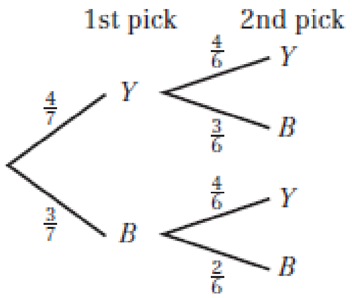
Chapter review 4

1 a



b P(both blue) $= \mathrm{P}(B \cap B)$

$$= \frac{3}{7} \times \frac{2}{6}$$

$$= \frac{1}{7}$$

c P(one of each colour) $= \mathrm{P}(Y \cap B) + \mathrm{P}(B \cap Y)$

$$= \frac{4}{7} \times \frac{3}{6} + \frac{3}{7} \times \frac{4}{6}$$

$$= \frac{12}{21} = \frac{4}{7}$$

2 a $\mathrm{P}(RRB \text{ or } RRG) = \left(\frac{7}{15} \times \frac{7}{15} \times \frac{3}{15}\right) + \left(\frac{7}{15} \times \frac{7}{15} \times \frac{5}{15}\right)$

$$= \frac{392}{3375}$$

b $\mathrm{P}(RBG) + \mathrm{P}(RGB) + \mathrm{P}(BGR) + \mathrm{P}(BRG) + \mathrm{P}(GBR) + \mathrm{P}(GRB)$

$$= \left(\frac{7}{15} \times \frac{3}{15} \times \frac{5}{15}\right) + \left(\frac{7}{15} \times \frac{5}{15} \times \frac{3}{15}\right) + \left(\frac{3}{15} \times \frac{5}{15} \times \frac{7}{15}\right) + \left(\frac{3}{15} \times \frac{7}{15} \times \frac{5}{15}\right) + \left(\frac{5}{15} \times \frac{3}{15} \times \frac{7}{15}\right) + \left(\frac{5}{15} \times \frac{7}{15} \times \frac{3}{15}\right)$$

$$= 6 \times \left(\frac{7 \times 3 \times 5}{15^3}\right) = \frac{630}{3375} = \frac{14}{75}$$

3 a $\mathrm{P}(HHH) = 0.341 \times 0.341 \times 0.341 = 0.0397$ (to 3 s.f.)

b $\mathrm{P}(NNN) = 0.659 \times 0.659 \times 0.659 = 0.286$ (to 3 s.f.)

c P(at least one H) $= 1 - \mathrm{P}(NNN) = 1 - 0.28619118 = 0.714$ (to 3 s.f.)

4 a $\text{P(Year 11)} = \dfrac{8+13+19+30+26+32}{250} = \dfrac{128}{250} = \dfrac{64}{125}$

b $\text{P}(s < 35) = \dfrac{7+8+15+13+18+19}{250} = \dfrac{80}{250} = \dfrac{8}{25}$

c $\text{P(Year 10 with score between 25 and 34)} = \dfrac{15+18}{250} = \dfrac{33}{250}$

d Using interpolation:

$$\text{Number of students passing} = \frac{40-37}{40-35}\times(25+30)+30+26+27+32$$
$$= \left(\frac{3}{5}\times 55\right) + 30 + 26 + 27 + 32 = 148$$

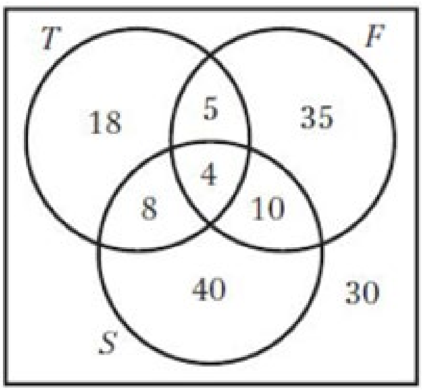
$\text{P(pass)} = \dfrac{148}{250} = \dfrac{74}{125}$

The assumption is that the marks between 35 and 40 are uniformly distributed.

5 a $\text{P(mass} > 3) = \dfrac{0.5\times 50+0.5\times 30+2\times 2}{1\times 6+0.5\times 50+0.5\times 30+2\times 2} = \dfrac{44}{50} = \dfrac{22}{25}$

b $\text{P(mass} < 3.75) = \dfrac{(1\times 6)+(0.5\times 50)+0.5\times(0.5\times 30)}{50} = \dfrac{38.5}{50} = 0.77$

6 a



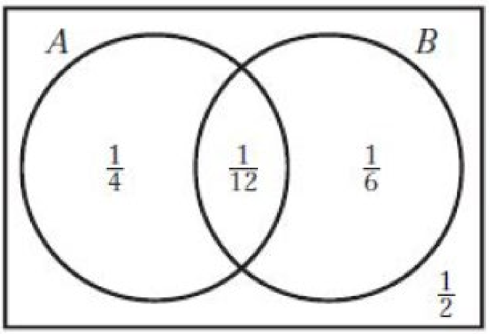
b i $\text{P(None)} = \dfrac{30}{150} = \dfrac{1}{5}$

ii $\text{P(No more than one)} = \dfrac{30+40+18+35}{150} = \dfrac{123}{150} = \dfrac{41}{50}$

7 a P(A and B) = P(A) + P(B) – P(A or B or both) = $\frac{1}{3}+\frac{1}{4}-\frac{1}{2}=\frac{1}{12}$

P(A and not B) = $\frac{1}{3}-\frac{1}{12}=\frac{1}{4}$

P(B and not A) = $\frac{1}{4}-\frac{1}{12}=\frac{1}{6}$



b P(A and B) = $\frac{1}{12}$

P(A) × P(B) = $\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}$

As P(A and B) = P(A) × P(B), A and B are independent events.

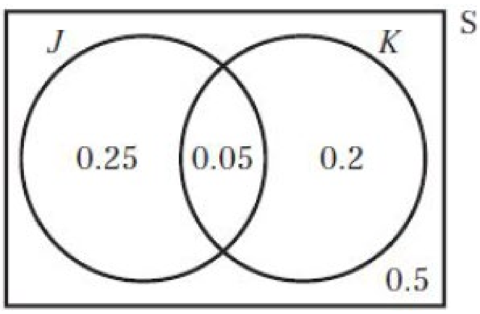
8 a Cricket and swimming do not overlap so are mutually exclusive.

b P(C and F) = $\frac{13}{38}$

P(C) × P(F) = $\frac{21}{38}\times\frac{22}{38}=\frac{462}{1444}=\frac{231}{722}$

As P(C and F) ≠ P(C) × P(F), the events 'likes cricket' and 'likes football' are not independent.

9 a



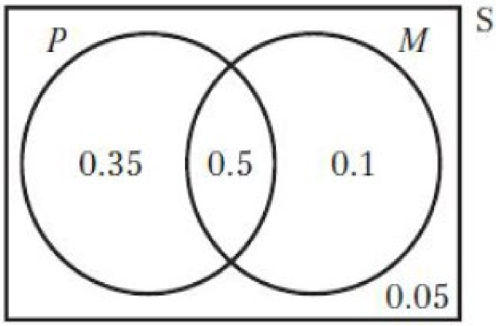
b P(J and K) = 0.05

P(J) × P(K) = 0.3 × 0.25 = 0.075

As P(J and K) ≠ P(J) × P(K), the events J and K are not independent.

10 a P(Phone and Tablet) = $0.85 + 0.6 - (1 - 0.05) = 0.5 = 50\%$

b



c P(only P) = 0.35

d P(P and T) = 0.5

P(P) × P(T) = $0.85 \times 0.6 = 0.51$

As P(P and T) ≠ P(P) × P(T), the events P and T are not independent.

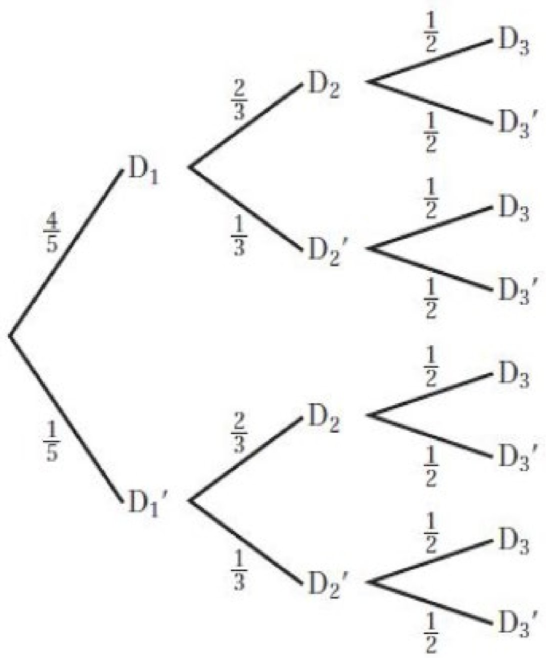
11 $x = 1 - (0.3 + 0.4 + 0.15) = 0.15$

P(A and B) = $x = 0.15$

P(A) × P(B) = $0.45 \times 0.55 = 0.2475$

As P(A and B) ≠ P(A) × P(B), the events A and B are not independent.

12 a



b i $P(D_1D_2D_3) = \frac{4}{5}\times\frac{2}{3}\times\frac{1}{2}=\frac{4}{15}$

ii Where D means a diamond and D' means no diamond,

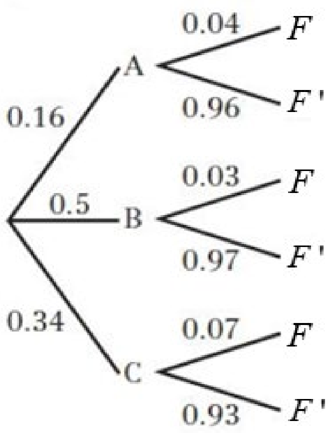
P (exactly one diamond) = $P(D, D', D') + P(D', D, D') + P(D', D', D)$

$$= \left(\frac{4}{5}\times\frac{1}{3}\times\frac{1}{2}\right)+\left(\frac{1}{5}\times\frac{2}{3}\times\frac{1}{2}\right)+\left(\frac{1}{5}\times\frac{1}{3}\times\frac{1}{2}\right)=\frac{7}{30}$$

c P(at least two diamonds) = 1 – P(at most one diamond) = 1 – (P(none) + P(exactly one diamond))

$$= 1 - \left(\frac{1}{5}\times\frac{1}{3}\times\frac{1}{2}+\frac{7}{30}\right)=1-\frac{4}{15}=\frac{11}{15}$$

13 a



b i P(B and faulty) = 0.5 × 0.03 = 0.015

ii P(faulty) = 0.16 × 0.04 + 0.5 × 0.03 + 0.34 × 0.07 = 0.0452

14 a $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B) = 0.4 + 0.35 - 0.2 = 0.55$

b $\mathrm{P}(A' \cap B') = 1 - \mathrm{P}(A \cup B) = 1 - 0.55 = 0.45$

c $\mathrm{P}(B|A) = \dfrac{\mathrm{P}(B \cap A)}{\mathrm{P}(A)} = \dfrac{0.2}{0.4} = 0.5$

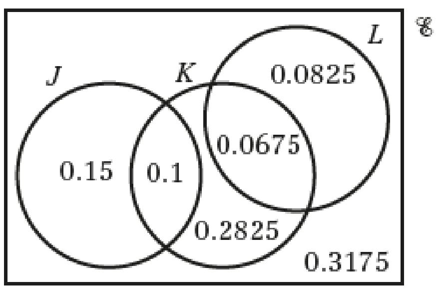
d $\mathrm{P}(A'|B) = \dfrac{\mathrm{P}(A' \cap B)}{\mathrm{P}(B)} = \dfrac{\mathrm{P}(B) - \mathrm{P}(A \cap B)}{\mathrm{P}(B)} = \dfrac{0.15}{0.35} = 0.429$ (3 s.f.)

15 a Work out each region of the Venn diagram from the information provided in the question.

First, as J and L are mutually exclusive, $J \cap L = \varnothing$ therefore $\mathrm{P}(J \cap L) = 0$
So $\mathrm{P}(J \cap K' \cap L') = \mathrm{P}(J) - \mathrm{P}(J \cap K) = 0.25 - 0.1 = 0.15$

As K and L are independent $\mathrm{P}(K \cap L) = \mathrm{P}(K) \times \mathrm{P}(L) = 0.45 \times 0.15 = 0.0675$
So $\mathrm{P}(L \cap K') = \mathrm{P}(L) - \mathrm{P}(L \cap K) = 0.15 - 0.0625 = 0.0825$
And $\mathrm{P}(K \cap J' \cap L') = \mathrm{P}(K) - \mathrm{P}(J \cap K) - \mathrm{P}(K \cap L) = 0.45 - 0.1 - 0.0675 = 0.2825$

Find the outer region by subtracting the sum of all the other regions from 1
$\mathrm{P}(J' \cap K' \cap L') = 1 - 0.15 - 0.1 - 0.2825 - 0.0675 - 0.0825 = 0.3175$



15 b i $P(J \cup K) = 0.15 + 0.1 + 0.2825 + 0.0675 = 0.6$

ii $P(J' \cap L') = 0.2825 + 0.3175 = 0.6$

iii $P(J|K) = \dfrac{P(J \cap K)}{P(K)} = \dfrac{0.1}{0.45} = 0.222$ (3 s.f.)

iv $P(K|J' \cap L') = \dfrac{P(K \cap (J' \cap L'))}{P(J' \cap L')} = \dfrac{0.2825}{0.6} = 0.471$ (3 s.f.)

16 a $P(F \cap S') + P(S \cap F') = P(F) - P(F \cap S) + P(F) - P(F \cap S)$

$$= \frac{35 - 27 + 45 - 27}{60} = \frac{26}{60} = 0.433 \text{ (3 s.f.)}$$

b $P(F|S) = \dfrac{P(F \cap S)}{P(S)} = \dfrac{27}{45} = 0.6$

c $P(S|F') = \dfrac{P(S \cap F')}{P(F')} = \dfrac{45 - 27}{60 - 35} = \dfrac{18}{25} = 0.72$

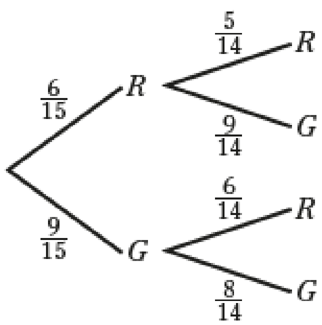
d There are 6 students that study just French and wear glasses ($8 \times 0.75 = 6$) and 9 students that study just Spanish and wear glasses ($18 \times 0.5 = 9$), so the required probability is

$$\text{P(studies one language and wears glasses)} = \frac{6+9}{60} = \frac{15}{60} = 0.25$$

e There are 26 students studying one language (from part **a**). Of these, 15 wear glasses (from part **d**).

$$\text{P(wears glasses|studies one language)} = \frac{15}{26} = 0.577 \text{ (3 s.f.)}$$

17 a



b i $P(GG) = \dfrac{9}{15} \times \dfrac{8}{14} = \dfrac{3}{5} \times \dfrac{4}{7} = \dfrac{12}{35} = 0.343$ (3 s.f.)

ii There are two different ways to obtain balls that are different colours:

$$P(RG) + P(GR) = \left(\frac{6}{15} \times \frac{9}{14}\right) + \left(\frac{9}{15} \times \frac{6}{14}\right) = \frac{2 \times 9}{5 \times 7} = \frac{18}{35} = 0.514 \text{ (3 s.f.)}$$

17 c There are 4 different outcomes:

$$\mathrm{P}(RRR)+\mathrm{P}(RGR)+\mathrm{P}(GRR)+\mathrm{P}(GGR)$$
$$=\left(\frac{6}{15}\times\frac{5}{14}\times\frac{4}{13}\right)+\left(\frac{6}{15}\times\frac{9}{14}\times\frac{5}{13}\right)+\left(\frac{9}{15}\times\frac{6}{14}\times\frac{5}{13}\right)+\left(\frac{9}{15}\times\frac{8}{14}\times\frac{6}{13}\right)$$
$$=\frac{120+270+270+432}{2730}=\frac{1092}{2730}=0.4$$

d The only way for this to occur is to draw a green ball each time. The corresponding probability is:

$$\mathrm{P}(GGGG)=\frac{9}{15}\times\frac{8}{14}\times\frac{7}{13}\times\frac{6}{12}=\frac{3\times2}{5\times13}=\frac{6}{65}=0.0923\text{ (3 s.f.)}$$

18 a Either Ty or Chimene must win both sets. Therefore the required probability is:

$$\mathrm{P}(\text{match over in two sets})=(0.7\times0.8)+(0.3\times0.6)=0.56+0.18=0.74$$

b $\mathrm{P}(\text{Ty wins}|\text{match over in two sets})=\frac{0.7\times0.8}{0.74}=\frac{0.56}{0.74}=0.757\text{ (3 s.f.)}$

c The three ways that Ty can win the match are: wins first set, wins second set; wins first set, loses second set, wins tiebreaker; loses first set, wins second set, wins tiebreaker.

$$\mathrm{P}(\text{Ty wins match})=(0.7\times0.8)+(0.7\times0.2\times0.55)+(0.3\times0.4\times0.55)$$
$$=0.56+0.077+0.066=0.703$$

19 a There are 20 kittens with neither black nor white paws (75 – 26 – 14 – 15 = 20).

$$\mathrm{P}(\text{neither white or black paws})=\frac{20}{75}=\frac{4}{15}=0.267\text{ (3 s.f.)}$$

b There are 41 kittens with some black paws (26 + 15 = 41).

$$\mathrm{P}(\text{black and white paws}|\text{some black paws})=\frac{15}{41}=0.366\text{ (3 s.f.)}$$

c This is selection without replacement (since the first kitten chosen is not put back).

$$\mathrm{P}(\text{both kittens have all black paws})=\frac{26}{75}\times\frac{25}{74}=\frac{13}{3\times37}=\frac{13}{111}=0.117\text{ (3 s.f.)}$$

d There are 29 kittens with some white paws (14 + 15 = 29).

$$\mathrm{P}(\text{both kittens have some white paws})=\frac{29}{75}\times\frac{28}{74}=\frac{812}{5550}=0.146\text{ (3 s.f.)}$$

20 a Using the fact that A and B are independent: $\mathrm{P}(A)\times\mathrm{P}(B)=\mathrm{P}(A\cap B)\Rightarrow\mathrm{P}(B)=\frac{0.12}{0.4}=0.3$

b Use the addition formula to find $\mathrm{P}(A\cup B)$

$$\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)=0.4+0.3-0.12=0.58$$
$$\mathrm{P}(A'\cap B')=1-\mathrm{P}(A\cup B)=1-0.58=0.42$$

20 c As A and C are mutually exclusive

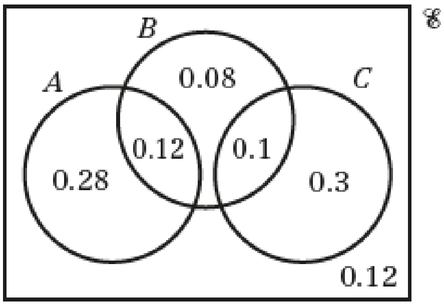
$\text{P}(A \cap B' \cap C') = P(A) - \text{P}(A \cap B) = 0.4 - 0.12 = 0.28$

$\text{P}(C \cap A' \cap B') = P(C) - \text{P}(B \cap C) = 0.4 - 0.1 = 0.3$

$\text{P}(B \cap A' \cap C') = P(B) - \text{P}(A \cap B) - P(B \cap C) = 0.3 - 0.12 - 0.1 = 0.08$

Find the outer region by subtracting the sum of all the other regions from 1

$\text{P}(A' \cap B' \cap C') = 1 - 0.28 - 0.12 - 0.08 - 0.1 - 0.3 = 0.12$



d i $\text{P}(B|C) = \dfrac{\text{P}(B \cap C)}{\text{P}(C)} = \dfrac{0.1}{0.4} = 0.25$

ii The required region must be contained within A, and not include B (the condition on C is irrelevant since A and C are mutually exclusive). Therefore, $\text{P}(A \cap (B' \cup C)) = 0.28$

21 a It may be that neither team scores in the match, and it is a 0–0 draw.

b $\text{P}(\text{team } A \text{ scores first}) = \text{P}(\text{team } A \text{ scores first and wins}) + \text{P}(\text{team } A \text{ scores first and does not win})$

So $\text{P}(\text{team } A \text{ scores first and does not win}) = 0.6 - 0.48 = 0.12$

c From the question $\text{P}(A \text{ wins}|B \text{ scores first}) = 0.3$. Using the multiplication formula gives

$$\text{P}(A \text{ wins}|B \text{ scores first}) = \frac{\text{P}(A \text{ wins} \cap B \text{ scores first})}{\text{P}(B \text{ scores first})} = 0.3$$

$\Rightarrow \text{P}(A \text{ wins} \cap B \text{ scores first}) = 0.3 \times 0.35 = 0.105$

Now find the required probability

$$\text{P}(B \text{ scores first}|A \text{ wins}) = \frac{\text{P}(A \text{ wins} \cap B \text{ scores first})}{\text{P}(A \text{ wins})} = \frac{0.105}{0.48 + 0.105} = \frac{0.105}{0.585} = 0.179 \text{ (3 s.f.)}$$

Challenge

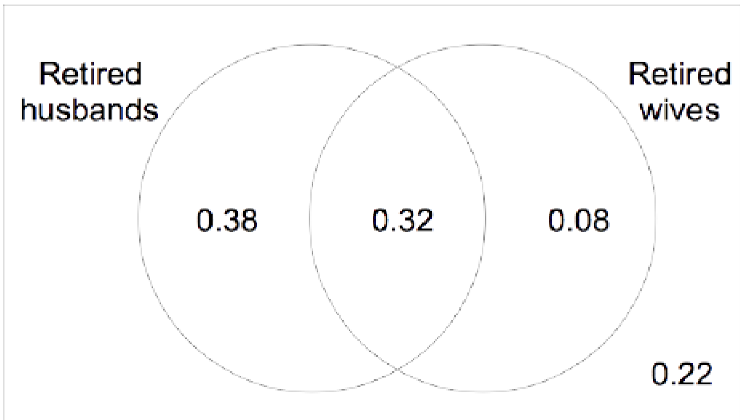
1 The probability that a wife is retired is 0.4.

Given that she is retired, the probability that her husband is also retired is 0.8.

Hence the probability that both are retired is $0.4 \times 0.8 = 0.32$.

The probability that a husband is retired is 0.7.

From this data you can deduce the following Venn diagram of the probabilities:



Let H = husband retired, H′ = husband not retired, W = wife retired, W′ = wife not retired.

The permutations where only one husband and only one wife is retired are:

Couple 1	Probability	Couple 2	Probability	Combined probability
H W′	0.38	H′ W	0.08	0.38×0.08
H′ W	0.08	H W′	0.38	0.08×0.38
H W	0.32	H′ W′	0.22	0.32×0.22
H′ W′	0.22	H W	0.32	0.22×0.32

P(only one husband and only one wife is retired) $= (0.38 \times 0.08 + 0.32 \times 0.22) \times 2 = 0.2016$

Challenge

2 a Let $P(A \cap B) = k$

As $P(A \cap B) \leqslant P(B) \Rightarrow k \leqslant 0.2$

A and B could be mutually exclusive, meaning $P(A \cap B) = 0$, so $0 \leqslant k \leqslant 2$

Now, $P(A \cap B') = P(A) - P(A \cap B)$, so $p = 0.6 - k \Rightarrow 0.4 \leqslant p \leqslant 0.6$

b Use the fact that $P(A \cap C) = P(A \cap B \cap C) + P(A \cap B' \cap C)$

So $P(A \cap B' \cap C) = P(A \cap C) - P(A \cap B \cap C) = P(A \cap C) - 0.1$

Consider the range of $P(A \cap C)$

$P(A \cap C) \leqslant P(A) \Rightarrow P(A \cap C) \leqslant 0.6$

By the multiplication formula $P(A \cup C) = P(A) + P(C) - P(A \cap C)$

So $P(A \cap C) = P(A) + P(C) - P(A \cup C) = 1.3 - P(A \cup C)$

As $P(A \cup C) \leqslant 1 \Rightarrow P(A \cap C) \geqslant 0.3$

So $0.3 \leqslant P(A \cap C) \leqslant 0.6$ and as $P(A \cap B' \cap C) = P(A \cap C) - 0.1$ this gives the result that $0.3 - 0.1 \leqslant P(A \cap B' \cap C) \leqslant 0.6 - 0.1$, so $0.2 \leqslant q \leqslant 0.5$

3 a $P(X = x) = kx,\ x = 1, 2, 3, 4, 5$

x	1	2	3	4	5
$P(X = x)$	k	$2k$	$3k$	$4k$	$5k$

The sum of the probabilities is 1, therefore, $15k = 1$ so $k = \frac{1}{15}$

b $$P(X = 5 \mid X > 2) = \frac{5k}{12k} = \frac{5}{12}$$

c $$P(X \text{ is odd} \mid X \text{ is prime}) = \frac{P(\text{odd} \cap \text{prime})}{P(\text{prime})} = \frac{\frac{8}{15}}{\frac{10}{15}} = \frac{8}{10} = \frac{4}{5}$$