Exercise 6G

1 For a discrete uniform distribution:

$$\begin{aligned}\mathrm{E}(X) &= \frac{n+1}{2}\\ &= \frac{5+1}{2}\\ &= 3\end{aligned}$$

$$\begin{aligned}\mathrm{Var}(X) &= \frac{(n+1)(n-1)}{12}\\ &= \frac{(5+1)(5-1)}{12}\\ &= 2\end{aligned}$$

2 a For a discrete uniform distribution:

$$\begin{aligned}\mathrm{E}(X) &= \frac{n+1}{2}\\ &= \frac{7+1}{2}\\ &= 4\end{aligned}$$

b $$\begin{aligned}\mathrm{Var}(X) &= \frac{(n+1)(n-1)}{12}\\ &= \frac{(7+1)(7-1)}{12}\\ &= 4\end{aligned}$$

3 a For a discrete uniform distribution:

$$\begin{aligned}\mathrm{E}(X) &= \frac{n+1}{2}\\ &= \frac{6+1}{2}\\ &= \frac{7}{2}\end{aligned}$$

$$\begin{aligned}\mathrm{Var}(X) &= \frac{(n+1)(n-1)}{12}\\ &= \frac{(6+1)(6-1)}{12}\\ &= \frac{35}{12}\end{aligned}$$

3 b $\text{Var}(X) = \frac{35}{12}$

Therefore

$\sigma = \sqrt{\frac{35}{12}}$

$= 1.707...$

$\bar{x} - \sigma = 1.792...$ and $\bar{x} + \sigma = 5.207...$

$\text{P}(1.792\ldots < x < 5.207\ldots) = \text{P}(2 \leqslant x \leqslant 5)$

$= \text{P}(2) + \text{P}(3) + \text{P}(4) + \text{P}(5)$

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

$= \frac{2}{3}$

4 a $\text{P}(X > 15) = 0.3$

b 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

x	2	4	6	8	10	12	14	16	18	20
$\text{P}(X = x)$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

$\text{E}(X) = \sum x\text{P}(X = x)$

$= 2\times0.1 + 4\times0.1 + 6\times0.1 + 8\times0.1 + 10\times0.1 + 12\times0.1 + 14\times0.1 + 16\times0.1 + 18\times0.1 + 20\times0.1$

$= 11$

$\text{Var}(X) = \text{E}(X^2) - (\text{E}(X))^2$

$\text{E}(X^2) = \sum x^2\text{P}(X = x)$

$= 2^2\times0.1 + 4^2\times0.1 + 6^2\times0.1 + 8^2\times0.1 + 10^2\times0.1 + 12^2\times0.1 + 14^2$

$\times0.1 + 16^2\times0.1 + 18^2\times0.1 + 20^2\times0.1$

$= 154$

$\text{Var}(X) = 154 - (11)^2$

$= 154 - 121$

$= 33$

Alternatively. Note that if you consider the set of numbers from 1 to 10 inclusive, this sequence is double that.

Taking $X \sim U(10)$

$\text{E}(2X) = 2\times\frac{10+1}{2}$

$= 11$

$\text{Var}(2X) = 2^2\times\text{Var}(X)$

$= 2^2\times\frac{(10+1)(10-1)}{12}$

$= 33$

5 A discrete uniform distribution is not likely to be a good a model for this distribution. The game depends on the skills of the player. The points are likely to cluster around the middle.

6 **a** A discrete uniform distribution.

b For a discrete uniform distribution

$$\mathrm{E}(X) = \frac{n+1}{2}$$
$$= \frac{8+1}{2}$$
$$= 4.5$$

c $$\mathrm{Var}(X) = \frac{(n+1)(n-1)}{12}$$
$$= \frac{(8+1)(8-1)}{12}$$
$$= 5.25$$

d The expected winnings are less than the 5 cents stake.