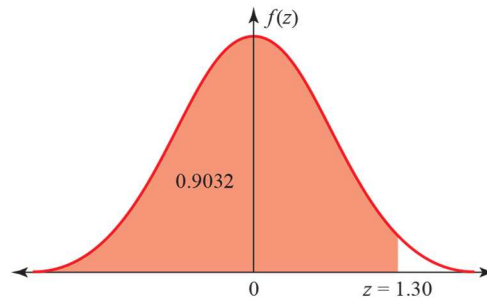
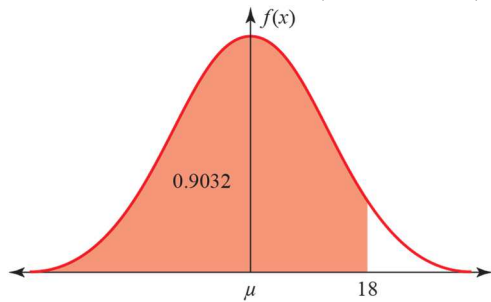


Exercise 7E

$$1 \quad P(X < 18) = 0.9032 \Rightarrow P\left(Z < \frac{18 - \mu}{5}\right) = 0.9032$$

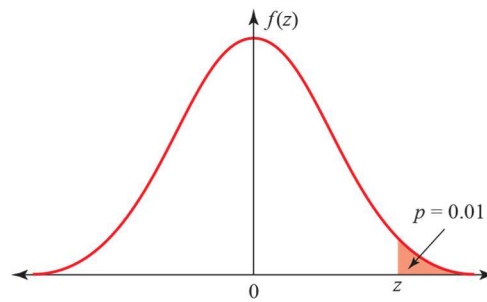
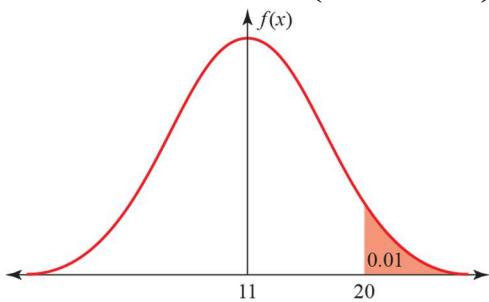


Using the inverse normal function, $z = 1.30$

$$\text{so } 1.30 = \frac{18 - \mu}{5}$$

$$\mu = 18 - 5 \times 1.30 = 11.5$$

$$2 \quad P(X > 20) = 0.01 \Rightarrow P\left(Z < \frac{20 - 11}{\sigma}\right) = 0.01$$



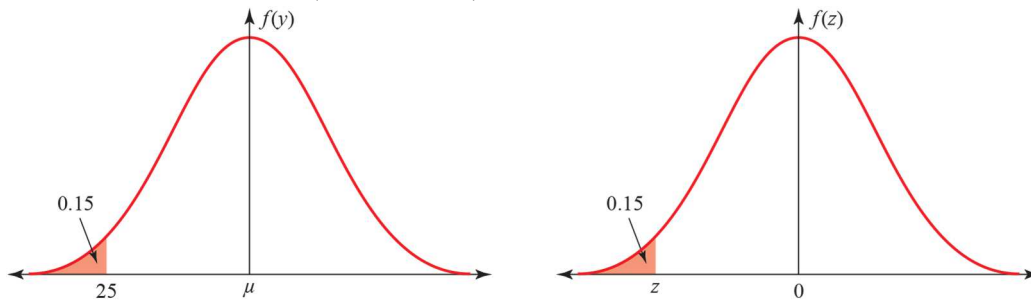
Using the inverse normal function, $z = 2.3263\dots$

$$\text{so } 2.3263\dots = \frac{20 - 11}{\sigma}$$

$$\sigma = \frac{9}{2.3263\dots}$$

$$= 3.8687\dots = 3.87 \text{ (3 s.f.)}$$

$$3 \quad P(Y < 25) = 0.15 \Rightarrow P\left(Z < \frac{25 - \mu}{\sqrt{40}}\right) = 0.15$$

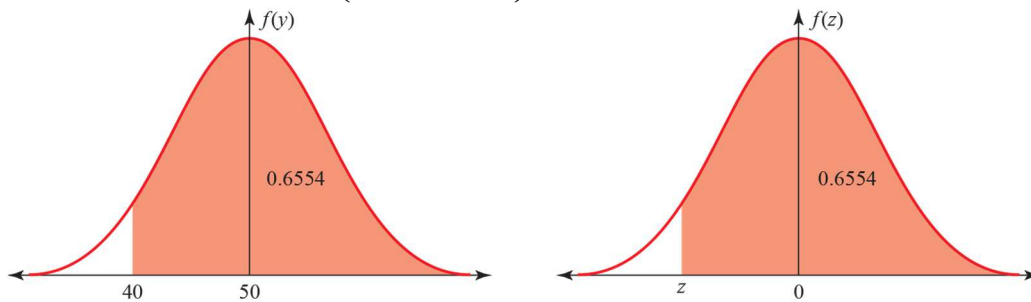


Using the inverse normal function, $z = -1.0364\dots$

$$\text{so } -1.0364\dots = \frac{25 - \mu}{\sqrt{40}}$$

$$\begin{aligned} \mu &= \sqrt{40} \times (-1.0364\dots) \\ &= 31.554\dots = 31.6 \text{ (3 s.f.)} \end{aligned}$$

$$4 \quad P(Y > 40) = 0.6554 \Rightarrow P\left(Z > \frac{40 - 50}{\sigma}\right) = 0.6554$$



Using the inverse normal function, $z = -0.3999\dots$

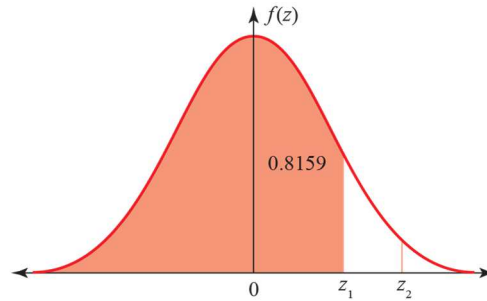
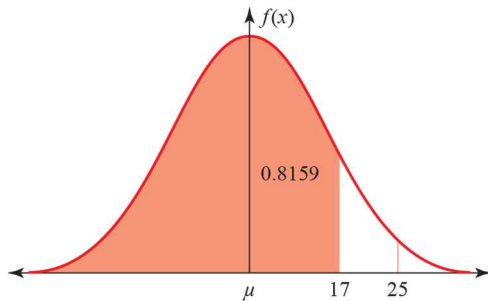
$$\text{so } -0.3999\dots = \frac{40 - 50}{\sigma}$$

$$\sigma = \frac{10}{0.3999\dots} = 25.0 \text{ (3 s.f.)}$$

5 Using the inverse normal function,

$$P(X < 17) = 0.8159 \Rightarrow P\left(Z < \frac{17 - \mu}{\sigma}\right) = 0.8159 \Rightarrow z_1 = 0.8998\dots$$

$$P(X < 25) = 0.9970 \Rightarrow P\left(Z < \frac{25 - \mu}{\sigma}\right) = 0.9970 \Rightarrow z_2 = 2.7477\dots$$



$$\text{So } 0.8998\sigma = 17 - \mu \quad (1)$$

$$\text{and } 2.7477\sigma = 25 - \mu \quad (2)$$

$$(2) - (1): 1.8479\sigma = 8$$

$$\sigma = 4.329\dots$$

Substituting into (2):

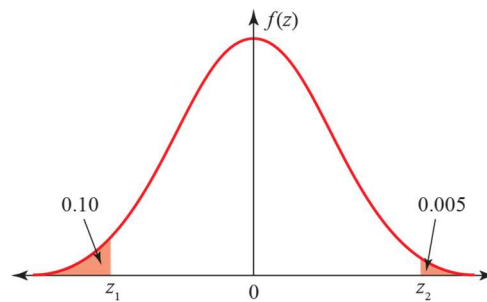
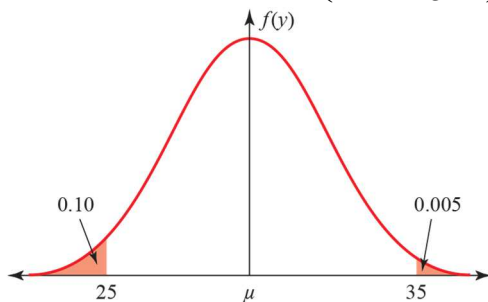
$$\mu = 17 - 0.8998 \times 4.329\dots = 13.104\dots$$

$$\text{So } \mu = 13.1 \text{ and } \sigma = 4.32 \text{ (3 s.f.)}$$

6 Using the inverse normal function (or the percentage points table),

$$P(Y < 25) = 0.10 \Rightarrow P\left(Z < \frac{25 - \mu}{\sigma}\right) = 0.10 \Rightarrow z_1 = -1.28155\dots$$

$$P(Y > 35) = 0.005 \Rightarrow P\left(Z > \frac{35 - \mu}{\sigma}\right) = 0.005 \Rightarrow z_2 = 2.57582\dots$$



$$\text{So } -1.2816\sigma = 25 - \mu \quad (1)$$

$$\text{and } 2.5758\sigma = 35 - \mu \quad (2)$$

$$(2) - (1): 3.8574\sigma = 10$$

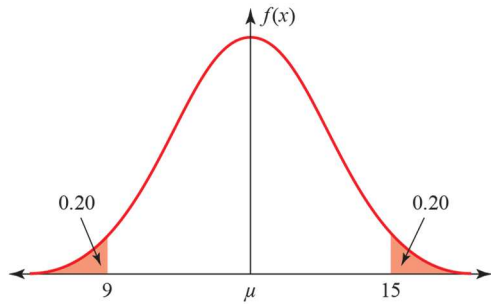
$$\sigma = 2.5924\dots$$

Substituting into (2):

$$\mu = 35 - 2.5758 \times 2.5924\dots = 28.322\dots$$

$$\text{So } \mu = 28.3 \text{ and } \sigma = 2.59 \text{ (3 s.f.)}$$

7



By symmetry, $\mu = \frac{1}{2}(9 + 15) = 12$

$$P(X > 15) = 0.20 \Rightarrow P\left(Z > \frac{15 - 12}{\sigma}\right) = 0.20$$

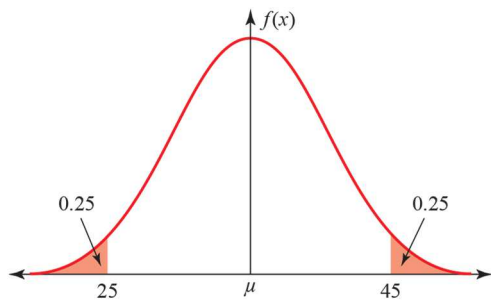
Using the inverse normal function (or the percentage points table), $z = 0.8416\dots$

$$\text{so } 0.8416 = \frac{3}{\sigma}$$

$$\sigma = \frac{3}{0.8416} = 3.564\dots$$

So $\mu = 12$ and $\sigma = 3.56$ (3 s.f.)

8



By symmetry, $\mu = \frac{1}{2}(25 + 45) = 35$

$$P(X > 45) = 0.25 \Rightarrow P\left(Z > \frac{45 - 35}{\sigma}\right) = 0.25$$

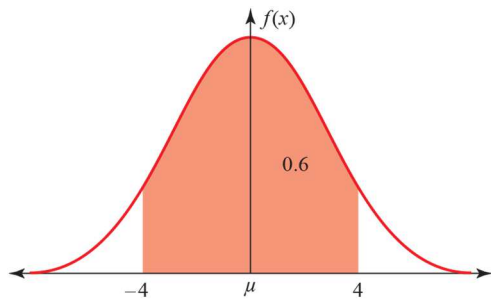
Using the inverse normal function, $z = 0.6744\dots$

$$\text{so } 0.6744 = \frac{10}{\sigma}$$

$$\sigma = \frac{10}{0.6744} = 14.82\dots$$

So $\mu = 35$ and $\sigma = 14.8$ (3 s.f.)

9 $\mu = 0$ (given) so $P(X > 4) = 0.2$ and $P\left(Z > \frac{4-0}{\sigma}\right) = 0.2$



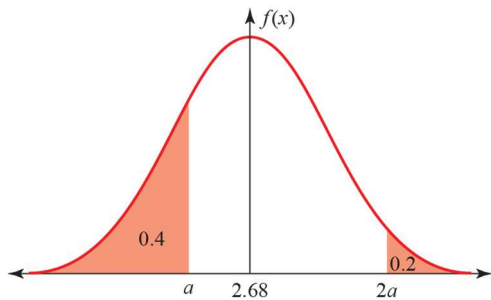
Using the inverse normal function (or the percentage points table), $z = 0.8416\dots$

$$\text{so } 0.8416 = \frac{4}{\sigma}$$

$$\sigma = \frac{4}{0.8416} = 4.752\dots$$

So $\sigma = 4.75$ (3 s.f.)

10



Using the inverse normal function (or the percentage points table),

$$P(X > 2a) = 0.2 \Rightarrow P\left(Z < \frac{2a - 2.68}{\sigma}\right) = 0.2 \Rightarrow z_1 = 0.8416\dots$$

$$P(X < a) = 0.4 \Rightarrow P\left(Z < \frac{a - 2.68}{\sigma}\right) = 0.4 \Rightarrow z_2 = -0.2533\dots$$

$$\text{So } 0.8416\sigma = 2a - 2.68 \quad (1)$$

$$\text{and } -0.2533\sigma = a - 2.68 \quad (2)$$

$$(2) \times 2: -0.5066\sigma = 2a - 5.36 \quad (3)$$

$$(1) - (3): 1.3482\sigma = 2.68$$

$$\sigma = 1.9878\dots$$

Substituting into (2):

$$a = 2.68 - 0.2533 \times 1.9878\dots = 2.176\dots$$

So $\sigma = 1.99$ and $a = 2.18$ (3 s.f.)

11 a The distribution is $D \sim N(\mu, 5^2)$.

$$P(D > 200) = 0.75 \Rightarrow P(D < 200) = 0.25 \Rightarrow P\left(Z < \frac{200 - \mu}{5}\right) = 0.25$$

Using the inverse normal function, $z = -0.6744\dots$

$$\text{so } -0.6744\dots = \frac{200 - \mu}{5}$$

$$\begin{aligned}\mu &= 200 + 5 \times 0.6744\dots \\ &= 203.37\dots = 203 \text{ mm (3 s.f.)}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad P(204 < D < 206) &= P(D < 206) - P(D < 204) \\ &= P\left(Z < \frac{206 - 203.37\dots}{5}\right) - P\left(Z < \frac{204 - 203.37\dots}{5}\right) \\ &= P(Z < 0.5256) - P(Z < 0.1256) \\ &= 0.70041\dots - 0.54997\dots \\ &= 0.15045 = 0.1504 \text{ (4 d.p.)}\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad P(D > 205) &= P(Z > 0.3256) = 1 - 0.62763\dots = 0.37237\dots \\ \text{So the probability that all three bowls are greater than 205 mm in diameter is } &P(D > 205)^3, \\ \text{i.e. } &(0.37237\dots)^3 = 0.05163\dots = 0.0516 \text{ (3 s.f.)}.\end{aligned}$$

12 a The distribution is $T \sim N(2.5, \sigma)$.

$$P(T < 2.55) = 0.65 \Rightarrow P\left(Z < \frac{2.55 - 2.5}{\sigma}\right) = 0.65$$

Using the inverse normal function, $z = 0.38532\dots$

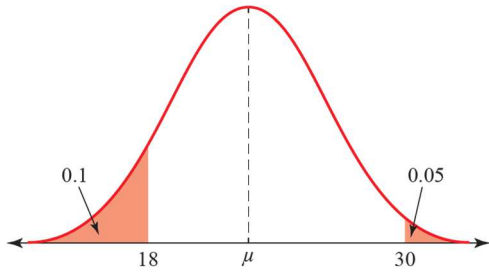
$$\text{so } \frac{2.55 - 2.5}{\sigma} = 0.38532\dots$$

$$\begin{aligned}0.05 &= \sigma \times 0.38532\dots \\ \sigma &= 0.12976\dots = 0.1298 \text{ (4 d.p.)}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad P(2.4 < T < 2.6) &= P(T < 2.6) - P(T < 2.4) \text{ (4 s.f.)} \\ &= P\left(Z < \frac{2.6 - 2.5}{0.12976\dots}\right) - P\left(Z < \frac{2.4 - 2.5}{0.12976\dots}\right) \\ &= P(Z < 0.77065\dots) - P(Z < -0.77065\dots) \\ &= 0.77954\dots - 0.22045\dots \\ &= 0.55908\dots = 0.5591 \text{ (4 d.p.)}\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad \text{Use the binomial distribution } X &\sim B(20, 0.5591). \\ \text{Using the binomial CD function,} \\ P(X \geq 15) &= 1 - P(X \leq 14) = 1 - 93501\dots = 0.06497\dots \\ \text{So the probability that at least 15 table cloths can be sold is } &0.0650 \text{ (4 d.p.)}\end{aligned}$$

13 a



$$\text{b } P(M < 18) = 0.10 \Rightarrow P\left(Z < \frac{18 - \mu}{\sigma}\right) = 0.10 \Rightarrow z_1 = -1.28155\dots$$

$$P(M > 30) = 0.05 \Rightarrow P\left(Z > \frac{30 - \mu}{\sigma}\right) = 0.05 \Rightarrow z_2 = 1.64485\dots$$

$$\text{So } -1.28155\dots \times \sigma = 18 - \mu \quad (1)$$

$$\text{and } 1.64485\dots \times \sigma = 30 - \mu \quad (2)$$

$$(2) - (1): 2.92640\dots \times \sigma = 12$$

$$\sigma = 4.10059\dots$$

Substituting into (2):

$$\mu = 30 - 1.64485\dots \times 4.10059\dots = 23.25512\dots$$

So $\mu = 23.26$ and $\sigma = 4.101$ (4 s.f.)

$$\text{c } P(M > 25) = 1 - P\left(Z > \frac{25 - 23.25512\dots}{4.10059\dots}\right) = 0.33522\dots$$

Use the binomial distribution $X \sim B(10, 0.33522\dots)$

Using the binomial CD function,

$$P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.55408\dots = 0.44591\dots$$

So the probability that at least 4 have a mass greater than 25 kg is 0.4459 (4 d.p.)

$$\text{14 a } P(L < 16) = 0.20 \Rightarrow P\left(Z < \frac{16 - \mu}{\sigma}\right) = 0.20 \Rightarrow z_1 = -0.84162\dots$$

$$P(L > 18) = 0.10 \Rightarrow P\left(Z > \frac{18 - \mu}{\sigma}\right) = 0.10 \Rightarrow z_2 = 1.28155\dots$$

$$\text{So } -0.84162\dots \times \sigma = 16 - \mu \quad (1)$$

$$\text{and } 1.28155\dots \times \sigma = 18 - \mu \quad (2)$$

$$(2) - (1): 2.12317\dots \times \sigma = 2$$

$$\sigma = 0.94198\dots$$

Substituting into (2):

$$\mu = 18 - 1.28155\dots \times 0.94198\dots = 16.79279\dots$$

So $\mu = 16.79$ and $\sigma = 0.9420$ (4 s.f.)

$$\text{b } P(L < Q_1) = 0.25 \Rightarrow z_1 = -0.67448\dots \Rightarrow \frac{Q_1 - 16.79279\dots}{0.94198\dots} = -0.67448\dots \Rightarrow Q_1 = 16.15743\dots$$

$$P(L < Q_3) = 0.75 \Rightarrow z_2 = 0.67448\dots \Rightarrow \frac{Q_3 - 16.79279\dots}{0.94198\dots} = 0.67448\dots \Rightarrow Q_3 = 17.42814\dots$$

The interquartile range is $Q_3 - Q_1 = 17.42814\dots - 16.15743\dots = 1.27071\dots = 1.27$ (2 d.p.)

Challenge

- a** Let the quartiles be $Q_3 = \mu + z\sigma$ and $Q_1 = \mu - z\sigma$.
Then the interquartile range is $Q_3 - Q_1 = q = (\mu + z\sigma) - (\mu - z\sigma) = 2z\sigma$
 z is such that $\Phi(z) = 0.75$ so, using the inverse normal function, $z = 0.67448\dots$
So $q = 2 \times 0.67448\dots \times \sigma = 1.34987\dots \times \sigma$
and hence $\sigma = 0.742 \times q = 0.741q$ (3 s.f.)
- b** Since $q = (\mu + z\sigma) - (\mu - z\sigma) = 2z\sigma$ (i.e. the μ s cancel), q is not dependent on μ .
So it is not possible to write an equation for q in terms of μ , and vice versa.