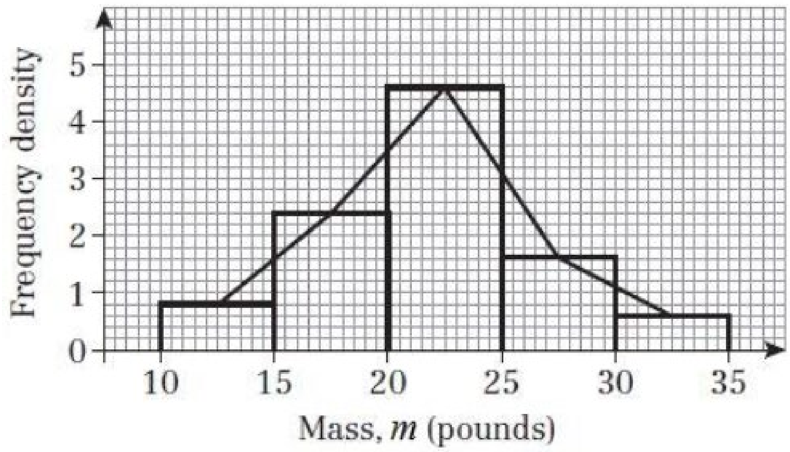
Exercise 3A

1 a, b Class widths are all 5.

Frequency densities are: 0.8, 2.4, 4.6, 1.6, 0.6



2 a Time is a continuous variable.

b Area of 60 to 70 seconds bar is 10 × 6 × 60 square units.

1 square unit = $\frac{90}{60}$ = 1.5 students

The area of the 40 to 60 seconds bar is 20 × 5 × 100 square units.

So there were 100 × 1.5 × 150 students who took between 40 and 60 seconds.

c Area for 80 seconds or less = 20 × 5 + 10 × 6 + 10 × 8.6 = 246 square units.

So there were 246 × 1.5 = 369 students who took 80 seconds or less.

d Total area = 246 + 5 × 14 + 5 × 12 + 30 × 3 = 466 square units.

So 466 × 1.5 = 699 students took part in the race in total.

3 a Distance is a continuous variable.

b Area for less than 20 m is 20 × 2 = 40 square units.

1 square unit = $\frac{80}{40}$ = 2 people

Total area = 20 × 2 + 15 × 5 + 10 × 10 + 15 × 6 + 5 × 1 = 310 square units.

So 310 × 2 = 620 people entered the competition.

c Area for 30 to 40 m is 5 × 5 + 5 × 10 = 75 square units.

So 75 × 2 = 150 people threw between 30 and 40 metres.

3 d Area for 45 to 65 m is $15 \times 6 + 5 \times 1 = 95$ square units.

So $95 \times 2 = 190$ people threw between 45 and 65 metres.

e Area for less than 25 m is $10 \times 2 + 10 \times 2 + 5 \times 5 = 65$ square units.

So $65 \times 2 = 130$ people threw less than 25 metres.

4 a The bar for $28 \leqslant m < 32$ has an area of $10 \times 10 = 100$ squares.
If 100 squares represents 32 lambs then $\frac{100}{4}$ squares represents $\frac{32}{4}$ lambs.
i.e. 25 squares represents 8 lambs.

b The class $24 \leqslant m < 26$ contains $5 \times 20 = 100$ squares.
As above, this represents 32 lambs.

c The class $20 \leqslant m < 24$ contains $10 \times 10 = 100$ squares which represents 32 lambs.

The class $24 \leqslant m < 26$ contains $5 \times 20 = 100$ squares which represents 32 lambs.

The class $26 \leqslant m < 28$ contains $5 \times 40 = 200$ squares which represents 64 lambs.

The class $28 \leqslant m < 32$ contains $10 \times 10 = 100$ squares which represents 32 lambs.

The class $32 \leqslant m < 34$ contains $5 \times 5 = 25$ squares which represents 8 lambs.

So in total we have $32 + 32 + 64 + 32 + 8 = 168$ lambs.

d Class $25 \leqslant m < 26$ is approximately $\frac{1}{2}$ of class $24 \leqslant m < 26$ which equates to 16 lambs.

Class $26 \leqslant m < 28$ represents 64 lambs.

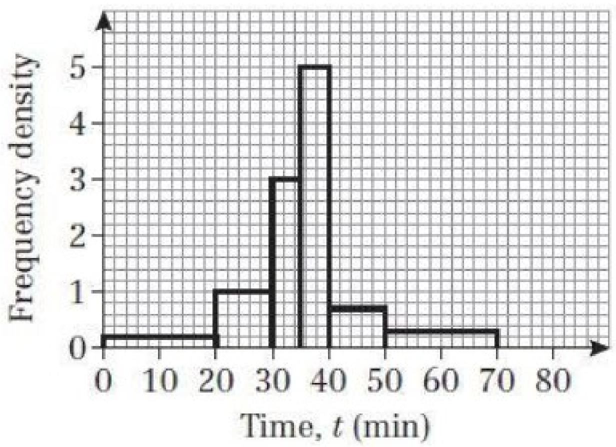
Class $28 \leqslant m < 29$ is approximately $\frac{1}{4}$ of class $28 \leqslant m < 32$ which equates to 8 lambs.

So in total we have $16 + 64 + 8 = 88$ lambs.

5 a i Use extra columns to help, using the frequency densities given in the histogram:

Time, t (min)	Frequency	Class width	Frequency density
$0 \leqslant t < 20$	4	20	0.2
$20 \leqslant t < 30$	$10 \times 1 = 10$	10	1
$30 \leqslant t < 35$	15	5	3
$35 \leqslant t < 40$	25	5	5
$40 \leqslant t < 50$	$10 \times 0.7 = 7$	10	0.7
$50 \leqslant t < 70$	$20 \times 0.3 = 6$	20	0.3

ii



b $\left(\frac{5}{10} \times 10\right) + 15 + \left(\frac{3}{5} \times 25\right) = 35$ passengers.

6 a 12.5 and 14.5 are the class boundaries, as we are dealing with continuous data.

b i The class boundaries for the 15–17 class are 14.5 and 17.5.
This width is 1.5 times the width of the 13–14 class, since $17.5 - 14.5 = 3 = 1.5 \times 2$.
So the width of the class is $1.5 \times 4 = 6$ cm.

ii The frequency density for the 13–14 class is $\frac{24}{2} = 12$.

The frequency density of this class is 6, which is 0.5 times the frequency density above: 12.
So the height of the class is $0.5 \times 6 = 3$ cm.

7 a The $10 \leqslant w < 11$ interval is half the width of the $8 \leqslant w < 10$ interval therefore it should be 0.5 cm wide.
The $8 \leqslant w < 10$ interval has a frequency of 8 and an area of 16, so the $10 \leqslant w < 11$ interval should be 12 cm high.

7 b $\bar{x} = \dfrac{\sum fx}{\sum f}$

$= \dfrac{4\times6+8\times9+6\times10.5+7\times11.5+5\times13.5+1\times15.5}{31}$

$= 10.403...$

$= €10.40$

$\sigma^2 = \dfrac{\sum fx^2}{\sum f} - \left(\dfrac{\sum fx}{\sum f}\right)^2$

$= \dfrac{\sum 4\times6^2+8\times9^2+6\times10.5^2+7\times11.5^2+5\times13.5^2+1\times15.5^2}{31} - (10.403...)^2$

$= 5.668...$

$\sigma = 2.380...$

$= €2.38$

c $Q_1 = \dfrac{31}{4} = 7.75$ therefore $Q_1 = 8$

$\dfrac{Q_1 - 8}{10-8} = \dfrac{8-4}{12-4}$

$Q_1 = €9$

d $\bar{x} + \sigma = 10.403... + 2.380...$

$= 12.783...$

$= 12.78$

12.78 lies in the interval $12 \leqslant w < 15$

$\dfrac{12.78-12}{15-12} = \dfrac{y-25}{30-25}$

$y = 26.3$

$31 - 26.3 = 4.7$

Therefore 5 employees (to the nearest whole number) earn an hourly wage higher than the mean plus one standard deviation.

Challenge

Length (cm)	5–10	10–20	20–25	25–30	30–40	40–60	60–90
Frequency	8	16	20	18	20	14	12
Frequency density	1.6	1.6	4.0	3.6	2.0	0.7	0.4

The ratio of the shortest and longest bars is 0.4 : 4 = 1 : 10