Chapter review 7

1 $H \sim \mathrm{N}(178, 4^2)$

a Using the normal CD function, $\mathrm{P}(H > 185) = 0.04059... = 0.0406$ (4 d.p.)

b Using the normal CD function, $\mathrm{P}(H < 180) = 0.69146...$
The probability that three men, selected at random, all satisfy this criterion is
$\mathrm{P}(H < 180)^3 = 0.33060... = 0.3306$ (4 d.p.).

c Using the inverse normal function, $\mathrm{P}(H > h) = 0.005 \Rightarrow h = 188.03...$
To the nearest centimetre, the height of a door frame needs to be at least 188 cm.

2 $W \sim \mathrm{N}(32.5,\ 2.2^2)$

a Using the normal CD function, $\mathrm{P}(W < 30) = 0.12790...$
The percentage of sheets weighing less than 30kg is 12.8% (3 s.f.).

b Using the normal CD function, $\mathrm{P}(31.6 < W < 34.8) = 0.51085...$
So 51.1% of sheets satisfy Bob's requirements (1 d.p.).

3 $T \sim \mathrm{N}(48,\ 8^2)$

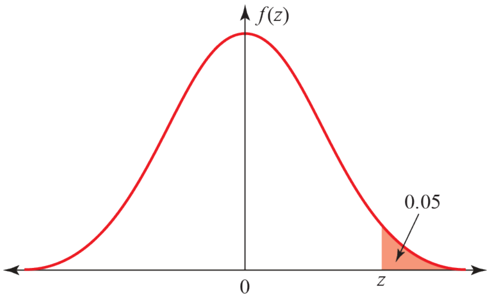
a Using the normal CD function, $\mathrm{P}(T > 60) = 0.06680...$
The probability that a battery will last for more than 60 hours is 0.0668 (4 d.p.).

b Using the normal CD function, $\mathrm{P}(T < 35) = 0.05208...$
The probability that a battery will last for less than 35 hours is 0.0521 (4 d.p.).

c Use the binomial distribution $X \sim \mathrm{B}(30, 0.05208...)$
Using the binomial CD function, $\mathrm{P}(X \leqslant 3) = 0.93145...$
The probability that three or fewer last less than 35 hours is 0.9315 (4 d.p.).

4 $X \sim N(24, \sigma^2)$

a $P(X > 30) = 0.05 \Rightarrow P\left(Z > \frac{30-\mu}{\sigma}\right) = 0.05$



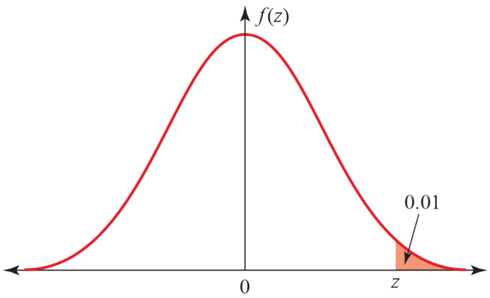
Using the inverse normal function, $z = -1.64485...$

so $1.64485... = \frac{30-24}{\sigma}$

$\sigma = \frac{6}{1.64485...} = 3.647... = 3.65$ (3 s.f.)

b Using the normal CD function, $P(X < 20) = 0.13636... = 0.136$ (3 d.p.)

c $P(X > d) = 0.01 \Rightarrow P\left(Z > \frac{d-\mu}{\sigma}\right) = 0.01$



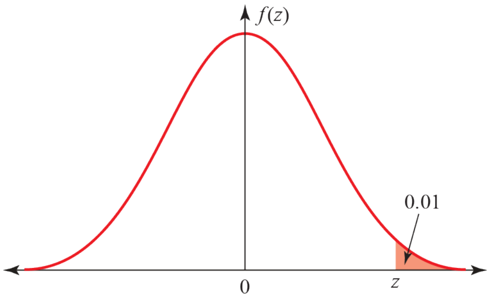
Using the inverse normal function, $z = 2.32634...$

so $2.32634... = \frac{d-24}{3.647...}$

$d = 32.485... = 32.5$ (3 s.f.)

5 $L \sim \text{N}(120,\ \sigma^2)$

a $\text{P}(L > 140) = 0.01 \Rightarrow \text{P}\left(Z > \frac{140 - \mu}{\sigma}\right) = 0.01$



Using the inverse normal function, $z = 2.32634...$

so $2.32634... = \frac{140 - 120}{\sigma}$

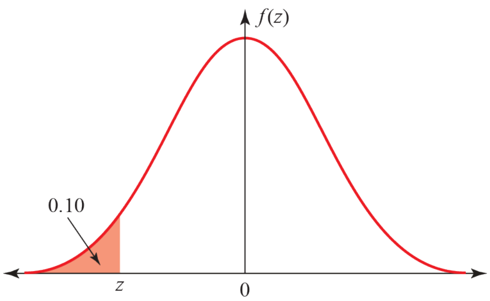
$\sigma = \frac{20}{2.32634...} = 8.59716...$

So the standard deviation of the volume dispensed is 8.60 ml (3 s.f.).

b Using the normal CD function, $\text{P}(L < 110) = 0.12237...$

The probability that the machine dispenses less than 110ml is 0.122 (3 s.f.).

c $\text{P}(L < c) = 0.10 \Rightarrow \text{P}\left(Z < \frac{c - \mu}{\sigma}\right) = 0.10$



Using the inverse normal function, $z = -1.28155...$

so $-1.2816 = \frac{c - 120}{8.59716...}$

$c = 108.982...$

To the nearest millilitre, the largest volume leading to a complaint is 109 ml.

6 a $P(X < 20) = 0.25$ and $P(X < 40) = 0.75$

Using the inverse normal function (or the percentage points table),

$$P(X < 20) = 0.25 \Rightarrow P\left(Z < \frac{20-\mu}{\sigma}\right) = 0.25 \Rightarrow z_1 = -0.67448...$$

$$P(X < 40) = 0.75 \Rightarrow P\left(Z < \frac{40-\mu}{\sigma}\right) = 0.75 \Rightarrow z_2 = 0.67448...$$

So $-0.6745\sigma = 20 - \mu$ **(1)**

and $0.6745\sigma = 40 - \mu$ **(2)**

(2) − **(1)**: $1.3489\sigma = 20$

$\sigma = 14.826...$

Substituting into **(2)**:

$\mu = 40 - 0.6745 \times 14.826... = 29.99...$

So $\mu = 30$ and $\sigma = 14.8$ (3 s.f.)

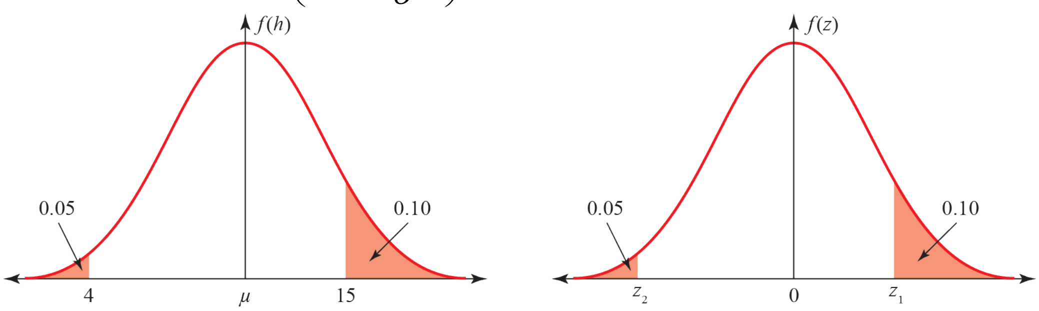
b Using the inverse normal CD function with $\mu = 30$ and $\sigma = 14.826...$,

$P(X < a) = 0.1 \Rightarrow a = 10.999...$ and $P(X < b) = 0.9 \Rightarrow b = 49.000...$

So the 10% to 90% interpercentile range is $49.0 - 11.0 = 38.0$

7 $$P(H > 15) = 0.10 \Rightarrow P\left(Z > \frac{15-\mu}{\sigma}\right) = 0.10 \Rightarrow z_1 = 1.28155...$$

$$P(H < 4) = 0.05 \Rightarrow P\left(Z > \frac{4-\mu}{\sigma}\right) = 0.05 \Rightarrow z_2 = -1.64485...$$



So $-1.6449\sigma = 4 - \mu$

$1.2816\sigma = 15 - \mu$

Subtract $2.9265\sigma = 11$

$\sigma = 3.7587... = 3.76\,\text{cm}$ (3 s.f.)

$\mu = 15 - 1.2816\sigma = 10.2\,\text{cm}$ (3 s.f.)

8 a $T \sim N(80, 10^2)$

Using the normal CD function, $P(T > 85) = 0.30853... = 0.3085$ (4 d.p.)

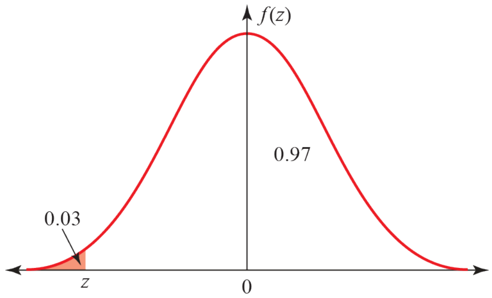
b $S \sim N(100, 15^2)$

Using the normal CD function, $P(S > 105) = 0.36944... = 0.3694$ (4 d.p.)

c The student's score on the first test was better, since fewer of the students got this score or higher.

9 $J \sim N(108,\ \sigma^2)$

a $P(J < 100) = 0.03 \Rightarrow P\left(Z < \dfrac{100-\mu}{\sigma}\right) = 0.03$



Using the inverse normal function, $z = -1.88079...$

so $-1.88079... = \dfrac{100-108}{\sigma}$

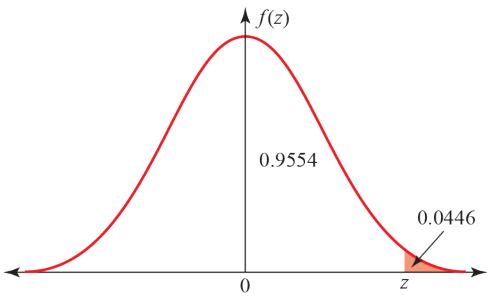
$\sigma = 4.2535... = 4.25\text{ g }(3\text{ s.f.}).$

The standard deviation is 4.25 g (3 s.f.).

b Using the normal CD function, $P(J > 115) = 0.0499... = 0.050$ (3 d.p.)

c Use the binomial distribution $X \sim B(25, 0.05)$
Using the binomial CD function,
$P(X \leqslant 2) = 0.87289... = 0.8729$ (4 d.p.)

10 $T \sim N(\mu, 3.8^2)$ and $P(T > 15) = 0.0446$



a $P(T > 15) = 0.0446 \Rightarrow P\left(Z > \dfrac{X-\mu}{\sigma}\right) = 0.0446 \Rightarrow z = 1.70$

so $1.70 = \dfrac{15-\mu}{3.8}$

$\mu = 15 - 3.8 \times 1.70$

$= 8.54$ minutes (3 s.f.)

b $P(T < 5) = P\left(Z < \dfrac{5-8.54}{3.8}\right)$

$= P(Z < -0.93...)$

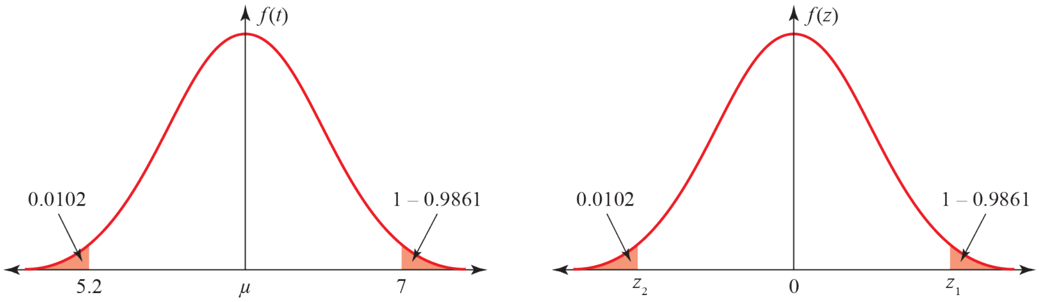
$= 0.17577... = 0.1758$ (4 d.p.)

11 $T \sim N(\mu, \sigma^2)$

Using the inverse normal function,

$P(T < 7) = 0.9861 \Rightarrow P\left(Z < \frac{7-\mu}{\sigma}\right) = 0.9861 \Rightarrow z_1 = 2.20009...$

$P(T < 5.2) = 0.0102 \Rightarrow P\left(Z < \frac{5.2-\mu}{\sigma}\right) = 0.0102 \Rightarrow z_2 = -2.31890...$



So $2.2001\sigma = 7 - \mu$ **(1)**

and $-2.3189\sigma = 5.2 - \mu$ **(2)**

(1) – **(2)**: $4.5190\sigma = 1.8$

$\sigma = 0.3983...$

Substituting into **(1)**:

$\mu = 7 - 2.2001 \times 0.3983... = 6.123...$

So the mean thickness of the shelving is 6.12 mm and the standard deviation is 0.398 mm (3 s.f.).

Challenge

1 $X \sim N(58, 10^2)$

$P(X < 36) = 1 - P(X < 80)$

$z = \frac{X - \mu}{\sigma}$

$= \frac{80-58}{10}$

$= 2.2$

$\Phi(2.2) = 0.9861$

$P(X < 36) = 1 - 0.9861$

$= 0.0139$

Since 2 000 000 televisions are made,

2 000 000 × 0.0139 = 27 800 televisions may be replaced.

2 a i The mean would remain unchanged at 5.2 hours as the mean of the extra people is also 5.2 hours.

ii The variance decreases, as the average deviation from the mean is less.

b The shape of the curve changes and $P(Z = 5.2) > 0$.