

### Exercise 6F

- 1 a Rearrange the formula  $Y = 4X - 6$  to get an expression for  $X$  in terms of  $Y$ :

$$Y = 4X - 6 \text{ gives}$$

$$X = \frac{Y+6}{4}$$

$$X = \frac{Y}{4} + \frac{3}{2}$$

$$\begin{aligned} E(X) &= E\left(\frac{Y}{4} + \frac{3}{2}\right) = \frac{1}{4}E(Y) + \frac{3}{2} \\ &= \frac{1}{4} \times 2 + \frac{3}{2} = \frac{1}{2} + \frac{3}{2} = 2 \end{aligned}$$

$$\begin{aligned} \text{b } \text{Var}(X) &= \text{Var}\left(\frac{Y}{4} + \frac{3}{2}\right) = \left(\frac{1}{4}\right)^2 \text{Var}(Y) \\ &= \frac{1}{16} \times 32 = 2 \end{aligned}$$

$$\text{c } \text{Standard deviation} = \sqrt{\text{Var}(X)} = \sqrt{2} = 1.4142 \text{ (4 d.p.)}$$

- 2 a Rearrange the formula get an expression for  $X$  in terms of  $Y$ :

$$2Y = 4 - 3X$$

$$3X = 4 - 2Y$$

$$X = \frac{4 - 2Y}{3}$$

$$X = \frac{4}{3} - \frac{2}{3}Y$$

$$\begin{aligned} E(X) &= E\left(\frac{4}{3} - \frac{2}{3}Y\right) = \frac{4}{3} - \frac{2}{3}E(Y) \\ &= \frac{4}{3} - \frac{2}{3}(-1) = 2 \end{aligned}$$

$$\begin{aligned} \text{b } \text{Var}(X) &= \text{Var}\left(\frac{4}{3} - \frac{2}{3}Y\right) = \left(-\frac{2}{3}\right)^2 \text{Var}(Y) \\ &= \frac{4}{9} \times 9 = 4 \end{aligned}$$

$$\begin{aligned} \text{c } \text{Var}(X) &= E(X^2) - (E(X))^2 \\ \text{So } E(X^2) &= \text{Var}(X) + (E(X))^2 \\ &= 4 + 2^2 = 8 \end{aligned}$$

3 Rearranging the formula for  $Y$  to get an expression for  $X$  gives:

$$X = \frac{Y}{2} - \frac{3}{2}$$

$$\begin{aligned} E(X) &= E\left(\frac{Y}{2} - \frac{3}{2}\right) = \frac{1}{2}E(Y) - \frac{3}{2} \\ &= \frac{1}{2} \times 8 - \frac{3}{2} = 4 - \frac{3}{2} = \frac{5}{2} = 2.5 \end{aligned}$$

$$E(X) = \sum xP(X = x) = 2.5$$

$$0.3 + 2a + 3b + 4 \times 0.2 = 2.5$$

$$2a + 3b + 1.1 = 2.5$$

$$2a + 3b = 1.4 \quad (1)$$

$$\sum P(X = x) = 1$$

$$\text{So } 0.3 + a + b + 0.2 = 1$$

$$a + b = 0.5 \quad (2)$$

$$2 \times (2) \Rightarrow 2a + 2b = 1 \quad (3)$$

$$(1) - (3) \Rightarrow b = 0.4$$

$$\text{From (2) } a + 0.4 = 0.5 \Rightarrow a = 0.1$$

Solution:

$$a = 0.1, \quad b = 0.4$$

4 a The probability distribution of  $Y$  is:

$y$	1	0	-1
$P(Y = y)$	$a$	$b$	0.3

$$E(Y) = a + 0 \times b - 1 \times 0.3 = a - 0.3$$

$$\text{As } a + b + 0.3 = 1, \quad 0 \leq a \leq 0.7$$

$$\max a = 0.7 \Rightarrow E(Y) = 0.4$$

$$\min a = 0 \Rightarrow E(Y) = -0.3$$

$$\text{So range of possible values for } E(Y) \text{ is } -0.3 \leq E(Y) \leq 0.4$$

b  $E(Y) = 0.2$  gives  $0.2 = a - 0.3 \Rightarrow a = 0.5$

As probabilities sum to 1:

$$a + b + 0.3 = 1 \Rightarrow b = 1 - 0.3 - 0.5 = 0.2$$