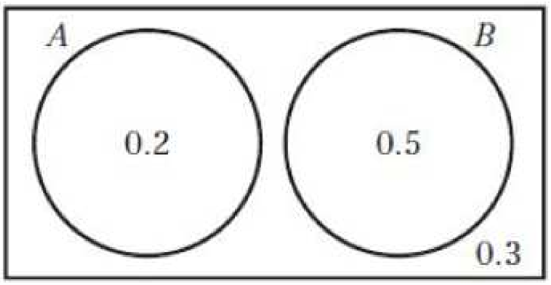
Exercise 4B

1 a



b $P(A \cup B) = 0.7$

c $P(A' \cap B') = 0.3$

2 P(Sum of 4) = $\frac{3}{36} = \frac{1}{12}$

P(Same number) = $\frac{6}{36} = \frac{1}{6}$

P(Sum of 4) + P(Same number) = $\frac{1}{12} + \frac{1}{6} = \frac{1}{4}$

P(Sum of 4 or same number) = $\frac{8}{36} = \frac{2}{9}$

P(Sum of 4) + P(Same number) ≠ P(Sum of 4 or same number),
So the events are not mutually exclusive.

Alternatively: A roll of 2 followed by another roll of 2 fits both conditions, so the intersection is not empty, and the events are not mutually exclusive.

3 $P(A \text{ and } B) = P(A) \times P(B) = 0.5 \times 0.3 = 0.15$

4 $P(A \text{ and } B) = P(A) \times P(B)$

$P(B) = P(A \text{ and } B) \div P(A) = 0.045 \div 0.15 = 0.3$

5 a The closed curves representing bricks and trains do not overlap and so they are mutually exclusive.

b $P(B \text{ and } F) = \dfrac{1}{3+1+4+6+2+5} = \dfrac{1}{21}$

$P(B) \times P(F) = \dfrac{3+1}{21} \times \dfrac{1+4+6}{21} = \dfrac{4}{21} \times \dfrac{11}{21} = \dfrac{44}{441}$

As $P(B \text{ and } F) \neq P(B) \times P(F)$, 'plays with bricks' and 'plays with action figures' are not independent events.

6 a $0.4 + x + 0.3 + 0.05 = 1$

$x = 0.25$

6 b $P(A \text{ and } B) = x = 0.25$

$P(A) \times P(B) = 0.65 \times 0.55 = 0.3575$

As $P(A \text{ and } B) \neq P(A) \times P(B)$, the two events 'like pasta' and 'like pizza' are not independent.

7 a $P(S \text{ and } T) = P(S) - P(S \text{ but not } T) = 0.3 - 0.18 = 0.12$

$P(S) \times P(T) = 0.3 \times 0.4 = 0.12$

As $P(S \text{ and } T) = P(S) \times P(T)$, S and T are independent events.

b i $P(S \text{ and } T) = 0.12$, as above.

ii $P(\text{neither } S \text{ nor } T) = 1 - P(S \text{ or } T) = 1 - (P(S \text{ but not } T) + P(T)) = 1 - (0.18 + 0.4) = 0.42$

8 $P(W \text{ and } X) = P(W) - P(W \text{ and not } X) = 0.5 - 0.25 = 0.25$

$P(X) = 1 - (P(W \text{ and not } X) + P(\text{neither } W \text{ nor } X)) = 1 - (0.25 + 0.3) = 0.45$

$P(W) \times P(X) = 0.5 \times 0.45 = 0.225$

As $P(W \text{ and } X) \neq P(W) \times P(X)$, the two events W and X are not independent.

9 a $P(A \text{ or } R) = P(A) + P(R) = 0.6$ because A and R are mutually exclusive.

$0.2 + 0.25 + x = 0.6$, so $x = 0.15$

$y = 1 - (0.2 + 0.25 + 0.15 + 0.1) = 0.3$

$(x, y) = (0.15, 0.3)$

b $P(R \text{ and } F) = x = 0.15$

$P(R) \times P(F) = 0.4 \times 0.45 = 0.18$

As $P(R \text{ and } F) \neq P(R) \times P(F)$, the two events R and F are not independent.

10 P(A and B) = p

P(A) × P(B) = (0.42 + p) × (p + 0.11)

= (p + 0.42)(p + 0.11)

As the events A and B are independent, P(A and B) = P(A) × P(B), so

$(p + 0.42)(p + 0.11) = p$

$p^2 + 0.53p + 0.0462 = p$

$p^2 - 0.47p + 0.0462 = 0$, a quadratic in p, which we can solve with the quadratic formula

$$p = \frac{0.47 \pm \sqrt{(-0.47)^2 - 4(1)(0.0462)}}{2(1)}$$

$$p = \frac{0.47 \pm 0.19}{2}$$

$p = 0.33$ or 0.14

When $p = 0.14$, $q = 1 - (0.42 + 0.14 + 0.11) = 0.33$

When $p = 0.33$, $q = 1 - (0.42 + 0.33 + 0.11) = 0.14$

$(p, q) = (0.14, 0.33)$ or $(0.33, 0.14)$

Challenge

a Set P(A) = p and P(B) = q

As A and B are independent events, P(A and B) = P(A) × P(B) = pq

P(A and not B) = P(A) − P(A and B) = $p - pq$, and notice P(not B) = 1 − P(B) = $1 - q$

Then P(A) × P(not B) = p($1 - q$) = $p - pq$ = P(A and not B)

As P(A and not B) = P(A) × P(not B), the events A and 'not B' are independent.

b Still using p and q as above,

P(not A and not B) = 1 − P(A or B)

P(A or B) = P(A) + P(B) − P(A and B)

Meaning P(not A and not B) = 1 − P(A) − P(B) + P(A and B) = $1 - p - q + pq = (1 - p)(1 - q)$

Remember P(not A) = $1 - p$ and P(not B) = $1 - q$

So P(not A) × P(not B) = $(1 - p)(1 - q)$ = P(not A and not B)

As P(not A and not B) = P(not A) × P(not B), the events 'not A' and 'not B' are independent.