Exercise 6B

1 a

x	1	2	3	4	5	6
$F(x)$	0.1	0.2	0.35	0.6	0.9	1.0

b $F(5) = 0.9$

c $F(2.2) = F(2) = 0.2$

2 a

x	0	1	2	3	4	5	6
$F(x)$	0	0.1	0.2	0.45	0.5	0.9	1.0

x	0	1	2	3	4	5	6
$P(X = x)$	0	0.1	0.1	0.25	0.05	0.4	0.1

b $P(X < 5) = 0 + 0.1 + 0.1 + 0.25 + 0.05$
$= 0.5$

c $P(2 \leqslant X < 5) = 0.1 + 0.25 + 0.05$
$= 0.4$

3 a $P(X = x) = \begin{cases} kx & x = 1,3,5 \\ k(x-1) & x = 2,4,6 \end{cases}$

x	1	2	3	4	5	6
$P(X = x)$	k	k	$3k$	$3k$	$5k$	$5k$

Since the sum of the probabilities is 1,
$18k = 1$

$k = \frac{1}{18}$

b

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{3}{18}$	$\frac{3}{18}$	$\frac{5}{18}$	$\frac{5}{18}$

c $P(2 \leqslant X < 5) = \frac{1}{18} + \frac{3}{18} + \frac{3}{18}$
$= \frac{7}{18}$

d $F(4) = \frac{1}{18} + \frac{1}{18} + \frac{3}{18} + \frac{3}{18}$
$= \frac{4}{9}$

e $F(1.6) = F(1) = \frac{1}{18}$

4 a Since the sum of the probabilities is 1,

$2(0.1)+2\alpha+0.3=1$

$\alpha=0.25$

b

x	-2	-1	0	1	2
$P(X=x)$	0.1	0.1	0.25	0.25	0.3

c $F(0.3)=F(0)$

$=0.1+0.1+0.25$

$=0.45$

5 a $F(X)=\frac{1+x}{6}$

$=\frac{1+4}{6}$

$=\frac{5}{6}$

b $P(X=4)=F(4)-F(3)$

$=\frac{1+4}{6}-\frac{1+3}{6}$

$=\frac{5}{6}-\frac{4}{6}$

$=\frac{1}{6}$

c

x	1	2	3	4	5
$P(X=x)$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

6 a $F(x)=\frac{(x+k)^2}{16}$

$F(3)=1$

Therefore

$\frac{(3+k)^2}{16}=1$

$(3+k)^2=16$

$3+k=\pm4$

$k=1$ or $k=-7$

When $k=-7$ and $x=1$

$P(1)=\frac{(x-7)^2}{16}=\frac{36}{16}$

As a probability cannot be greater than 1, $k\neq-7$

Therefore $k=1$.

6 b $F(x) = \dfrac{(x+1)^2}{16}$

x	1	2	3
$F(x)$	$\frac{4}{16}$	$\frac{9}{16}$	1

x	1	2	3
$P(X = x)$	$\frac{4}{16}$	$\frac{5}{16}$	$\frac{7}{16}$