Chapter review 2

1 $(8 \times 65) + (12 \times 72) = 1384$

$\frac{1384}{20} = 69.2$ marks

2 a 10, 12, 9, 2, 2.5, 9.5

b Coded mean $= \frac{45}{6} = 7.5$

c Mean $= 7.5 \times 80 + 7 = 607$

3 a Group A:

$(1 \times 24.5) + (3 \times 34.5) + (6 \times 44.5) + (6 \times 54.5) + (11 \times 64.5) + (10 \times 74.5) + (8 \times 84.5)$

Mean $= \frac{2852.5}{45} = 63.39$ marks

Group B:

$(1 \times 24.5) + (2 \times 34.5) + (4 \times 44.5) + (13 \times 54.5) + (15 \times 64.5) + (6 \times 74.5) + (3 \times 84.5)$

Mean $= \frac{2648}{44} = 60.18$ marks

b The method used to teach group A is best as the mean mark is higher.

4 a Modal class is 21–25 hours

b We need the 40th value. This is in the 21–25 class.

$$\frac{m - 20.5}{25.5 - 20.5} = \frac{40 - 30}{75 - 30}$$

$45m = 50 + 922.5 = 972.5$
Median = 21.6 hours

c Mean = 20.6 hours

d $12 \times 22.3 = 267.6$ hours

Total hours for all 92 batteries is $267.6 + 1645 = 1912.6$
Mean life for 92 batteries is 20.8 hours.

5 CF = 5 15 30 42 50

Data is continuous, so:

Q_1 : $\frac{50}{4}$ = 12.5th value, so Q_1 is in class 21–40

$$\frac{Q_1 - 20.5}{40.5 - 20.5} = \frac{12.5 - 5}{15 - 5}$$

$$\frac{Q_1 - 20.5}{20} = \frac{7.5}{10}$$

$Q_1 = 35.5$

Q_3 : $\frac{3 \times 50}{4}$ = 37.5th value, so Q_3 is in class 61–80

$$\frac{Q_3 - 60.5}{80.5 - 60.5} = \frac{37.5 - 30}{42 - 30}$$

$Q_3 = 73$

IQR = 73 − 35.5 = 37.5

6 a 30th: $\frac{30}{100} \times 100 = 30$

$P_{30} = 20.5$

b 70th: $\frac{70}{100} \times 100 = 70$

$$\frac{P_{70} - 30.5}{40.5 - 30.5} = \frac{70 - 60}{84 - 60}$$

$P_{66} = 34.7$

c 30% to 70% interpercentile = 34.7 − 20.5 = 14.2

7 a Q_1: $\frac{80}{4} = 20$th value, so Q_1 is in class 40–49

$$\frac{Q_1 - 39.5}{49.5 - 39.5} = \frac{20 - 15}{51 - 15}$$

$$\frac{Q_1 - 39.5}{10} = \frac{5}{36}$$

$Q_1 = 40.9$

Q_3: $\frac{3 \times 80}{4} = 60$th value, so Q_3 is in class 50–59

$$\frac{Q_3 - 49.5}{59.5 - 49.5} = \frac{60 - 51}{71 - 51}$$

$Q_3 = 54$

IQR = 54 − 40.9 = 13.1

b Variance = $\frac{183\,040}{80} - \left(\frac{3740}{80}\right)^2 = 102.4375 = 102$

Standard deviation = $\sqrt{102.4375} = 10.1$

8 CF = 5 15 41 49 50

a Data is continuous, so:

Q_1: $\frac{50}{4} = 12.5$th value, so Q_1 is in class 95–100

$$\frac{Q_1 - 95}{100 - 95} = \frac{12.5 - 5}{15 - 5}$$

$Q_1 = 98.75$

b Q_3: $3 \times \frac{50}{4} = 37.5$th value, so Q_3 is in class 100–105

$$\frac{Q_3 - 100}{105 - 100} = \frac{37.5 - 15}{41 - 15}$$

$Q_3 = 104.33$

IQR = 104.33 − 98.75 = 5.58

c Standard deviation = $\sqrt{\frac{516\,112.5}{50} - \left(\frac{5075}{50}\right)^2} = 4.47$

9 a $\bar{x} = \dfrac{\sum fx}{\sum f}$

$= \dfrac{12\times 26 + 14\times 28 + 4\times 30}{30}$

$= 27.4667...$

$= 27.5°C$

$\sigma^2 = \dfrac{\sum fx^2}{\sum f} - \left(\dfrac{\sum fx}{\sum f}\right)^2$

$= \dfrac{12\times 26^2 + 14\times 28^2 + 4\times 30^2}{30} - (27.4667...)^2$

$= 1.8489...$

$\sigma = 1.3597...$

$= 1.36°C$

b 10th percentile = 3rd data value

$\dfrac{P_{10} - 25}{27 - 25} = \dfrac{3 - 0}{12 - 0}$

$P_{10} = 25.5°C$

90th percentile = 27th data value

$\dfrac{P_{90} - 29}{31 - 29} = \dfrac{27 - 26}{30 - 26}$

$P_{90} = 29.5°C$

The 10th – 90th interpercentile range is 29.5 – 25.5 = 4.0°C

c $\bar{x} + \sigma = 27.464... + 1.412$

$= 28.876...$

$= 28.9$

28.9 lies in the interval $27 \leq t < 29$

$\dfrac{28.9 - 27}{29 - 27} = \dfrac{y - 12}{26 - 12}$

$y = 25.3$

30 – 25.3 = 4.7

Therefore 5 days (to the nearest day)

10 a Coded mean $= \frac{106}{31} = 3.419\ldots$

Coded standard deviation $= \sqrt{\frac{80.55}{31}} = 1.6119\ldots$

b Mean $= 3.419\ldots \times 2 + 3 = 9.84$ kn

Standard deviation $= 1.6119 \times 2 = 3.22$ kn

11 a Mean $= \frac{316}{20} = 15.8$

Standard deviation $= \sqrt{\frac{5078}{20} - \left(\frac{316}{20}\right)^2} = 2.06$

b It will decrease the mean wing span since $13 < 15.8$

c Coded mean $= \frac{104}{20} = 5.2$

Mean $= 5.2 \times 10 + 5 = 57$ cm

Coded standard deviation $= \sqrt{\frac{1.8}{20}} = 0.3$

Standard deviation $= 0.3 \times 10 = 3$

Challenge

Total $= 3.1 \times 20 = 62$

New total $= 62 - 2.3 + 3.2 = 62.9$

New mean $= \frac{62.9}{20} = 3.145$

$\sigma = 1.4$, $\sigma^2 = 1.96$

$1.96 = \frac{\sum x^2}{20} - \left(\frac{62}{20}\right)^2$

$\sum x^2 = 231.4$

New $\sum x^2 = 231.4 - 2.3^2 + 3.2^2 = 236.35$

New standard deviation $= \sqrt{\frac{236.35}{20} - \left(\frac{62.9}{20}\right)^2} = 1.39$