.6449}$ **(2)**

Equating **(1)** and **(2)** gives:

$\frac{r - 499}{2.3263} = \frac{505 - r}{1.6449}$

$1.6449(r - 499) = 2.3263(505 - r)$

$3.9712r = 1995.5866$

$r = 502.5147$

$q = \frac{505 - 502.5147}{1.6449}$

$= 1.5108$

r = 503, q = 1.51 (3 s.f)