Review exercise 2

1 Diagram *A*: as *x* increases, *y* decreases. There is a negative correlation.
So this corresponds to $r = -0.79$.

Diagram *B*: There is no real pattern. There are several values of *v* for one value of *u*. There is very weak or no correlation. So this corresponds to $r = 0.08$.

Diagram *C*: As *s* increases, *t* increases. There is a positive correlation.
So this corresponds to $r = 0.68$.

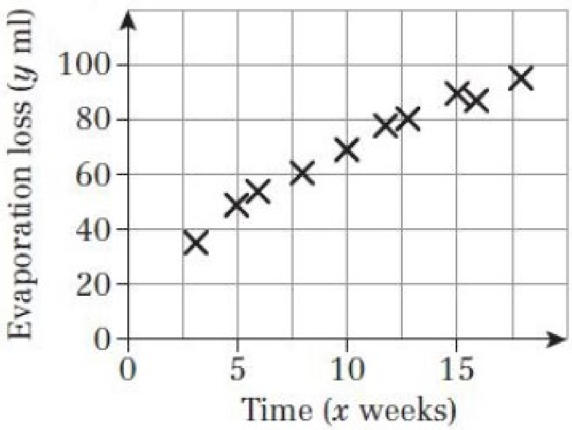
2 a Mean + 2 standard deviations = $15.3 + 2 \times 10.2 = 35.7$
$45 > 35.7$ so $t = 45$ is an outlier

b A temperature of 45 °C is very high so it is likely this value was recorded incorrectly. Therefore, this outlier should be omitted from the data.

c In the regression equation, 2.81 represents the number of additional ice creams (in hundreds) sold each month for each degree Celsius increase in average temperature.

d A temperature of 2 °C is outside the range of the data so a value calculated using the equation for the regression line involves extrapolation and is likely to be inaccurate.

3 a



b Points lie close to a straight line.

3 c $y = ax + b$ where

$b = \frac{S_{xy}}{S_{xx}}$ and $a = \bar{y} - b\bar{x}$

$$S_{xy} = \sum xy - \frac{\sum x \sum y}{n}$$

$$= 8354 - \frac{106 \times 704}{10}$$

$$= 891.6$$

$$S_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n}$$

$$= 1352 - \frac{106^2}{10}$$

$$= 228.4$$

$$b = \frac{891.6}{228.4}$$

$$= 3.903...$$

$$= 3.90 \text{ (3 s.f.)}$$

Since $a = \bar{y} - b\bar{x}$

When

$\bar{x} = 10.6$, $\bar{y} = 70.4$ and $b = 3.90$

$a = 70.4 - 3.90 \times 10.6$

$= 29.021\ldots$

$= 29.0$ (3 s.f.)

d 3.90 ml of the chemicals evaporate each week.

e i $y = 3.90x + 29.0$

When $x = 19$

$y = 3.90(19) + 29.0$

$= 103.1$

$= 103$ ml (3 s.f.)

ii $y = 3.90x + 29.0$

When $x = 35$

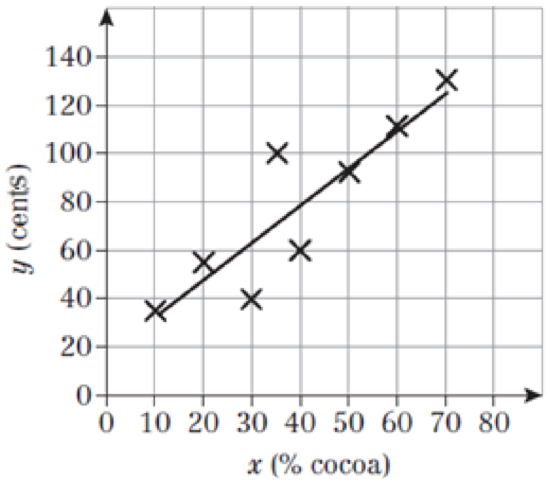
$y = 3.90(35) + 29.0$

$= 165.5$

$= 166$ ml (3 s.f.)

f The prediction for 19 weeks is likely to be reasonably reliable as it is close to the range investigated.
The prediction for 35 weeks is likely to be unreliable, since the time is well outside range of x and there is no evidence that model will continue to hold.

4 a This is a scatter diagram of the data. (The diagram also shows the regression line, see part **d**.)



b $S_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 28\,750 - \frac{315 \times 620}{8} = 4337.5$

$S_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 15\,225 - \frac{315^2}{8} = 2821.875$

c $b = \frac{S_{xy}}{S_{xx}} = 1.537\,09\ldots = 1.54 \text{ (3 s.f.)}$

$a = \bar{y} - b\bar{x} = \frac{620}{8} - b\frac{315}{8} = 16.976\ldots = 17.0 \text{ (3 s.f.)}$

d From part **c**, the equation of the regression line is: $y = 17.0 + 1.54x$
This line is shown on the scatter diagram (see answer for part **a**).

e i Brand D is overpriced, since this data point is a long way above the regression equation line.

ii Using the equation of the regression line: $y = 17 + 35 \times 1.54 = 70.9$.
So a fair price would be 71 cents.

5 a $S_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 8880 - \frac{130 \times 48}{8} = 8100$

$b = \frac{S_{xy}}{S_{xx}} = \frac{8100}{20\,487.4} = 0.395363\ldots = 0.395 \text{ (3 s.f.)}$

$a = \bar{y} - b\bar{x} = \frac{48}{8} - \frac{8100}{20\,487.4} \times \frac{130}{8} = -0.424\,68\ldots = -0.425 \text{ (3 s.f.)}$

So the equation of the regression line is: $y = -0.425 + 0.395x$

b $f - 100 = -0.42468\ldots + 0.39536\ldots(m - 250)$

$\Rightarrow f = 0.7353\ldots + 0.3953\ldots m$

$\Rightarrow f = 0.735 + 0.395m$ (giving the equation parameters to 3 s.f.)

c $f = 0.7353 + 0.3953 \times 235 = 93.6$ litres (3 s.f.)

6 **a** Find the y values by subtracting 2460 form all the l values. The summary data for x and y are:

$\sum x = \sum t = 337.1 \qquad \sum y = 16.28$

$$S_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 757.467 - \frac{337.1 \times 16.28}{8} = 71.4685$$

$$S_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 15965.01 - \frac{337.1^2}{8} = 1760.45875$$

b $b = \frac{S_{xy}}{S_{xx}} = \frac{71.4685}{1760.45875} = 0.04059652 = 0.0406 \text{ (3 s.f.)}$

$a = \bar{y} - b\bar{x} = \frac{16.28}{8} - 0.04059652 \times \frac{337.1}{8} = 0.324364 = 0.325 \text{ (3 s.f.)}$

The equation of the regression line is: $y = 0.324 + 0.0406x$

c $t = 40$, therefore $x = 40$

$y = 0.3243 + 0.0406 \times 40 = 1.9483$

$l = 2460 + 1.9483 = 2461.95 \text{ mm (2 d.p.)}$

d $l - 2460 = 0.324 + 0.0406t$

$\Rightarrow l = 2460.324 + 0.0406t$

e At $t = 90$, $l = 2460.324 + 0.0406 \times 90 = 2463.98 \text{ mm (2 d.p.)}$

f As 90 °C is well outside the range of data, the estimate is unlikely to be reliable.

7 **a** $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{-808.917}{\sqrt{113573 \times 8.657}} = -0.8157\ldots = -0.816 \text{ (3 s.f.)}$

b There is a negative correlation. The survey suggests that houses are cheaper the further they are from the railway station.

c To change miles to kilometres, multiply by 1.6. The coding is linear, so the product moment correlation coefficient will be unaffected by the coding. So the product moment correlation coefficient will still be –0.816.

8 **a** The shopper in the supermarket for 15 minutes spent 20 – 3 = $17

b The summary statistics for s and t are:

$\sum t = 212 \qquad \sum m = 61$

$$S_{tm} = \sum tm - \frac{\sum t \sum m}{n} = 2485 - \frac{61 \times 212}{10} = 1191.8$$

$$S_{tt} = \sum t^2 - \frac{\left(\sum t\right)^2}{n} = 5478 - \frac{212^2}{10} = 983.6$$

$$S_{mm} = \sum m^2 - \frac{\left(\sum m\right)^2}{n} = 2101 - \frac{61^2}{10} = 1728.9$$

8 c $r = \frac{S_{tm}}{\sqrt{S_{tt}S_{mm}}} = \frac{1191.8}{\sqrt{983.6 \times 1728.9}} = 0.91392\ldots = 0.914$ (3 s.f.)

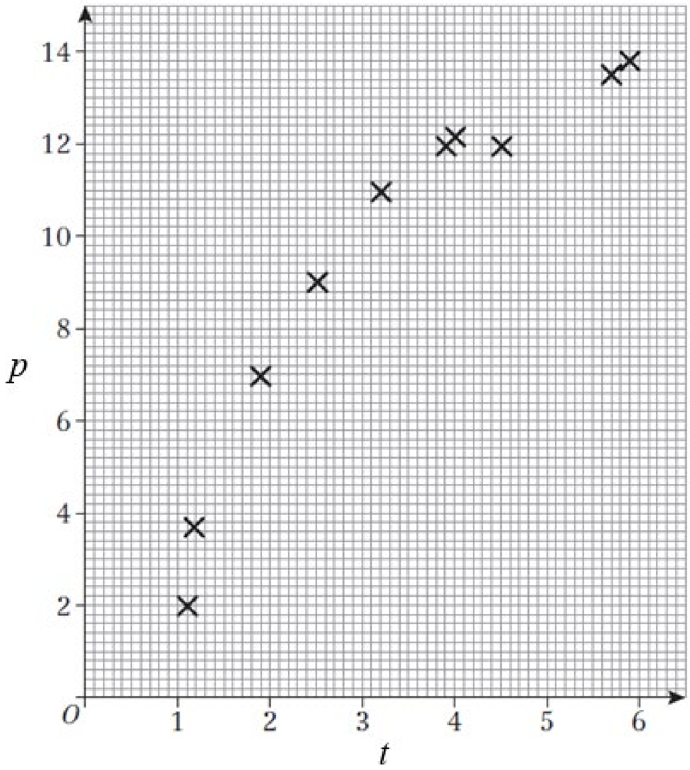
d The coding is linear (m = *amount spent* – 20) so the product moment correlation coefficient will be unaffected by the coding. So the product moment correlation coefficient will still be 0.914.

e The product moment correlation coefficient of 0.914 suggests that the longer spent shopping the more money the customer spends. It would suggest a relationship between time spent shopping and money spent.

The product moment correlation coefficient of 0.178 suggests that there is no relationship between time spent shopping and money spent.

f The two sets of data might be very different because the data was collected at different times of the day – or on different days of the week – when shopping behaviour is not the same.

9 a



b The product moment correlation coefficient measures the linear correlation between two variables, i.e. it is a measure of the strength of the linear link between the variables.

9 c The summary statistics for t and p are:

$$\sum t = 33.9 \qquad \sum p = 96.4$$

$$S_{tt} = \sum t^2 - \frac{\left(\sum t\right)^2}{n} = 141.51 - \frac{33.9^2}{10} = 26.589$$

$$S_{pp} = \sum p^2 - \frac{\left(\sum p\right)^2}{n} = 1081.74 - \frac{96.4^2}{10} = 152.444$$

$$S_{tp} = \sum tp - \frac{\sum t \sum p}{n} = 386.32 - \frac{33.9 \times 96.4}{10} = 59.524$$

d $r = \dfrac{59.524}{\sqrt{152.444 \times 26.589}} = 0.93494\ldots = 0.935$ (3 s.f.)

10 a For four coin tosses there are $2 \times 2 \times 2 \times 2 = 2^4 = 16$ possible outcomes.
There is only way of achieving 0 or 4 heads (= 0 tails).
1 head or 3 heads (= 1 tail) may appear on the 1st, 2nd, 3rd or 4th toss; i.e. in one of 4 ways.

Therefore, if X is the number of heads in the outcome:

No. of heads, x	0	1	2	3	4
P($X = x$)	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{n}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

$$\frac{1}{16} + \frac{4}{16} + \frac{n}{16} + \frac{4}{16} + \frac{1}{16} = 1$$
$$10 + n = 1$$
$$n = 6$$

The probability of an equal number of heads and tails is $\frac{6}{16} = \frac{3}{8} = 0.375$

Alternative solution:
Let X be the random variable 'the number of heads'.
$X \sim B(4, 0.5)$

$$\text{P}(X = 2) = \binom{4}{2} 0.5^2 \times 0.5^2$$
$$= \frac{4!}{2!2!} 0.5^2 \times 0.5^2$$
$$= 0.375$$

b $\text{P}(X = 0 \text{ or } 4) = \text{P}(X = 0) + \text{P}(X = 4)$
$= \frac{1}{16} + \frac{1}{16} = \frac{1}{8}$
$= 0.125$

c $\text{P}(HHT) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
$= 0.125$

11 a

x	1	2	3	4	5	6
P ($X = x$)	$\frac{1}{36}$ $= 0.0278$	$\frac{3}{36}$ $= 0.0833$	$\frac{5}{36}$ $= 0.1389$	$\frac{7}{36}$ $= 0.1944$	$\frac{9}{36}$ $= 0.25$	$\frac{11}{36}$ $= 0.3056$

b $P(2 < X \leqslant 5) = P(Y = 3) + P(Y = 4) + P(Y = 5)$

$P(Y = 3) + P(Y = 4) + P(Y = 5) = \frac{21}{36} = \frac{7}{12}$

$= 0.5833... = 0.583$ (3 s.f.)

12 a Probabilities sum to 1, so:

$2a + 2b + c = 1$ (1)

$E(Y) = \frac{1}{4} - \frac{1}{2}E(X) = 0.25 - 0.5E(X)$

So $0.25 - 0.5(-3a - 2b + a + 3c) = -0.05$

$\Rightarrow -2a - 2b + 3c = 0.6$ (2)

$Y > 0 \Rightarrow \frac{1-2X}{4} > 0 \Rightarrow X < 0.5$

So $P(Y > 0) = P(X < 0.5) = 0.5$

$\Rightarrow a + 2b = 0.5$ (3)

Add equations (1) and (2) to get :

$4c = 1.6 \Rightarrow c = 0.4$

Substitute for c in equation (1), and subtract (3) from (1):

$a + 0.4 = 0.5 \Rightarrow a = 0.1$

Substitute for a in equation (3):

$0.1 + 2b = 0.5 \Rightarrow b = 0.2$

$\Rightarrow b = 0.2$

So the probability distribution for X is:

x	–3	–2	0	1	3
P($X = x$)	0.1	0.2	0.2	0.1	0.4

b $-3X < 5Y \Rightarrow -3X < 5\left(\frac{1-2X}{4}\right) \Rightarrow -12X < 5 - 10X \Rightarrow X > -\frac{5}{2}$

So $P(-3X < 5Y) = P(X > -2.5) = 1 - P(X = -3) = 0.9$

13 a The probability distribution for X is:

x	1	2	3	4	5	6
$\mathrm{P}(X=x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

b $\mathrm{P}(2 < X \leqslant 5) = \mathrm{P}(X=3) + \mathrm{P}(X=4) + \mathrm{P}(X=5)$

$$= \frac{5+7+9}{36} = \frac{21}{36} = \frac{7}{12} = 0.583 \text{ (3 s.f.)}$$

c $\mathrm{E}(X) = \frac{1}{36}(1 + 2\times3 + 3\times5 + 4\times7 + 5\times9 + 6\times11) = \frac{161}{36}$

d Show all the steps when asked to show that Var(X) = 1.97

$\mathrm{E}(X^2) = \sum x^2\,\mathrm{P}(X=x)$

$= \frac{1}{36}(1 + 2^2\times3 + 3^2\times5 + 4^2\times7 + 5^2\times9 + 6^2\times11)$

$= \frac{791}{36}$

$\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$

$= \frac{791}{26} - \frac{25\,291}{1296} = \frac{28\,476}{1296} - \frac{25\,291}{1296} = \frac{2555}{1296} = 1.97 \text{ (3 s.f.)}$

e Using $\mathrm{Var}(aX+b) = a^2\mathrm{Var}(X)$, so:

$\mathrm{Var}(2-3X) = (-3)^2\,\mathrm{Var}(X) = 9\times\frac{2555}{1296} = 17.7 \text{ (3 s.f.)}$

14 The probability distribution for X is:

x	1	2	3	4	5
$\mathrm{P}(X=x)$	k	$2k$	$3k$	$5k$	$6k$

a Probabilities sum to 1, so:

$k + 2k + 3k + 5k + 6k = 1$

$17k = 1$

$k = \frac{1}{17} = 0.0588 \text{ (3 s.f.)}$

b As the question requires an exact answer work in fractions

$\mathrm{E}(X) = \frac{1}{17}(1 + 2\times2 + 3\times3 + 4\times5 + 5\times6) = \frac{64}{17}$

c Show all the steps when asked to show that Var(X) = 1.47

$\mathrm{E}(X^2) = \sum x^2\,\mathrm{P}(X=x)$

$= \frac{1}{17}(1 + 2^2\times2 + 3^2\times3 + 4^2\times5 + 5^2\times6)$

$= \frac{266}{17}$

$\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$

$= \frac{266}{17} - \frac{4906}{289} = \frac{4522}{289} - \frac{4906}{289} = \frac{426}{289} = 1.47 \text{ (3 s.f.)}$

d Using $\mathrm{Var}(aX+b) = a^2\mathrm{Var}(X)$, so:

$\mathrm{Var}(4-3X) = (-3)^2\,\mathrm{Var}(X) = 9\times\frac{426}{289} = 13.3 \text{ (1 d.p.)}$

15 a Probabilities sum to 1, so:

$0.1 + p + 0.2 + q + 0.3 = 1$

$\Rightarrow p + q = 0.4 \qquad (1)$

$\mathrm{E}(X) = \sum x \mathrm{P}(X = x) = 3.5$

So $1 \times 0.1 + 2 \times p + 3 \times 0.2 + 4 \times q + 5 \times 0.3 = 3.5$

$\Rightarrow 2p + 4q = 1.3 \qquad (2)$

b Multiply equation (1) by 2

$2p + 2q = 0.8 \qquad (3)$

Subtract equation (3) from equation (2) gives

$2q = 0.5 \Rightarrow q = 0.25$

Substitute value for q into equation (1)

$p + 0.25 = 0.4 \Rightarrow p = 0.15$

Solution: $p = 0.15,\ q = 0.25$

c $\mathrm{E}(X^2) = \sum x^2 \mathrm{P}(X = x) = 1^2 \times 0.10 + 2^2 \times 0.15 + 3^2 \times 0.2 + 4^2 \times 0.25 + 5^2 \times 0.30 = 14$

$\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2 = 14 - 3.5^2 = 1.75$

d Using $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$, so:

$\mathrm{Var}(3 - 2X) = (-3)^2\ \mathrm{Var}(X) = 4 \times 1.75 = 7$

16 a Probabilities sum to 1, so:

$0.2 + p + 0.2 + q + 0.15 = 1$

$\Rightarrow p + q = 0.45 \qquad (1)$

$\mathrm{E}(X) = \sum x \mathrm{P}(X = x) = 4.5$

So $1 \times 0.2 + 3 \times p + 5 \times 0.2 + 7 \times q + 9 \times 0.15 = 4.5$

$\Rightarrow 3p + 7q = 1.95 \qquad (2)$

b Multiply equation (1) by 3

$3p + 3q = 1.35 \qquad (3)$

Subtract equation (3) from equation (2) gives

$4q = 0.6 \Rightarrow q = 0.15$

Substitute value for q into equation (1)

$p + 0.15 = 0.45 \Rightarrow p = 0.3$

Solution: $p = 0.3,\ q = 0.15$

c $\mathrm{P}(4 < X \leqslant 7) = \mathrm{P}(X = 5) + \mathrm{P}(X = 7)$

$= 0.2 + q = 0.35$

16 d $\text{Var}(X) = \text{E}(X^2) - (\text{E}(X))^2 = 27.4 - 4.5^2 = 7.15$

e Using $\text{E}(aX + b) = a\text{E}(X) + b$ so:
$\text{E}(19 - 4X) = -4\text{E}(X) + 19 = 19 - (4 \times 4.5) = 1$

f Using $\text{Var}(aX + b) = a^2\text{Var}(X)$, so:
$\text{Var}(19 - 4X) = (-4)^2 \text{Var}(X) = 16 \times 7.15 = 114.4$

17 a Probabilities sum to 1, so:
$0.2 + 0.3 + a + b = 1$
$\Rightarrow a + b = 0.5$
$\text{E}(Y) = 2 - 3\text{E}(X) = 2.9$
$\Rightarrow \text{E}(X) = -0.3$
So $-2(0.2) - 1(0.3) + b = -0.3$
$\Rightarrow b = 0.4$
$\Rightarrow a = 0.1$

b $\text{E}\left(X^2\right) = 0.2(-2)^2 + 0.3(-1)^2 + 0.4(1)^2 = 1.5$
$\text{Var}(X) = \text{E}\left(X^2\right) - (\text{E}(X))^2 = 1.5 - (-0.3)^2 = 1.41$

c Using $\text{Var}(aX + b) = a^2\text{Var}(X)$:
$\text{Var}(Y) = \text{Var}(2 - 3X) = (-3)^2 \text{Var}(X) = 9 \times 1.41 = 12.69$

d $Y + 1 = 3 - 3X$
So $Y + 1 < X \Rightarrow 3 - 3X < X \Rightarrow X > \frac{3}{4}$
$\text{P}(Y + 1 < X) = \text{P}\left(X > \frac{3}{4}\right) = \text{P}(X = 1) = 0.4$

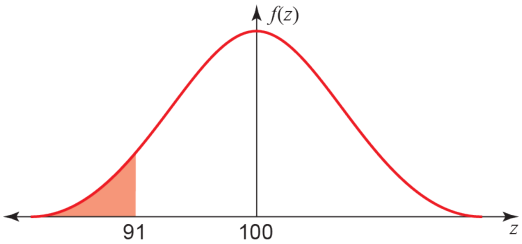
18 a $\text{P}(X = x) = 0.2$

b The spinner has 3 odd numbers and two even numbers so:
$\text{P}(X = \text{even}) = 0.4$
$\text{P}(X = \text{odd}) = 0.6$

y	1	2	3	4
P($Y = y$)	0.6	$0.4 \times 0.6 = 0.24$	$0.4^2 \times 0.6 = 0.096$	$0.4^3 \times 0.6 + 0.4^4 = 0.064$

c $\text{P}(Y > 2) = \text{P}(Y = 3) + \text{P}(Y = 4) = 0.096 + 0.064 = 0.16$

19 a Drawing a diagram will help you to work out the correct area:

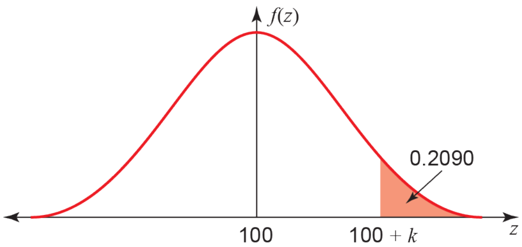


Using $z = \frac{x-\mu}{\sigma}$. As 91 is to the left of 100, your z value should be negative.

$$\text{P}(X < 91) = \text{P}\left(Z < \frac{91-100}{15}\right)$$
$$= \text{P}(Z < -0.6)$$
$$= 1 - 0.7257$$
$$= 0.2743$$

(The tables give $\text{P}(Z < 0.6) = \text{P}(Z > -0.6)$, so you want $1 -$ this probability.)

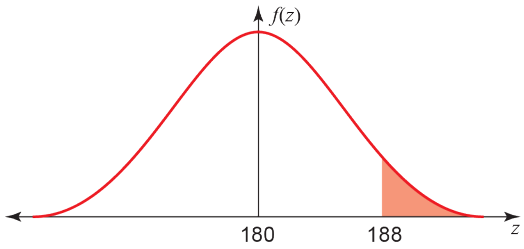
b



As 0.2090 is not in the table of percentage points you must work out the larger area:
$1 - 0.2090 = 0.7910$
Use the first table or calculator to find the z value. It is positive as $100 + k$ is to the right of 100.
$\text{P}(X > 100 + k) = 0.2090$ or $\text{P}(X < 100 + k) = 0.791$

$$\frac{100 + k - 100}{15} = 0.81$$
$$k = 12$$

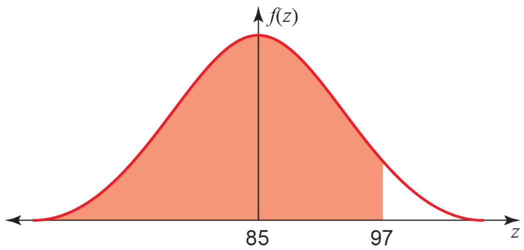
20 a Let H be the random variable ~ height of athletes, so $H \sim \mathrm{N}(180,\ 5.2^2)$
Drawing a diagram will help you to work out the correct area:



Using $z = \frac{x-\mu}{\sigma}$. As 188 is to the right of 180 your z value should be positive. The tables give $\mathrm{P}(Z < 1.54)$ so you want $1 -$ this probability:

$$\begin{aligned}\mathrm{P}(H > 188) &= \mathrm{P}\left(Z > \frac{188-100}{5.2}\right)\\ &= \mathrm{P}(Z > 1.54)\\ &= 1 - 0.9382\\ &= 0.0618\end{aligned}$$

b Let W be the random variable ~ weight of athletes, so $W \sim \mathrm{N}(85,\ 7.1^2)$



Using $z = \frac{x-\mu}{\sigma}$. As 97 is to the right of 85, your z value should be positive.

$$\begin{aligned}\mathrm{P}(W < 97) &= \mathrm{P}\left(Z < \frac{97-85}{7.1}\right)\\ &= \mathrm{P}(Z < 1.69)\\ &= 0.9545\end{aligned}$$

c $\mathrm{P}(W > 97) = 1 - \mathrm{P}(W < 97)$, so

$$\begin{aligned}\mathrm{P}(H > 188\ \&\ W > 97) &= 0.618(1 - 0.9545)\\ &= 0.00281\end{aligned}$$

d Use the context of the question when you are commenting:
The evidence suggests that height and weight are positively correlated/linked, so assumption of independence is not sensible.

21 a Use the table of percentage points or calculator to find z. You must use at least the four decimal places given in the table.

$$P(Z > a) = 0.2$$
$$a = 0.8416$$
$$P(Z < b) = 0.3$$
$$b = -0.5244$$

0.5244 is negative since 1.65 is to the left of the centre. 0.8416 is positive as 1.78 is to the right of the centre.

Using $z = \dfrac{x-\mu}{\sigma}$:

$$\frac{1.78-\mu}{\sigma} = 0.8416 \Rightarrow 1.78 - \mu = 0.8416\sigma \quad (\mathbf{1})$$
$$\frac{1.65-\mu}{\sigma} = -0.5244 \Rightarrow 1.65 - \mu = 0.5244\sigma \quad (\mathbf{2})$$

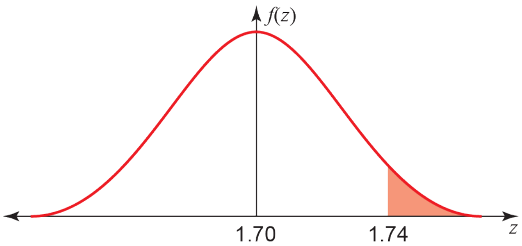
Solving simultaneously, $(\mathbf{1}) - (\mathbf{2})$:

$$0.13 = 1.366\sigma$$
$$\sigma = 0.095 \text{ m}$$

Substitute in (**1**): $1.78 - \mu = 0.8416 \times 0.095$

$$\mu = 1.70 \text{ m}$$

b



Using $z = \dfrac{x-\mu}{\sigma}$:

$$P(\text{height} > 1.74) = P\left(z > \frac{1.74-1.70}{0.095}\right)$$
$$= P(z > 0.42) \text{ (the tables give } P(Z < 0.42) \text{ so you need } 1 - \text{this probability)}$$
$$= 1 - 0.6628$$
$$= 0.3372 \text{ (calculator gives 0.3369)}$$

22 a $P(D < 21.5) = 0.32$ and $P(Z < a) = 0.32 \Rightarrow a = -0.467$. Therefore

$$\frac{21.5-\mu}{\sigma} = -0.467 \Rightarrow 21.5 - 22 = -0.467\sigma \Rightarrow \sigma = \frac{0.5}{0.467} = 1.071 \text{ (4 s.f.).}$$

b $P(21 < D < 22.5) = P(D < 22.5) - P(D < 21) = 0.5045$ (4 s.f.).

c $P(B \geq 10) = 1 - P(B \leqslant 9) = 1 - 0.01899 = 0.98101$

(using 4 s.f. for the value given by the binomial distribution) or 0.981 (4 s.f.).

23 $X \sim \mathrm{N}(14, 3^2)$

a $\mathrm{P}(X \geq 11) = \mathrm{P}(X < 17)$

$$z = \frac{X - \mu}{\sigma}$$

$$= \frac{17 - 14}{3}$$

$$= 1$$

$\Phi(1) = 0.8413$

$\mathrm{P}(X \geq 11) = \mathrm{P}(Z < 1)$

$\mathrm{P}(X \geq 11) = 0.8413$

b $\mathrm{P}(9 < X < 11)$

$\mathrm{P}(X < 9) = 1 - \mathrm{P}(X < 19)$

$$z = \frac{X - \mu}{\sigma}$$

$$= \frac{19 - 14}{3}$$

$$= 1.667$$

$\Phi(1.667) = 0.9522$

$\mathrm{P}(X < 9) = 1 - \mathrm{P}(Z < 1.667)$

$\mathrm{P}(X < 9) = 0.0478$

$\mathrm{P}(X < 11) = 1 - 0.8413$ (from part **a**)

$= 0.1587$

Therefore

$\mathrm{P}(9 < X < 11) = 0.1587 - 0.0478$

$= 0.1109$

24 a $X \sim \mathrm{N}(20, 5^2)$

$\mathrm{P}(X \leqslant 16) = 1 - \mathrm{P}(X \leqslant 24)$

$$z = \frac{X - \mu}{\sigma}$$

$$= \frac{24 - 20}{5}$$

$$= 0.8$$

$\Phi(0.8) = 0.7881$

$\mathrm{P}(X \leqslant 16) = 1 - \mathrm{P}(Z \leqslant 0.8)$

$\mathrm{P}(X \leqslant 16) = 0.2119$

24 b $P(X < d) = 0.95$

$$P\left(z < \frac{d-20}{5}\right) = 0.95$$

$$\frac{d-20}{5} = 1.645$$

$$d = 28.225$$
$$= 28.2$$

25 a $S \sim N\left(850, 50^2\right)$

$P(S < 830) = 1 - P(S < 870)$

$$z = \frac{X-\mu}{\sigma}$$
$$= \frac{870-850}{50}$$
$$= 0.4$$

$\Phi(0.4) = 0.6554$

$P(S < 830) = 1 - P(Z \leqslant 0.4)$

$P(S < 830) = 1 - 0.6554$

$= 0.3446$

b $500 \times 0.3446 = 172.3$

so 172 batteries

c $P(P < 818) = 1 - P(P < 902)$

Therefore

$1 - P(P < 902) = 0.2$

$P(P < 902) = 0.8$

$$P\left(z < \frac{902-860}{\sigma}\right) = 0.8$$

$$\frac{902-860}{\sigma} = 0.8412$$

$$\sigma = 49.928\ldots$$
$$= 49.9 \text{ (3 s.f.)}$$

d Power Batteries, as the mean is greater than Strong Batteries.
They also have approximately the same standard deviation.

26 $T \sim N\left(25, 4^2\right)$

a $P(T < 28)$

$$z = \frac{X-\mu}{\sigma}$$
$$= \frac{28-25}{4}$$
$$= 0.75$$

$\Phi(0.75) = 0.7734$

$P(T < 28) = 0.7734$

26 b P(|T – 25| < 5) = P(20 < T < 30)

For P(T < 30)

$$z = \frac{X - \mu}{\sigma}$$
$$= \frac{30 - 25}{4}$$
$$= 1.25$$

Φ(1.25) = 0.8944

P(T < 30) = 0.8944

Therefore P(25 < T < 30) = 0.8944 – 0.5
= 0.3944

and

P(20 < T < 25) = P(25 < T < 30) = 0.3944

so

P(20 < T < 30) = 0.3944 × 2
= 0.7888

c P(T < 23) = 1 – P(T < 27)

$$z = \frac{X - \mu}{\sigma}$$
$$= \frac{27 - 25}{4}$$
$$= 0.5$$

Φ(0.5) = 0.6915

P(T < 23) = 1 – 0.6915 = 0.3085

P(T < 23 on all three laptops) = 0.3085^3
= 0.0294

Challenge

1 a i $y = -2.63 + 2.285x$

ii $y = 1.04 + 0.1206x + 0.2353x^2$

iii $y = 1.1762e^{0.3484x}$

b For $y = -2.63 + 2.285x$ the residuals are:

when $x = 1$

$y = -2.63 + 2.285(1) = -0.345 \Rightarrow$ residual = 1.5 − (−0.345) = 1.845

when $x = 3$

$y = -2.63 + 2.285(3) = 4.225 \Rightarrow$ residual = 3.3 − 4.225 = −0.925

when $x = 4$

$y = -2.63 + 2.285(4) = 6.510 \Rightarrow$ residual = 5.3 – 6.510 = −1.210

when $x = 5$

$y = -2.63 + 2.285(5) = 8.795 \Rightarrow$ residual = 7.5 – 8.795 = −1.295

when $x = 7$
$y = -2.63 + 2.285(7) = 13.635 \Rightarrow$ residual = 13.8 – 13.635 = 0.165

when $x = 8$
$y = -2.63 + 2.285(8) = 15.65 \Rightarrow$ residual = 16.8 – 15.65 = 1.15

For $y = 1.04 + 0.1206x + 0.2353x^2$ the residuals are:
when $x = 1$
$y = 1.04 + 0.1206(1) + 0.2353(1)^2 = 1.3959 \Rightarrow$ residual = 1.5 – 1.3959 = 0.1041

when $x = 3$
$y = 1.04 + 0.1206(3) + 0.2353(3)^2 = 3.5195 \Rightarrow$ residual = 3.3 – 3.5195 = −0.2195

when $x = 4$
$y = 1.04 + 0.1206(4) + 0.2353(4)^2 = 5.2872 \Rightarrow$ residual = 5.3 – 5.2872 = 0.0128

when $x = 5$
$y = 1.04 + 0.1206(5) + 0.2353(5)^2 = 7.5255 \Rightarrow$ residual = 7.5 – 7.5255 = −0.0255

when $x = 7$
$y = 1.04 + 0.1206(7) + 0.2353(7)^2 = 13.4139 \Rightarrow$ residual = 13.8 – 13.4139 = 0.3861

when $x = 8$
$y = 1.04 + 0.1206(8) + 0.2353(8)^2 = 17.064 \Rightarrow$ residual = 16.8 – 17.064 = −0.264

For $y = 1.1762e^{0.3484x}$
when $x = 1$
$y = 1.1762e^{0.3484(1)} = 1.6664 \Rightarrow$ residual = 1.5 – 1.6664 = –0.1664

when $x = 3$
$y = 1.1762e^{0.3484(3)} = 3.3451 \Rightarrow$ residual = 3.3 – 3.3451 = −0.0451

when $x = 4$
$y = 1.1762e^{0.3484(4)} = 4.7393 \Rightarrow$ residual = 5.3 – 4.7393 = 0.5607

when $x = 5$
$y = 1.1762e^{0.3484(5)} = 6.7146 \Rightarrow$ residual = 7.5 – 6.7146 = −0.0255

when $x = 7$
$y = 1.1762e^{0.3484(7)} = 13.4784 \Rightarrow$ residual = 13.8 – 13.4784 = 0.7854

when $x = 8$
$y = 1.1762e^{0.3484(8)} = 19.0962 \Rightarrow$ residual = 16.8 – 19.0962 = −2.2962

Hence the quadratic model is most suitable as the residuals are smaller and are randomly scattered around zero.

2 a P(difference is 0) = P(1,1,1) + P(2,2,2) + P(3,3,3) + P(4,4,4)

$$= 4\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right)$$

$$= \frac{1}{16}$$

P(difference is 1) = P(1,1,2) + P(1,2,1) + P(2,1,1) + P(1,2,2) + P(2,1,2) + P(2,2,1) + P(2,2,3) + P(2,3,2) + P(3,2,2) + P(2,3,3) + P(3,2,3) + P(3,3,2) + P(3,3,4) + P(3,4,3) + P(4,3,3) + P(3,4,4) + P(4,3,4) + P(4,4,3)

$$= 18\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right)$$

$$= \frac{9}{32}$$

P(difference is 3) = P(1,1,4) + P(1,4,1) + P(4,1,1) + P(1,4,4) + P(4,1,4) + P(4,4,1) + P(1,2,4) + P(1,4,2) + P(4,2,1) + P(2,1,4) + P(2,4,1) + P(4,1,2) + P(1,3,4) + P(1,4,3) + P(4,3,1) + P(3,1,4) + P(34,1) + P(4,1,3)

$$= 18\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right)$$

$$= \frac{9}{32}$$

Since the sum of the probabilities is 1, P(difference is 2) = $\frac{12}{32}$

x	0	1	2	3
$P(X = x)$	$\frac{2}{32}$	$\frac{9}{32}$	$\frac{12}{32}$	$\frac{9}{32}$

b $E(X) = \sum xP(X = x)$

$$= 0\times\frac{2}{32} + 1\times\frac{9}{32} + 2\times\frac{12}{32} + 3\times\frac{9}{32}$$

$$= \frac{15}{8} \text{ as required}$$