Exercise 4A

1

	Coin 1	
Coin 2	**H**	**T**
H	HH	TH
T	HT	TT

P(same) = $\frac{2}{4} = \frac{1}{2}$

2 a

First roll \ Second roll	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

b i $\mathrm{P}(X = 24) = \frac{2}{36} = \frac{1}{18}$

ii $\mathrm{P}(X < 5) = \frac{8}{36} = \frac{2}{9}$

iii $\mathrm{P}(X \text{ is even}) = \frac{27}{36} = \frac{3}{4}$

3 a $\mathrm{P}(m \geqslant 54) = \frac{33+21+2}{140} = \frac{56}{140} = \frac{2}{5}$

b $\mathrm{P}(48 \leqslant m < 57) = \frac{25+42+33}{140} = \frac{100}{140} = \frac{5}{7}$

3 c Let B = the number of Bullmastiffs with mass less than 53 kg.

Using interpolation:

$$\frac{53-51}{54-51}=\frac{B-(17+25)}{42}$$

So $B = 70$

$P(m < 53) = \frac{70}{140} = 0.5$, so half of the Bullmastiffs are estimated to have a mass less than 53 kg.

This probability is lower than the probability of 0.54 for Rottweilers, and so it is less likely.

The assumption made is that the frequency is uniformly distributed throughout the class.

4 a $\text{P(female)} = \frac{14+15+32+27+26}{240}=\frac{114}{240}=\frac{19}{40}$

b $P(l < 80) = \frac{4+14+20+15+24+32}{240}=\frac{109}{240}$

c $P(\text{male and } 75 \leqslant l < 85) = \frac{24+47}{240}=\frac{71}{240}$

d Using interpolation for males:

The number of male juvenile koalas is approximately $4 + \frac{72-70}{75-70} \times 20 = 4 + 8 = 12$.

The number of female juvenile koalas is approximately $14 + \frac{72-70}{75-70} \times 15 = 14 + 6 = 20$.

So $\text{P(juvenile)} = \frac{12+20}{240}=\frac{32}{240}=\frac{2}{15}$

The assumption is that the distribution of lengths of koalas between 70 and 75 cm is uniform.

5 a $P(m > 5) = \frac{(1\times 24)+(2\times 4)}{70}=\frac{32}{70}=\frac{16}{35}$

b Start with the probability that the cat has a mass *greater* than 6.5.

$$P(m > 6.5) = \frac{\frac{3}{4}\times(2\times 4)}{70}=\frac{6}{70}=\frac{3}{35}$$

So $P(m < 6.5) = 1 - \frac{3}{35}=\frac{32}{35}$

The fact that we have ignored the case 6.5 is not a problem in this estimate. We are assuming that the class is continuous when we interpolate, and that the probability of being exactly equal to any individual value is negligible.

Challenge

	A			
B	×	**2**	**7**	**5**
	4	8	28	20
	$\boldsymbol{x}$	$2x$	$7x$	$5x$

If x is even, all the products are even, so P(Y is even) = 1

But P($Y \geqslant 20$) = 1 is impossible, as the product of 2 and 4 is only 8, so x cannot be even.

If x is odd, there are four even values of Y: 8, 28, 20 and $2x$.

But P(Y is even) = P($Y \geqslant 20$), so there must also be four values where $Y \geqslant 20$.

Two of them are in the top row: 28 and 20, leaving two in the bottom row.

Given that exactly two of these three values are greater than or equal to 20:

$2x < 20$ and $5x \geqslant 20$, i.e. $x < 10$ and $x \geqslant 4$.

Hence $4 \leqslant x < 10$ and x is odd so the possible values of x are 5, 7 and 9.