Question			Answer	Marks	Guidance	
1	(i)		P(All blue) $= \frac{30}{50} \times \frac{29}{49} \times \frac{28}{48} = 0.2071$	M1	For $\frac{30}{50} \times$ (as part of a triple product)	$(30/50)^3 = 0.216$ scores M1M0A0 $\frac{k}{50} \times \frac{(k-1)}{49} \times \frac{(k-2)}{48}$ for values of k other than 30 scores M1M0A0 Zero for binomial unless simplifies to $(3/5)^3$
				M1	For product of other two fractions	Correct working but then multiplied or divided by some factor scores M1M0A0
			OR $\binom{30}{3} / \binom{50}{3} = 4060/19600 = 29/140 = 0.2071$ M2 for the complete method	A1 **[3]**	CAO SC2 for P(All red) = 0.0582	Accept 0.21 with working and 0.207 without working Allow unsimplified fraction as final answer 24360/117600 oe
1	(ii)		P(All red) $= \frac{20}{50} \times \frac{19}{49} \times \frac{18}{48} = 0.0582$ or $\binom{20}{3} / \binom{50}{3} = 0.0582$	M1	For P(All red)	SC2 for $1 - (30/50)^3 - (20/50)^3 = 1 - 0.216 - 0.064 = 0.72$, providing consistent with (i) . If not consistent with (i) M0M0A0
			P(At least one of each colour) $= 1 - (0.2071 + 0.0582) = 0.7347$ or $1 - \left(\frac{29}{140} + \frac{57}{980}\right) = 1 - \frac{260}{980} = 1 - \frac{13}{49} = \frac{36}{49}$	M1	For $1 - (0.2071 + 0.0582)$	
				A1 **[3]**	CAO	Allow 0.73 with working Allow unsimplified fraction as final answer 86400/117600 oe
			OR P(2b,1r)+P(1b,2r)	(M1)	For either $\frac{30}{50} \times \frac{29}{49} \times \frac{20}{48}$ or $\frac{20}{50} \times \frac{19}{49} \times \frac{30}{48}$	Allow M1 for $3 \times (30/50)^2 \times (20/50)$ or $3 \times (30/50) \times (20/50)^2$ and second M1 for sum of both if = 0.72 If not consistent with (i) M0M0A0

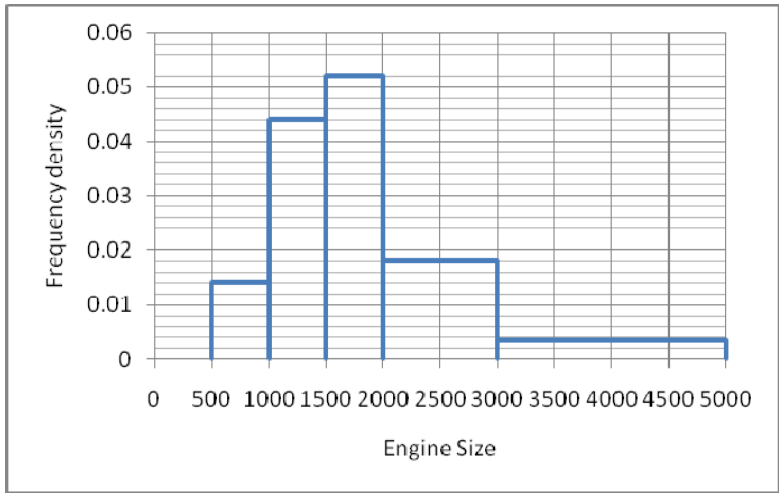
Question			Answer	Marks	Guidance	
			$= 3\times\frac{30}{50}\times\frac{29}{49}\times\frac{20}{48} + 3\times\frac{20}{50}\times\frac{19}{49}\times\frac{30}{48}$	(M1)	For sum of both or for 3× either	NB M2 also for $\frac{30}{50}\times\frac{20}{49}\left(\times\frac{48}{48}\right)$ even if not multiplied by 3
			= 3 × 0.1480 + 3 × 0.0969 = 0.7347	(A1)	CAO	Allow 0.73 or better with working
			OR Either $\binom{30}{2}\times\binom{20}{1}/\binom{50}{3}$ or $\binom{30}{1}\times\binom{20}{2}/\binom{50}{3}$	(M1)		
				(M1) (A1)	For sum of both CAO	
2	**(i)**		$^9C_3 \times {}^5C_3 = 84 \times 10 = 840$	M1 M1 A1 **[3]**	For either $^9C_3$ or $^5C_3$ For product of both correct combinations CAO	Zero for permutations
2	**(ii)**		Total number of ways of answering 6 from 14 is $^{14}C_6 = 3003$ Probability $= \frac{840}{3003} = \frac{40}{143} = 0.27972 = 0.280$	M1 M1 A1 **[3]**	For $^{14}C_6$ seen in part (ii) For their 840/ 3003 or their 840/$^{14}C_6$ FT their 840	Allow full marks for unsimplified fractional answers
			OR $^6C_3 \times 5/14 \times 4/13 \times 3/12 \times 9/11 \times 8/10 \times 7/9 = 0.280$	(M1)	For product of fractions	SC1 for $^6C_3 \times (5/14)^3 \times (9/14)^3 = 0.2420$
				(M1) (A1)	For $^6C_3\times$ correct product	

Question			Answer	Marks	Guidance	
3	(i)		X ~ B(30, 0.85) $P(X = 29) = \binom{30}{29} \times 0.85^{29} \times 0.15^{1} = 30 \times 0.0013466 = 0.0404$	M1 M1 A1 **[3]**	For $0.85^{29} \times 0.15^{1} = 0.0013466$ For $\binom{30}{29} \times p^{29} \times q^{1}$ CAO	With $p + q = 1$ Allow 0.04 www If further working (EG P(X=29) –P(X=28)) give M2A0
3	(ii)		$P(X = 30) = 0.85^{30} = 0.0076$ $P(X \geq 29) = 0.0404 + 0.0076 = 0.0480$	M1 M1 A1 **[3]**	For 0.85^{30} For P(X = 29) + P(X = 30) (not necessarily correct, but both attempts at binomial, including coefficient in (i)) CAO	Allow eg 0.04+0.0076=0.0476 Allow 0.05 with working
3	(iii)		Expected number = 10 × 0.0480 = 0.480	M1 A1 **[2]**	For 10 × their **(ii)** FT their **(ii)** but if answer to (ii) leads to a whole number for (iii) give M1A0	**provided (ii) between 0 and 1** Do not allow answer rounded to 0 or 1.

Question			Answer	Marks	Guidance	
4	(i)	(*A*)	P(third selected) = $0.92^2 \times 0.08 = 0.0677$ Or = 1058/15625	M1 M1 A1 **[3]**	For 0.92^2 For $p^2 \times q$ CAO SC1 for 'without replacement' method 92/100×91/99×8/98 =0.0690	With $p + q = 1$ With no extra terms Allow 0.068 but not 0.067 nor 0.07
4	(i)	(*B*)	P (second) + P(third) = (0.92 × 0.08) + ($0.92^2 \times 0.08$) = 0.0736 + 0.0677 = 0.1413 = 2208/15625	M1 A1 **[2]**	For 0.92 × 0.08 FT their 0.0677 SC1 for answer of 0.143 from 'without replacement' method	With no extra terms Allow 0.141 to 0.142 and allow 0.14 with working
4	(ii)		P(At least one of first 20) = 1 - P(None of first 20) = $1 - 0.92^{20} = 1 - 0.1887 = 0.8113$	M1 M1 A1 **[3]**	0.92^{20} $1 - 0.92^{20}$ CAO	Accept answer of 0.81 or better from P(1) + P(2) + ... , or SC2 if all correct working shown but wrong answer No marks for 'without replacement' method' Allow 0.81 with working but not 0.812

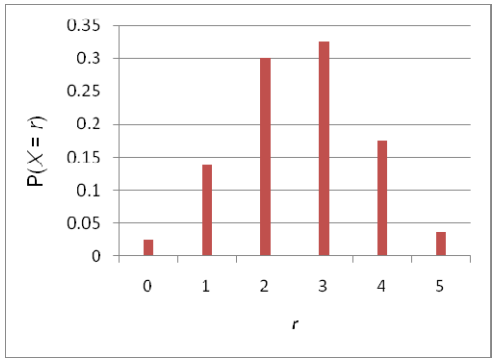
Question			Answer	Marks	Guidance	
5			Let p = probability that a randomly selected frame is faulty	B1	For definition of p in context Minimum needed for B1 is p = probability that frame/bike is faulty. Do not allow is p = probability that it is faulty Allow p = P(frame faulty) Definition of p must include word probability (or chance or proportion or percentage or likelihood but NOT possibility). Preferably as a separate comment. However can be at end of H_0 as long as it is a clear definition ' p = the probability that frame is faulty, NOT just a sentence 'probability is 0.05' Do NOT allow 'p = the probability that faulty frames have increased'	
			H_0: $p = 0.05$	B1	H_0 : p(frame faulty) = 0.05, H_1 : p(frame faulty) > 0.05 gets B0B1B1 Allow p=5%, allow θ or π and ρ but not x. However allow any single symbol <u>if defined</u> Allow $H_0 = p$=0.05, Allow $H_0 : p = {}^1/_{20}$ Do not allow $H_0 : P(X=x) = 0.05$, $H_1 : P(X=x) > 0.05$ Do not allow H_0: =0.05, =5%, P(0.05), p(0052), p(x)=0.05, x=0.05 (unless x correctly defined as a probability) Do not allow H_1:$p \geq 0.05$, Do not allow H_0 and H_1 reversed Allow NH and AH in place of H_0 and H_1 For hypotheses given in words allow Maximum B0B1B1 Hypotheses in words must include probability (or chance or proportion or percentage) and the figure 0.05 oe.	
			H_1: $p > 0.05$	B1		
			P($X \geq 4$)	B1	For notation P($X \geq 4$) or 1- P($X \leq 3$) This mark may be implied by 0.0109 as long as no incorrect notation.	No further marks if point probs used - P($X = 4$) = 0.0094 DO NOT FT wrong H_1 But if H_1 is $p \geq 0.05$ allow the rest of the marks if earned so max 7/8
			= 1- P($X \leq 3$) = 1 – 0.9891 = 0.0109	B1*	For 0.0109, indep of previous mark	Or for 1 – 0.9891

Question	Answer	Marks	Guidance	
	0.0109 < 0.05	M1* dep	For comparison with 5%	
	So reject H_0	A1*	or significant or 'accept H_1'	
	There is evidence to suggest that the proportion of faulty frames has increased.	E1* Dep on A1	Must include 'sufficient evidence' or something similar such as 'to suggest that' ie an element of doubt for E1. 'Sufficient evidence' or similar can be seen in the either the A mark or the E mark.	
		[8]		
	OR *Critical region method:*			No marks if CR not justified
	Let $X \sim B(18, 0.05)$			Do not insist on correct notation
	$P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.9419 = 0.0581 > 5\%$	(B1)	For 0.0581	as candidates have to work out two probabilities for full marks
	$P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.9891 = 0.0109 < 5\%$	(B1)	For 0.0109	
		(M1)	For at least one correct comparison with 5%	
	So critical region is {4,5,6,7,8,9,10,11,12,13,14,15,16,17,18}	(A1)	CAO for critical region and significant oe	Condone {4,5 ..}, $X \geq 4$, oe but not $P(X \geq 4)$
	4 lies in the critical region, so significant,			
	There is evidence to suggest that the proportion of faulty frames has increased.	(E1)		

Question	Answer	Marks	Guidance
6 (i)	<table><tr><th>Engine size</th><th>Frequency</th><th>Group width</th><th>Frequency density</th></tr><tr><td>$500 \le x \le 1000$</td><td>7</td><td>500</td><td>0.014</td></tr><tr><td>$1000 < x \le 1500$</td><td>22</td><td>500</td><td>0.044</td></tr><tr><td>$1500 < x \le 2000$</td><td>26</td><td>500</td><td>0.052</td></tr><tr><td>$2000 < x \le 3000$</td><td>18</td><td>1000</td><td>0.018</td></tr><tr><td>$3000 < x \le 5000$</td><td>7</td><td>2000</td><td>0.0035</td></tr></table>	M1	At least 4 fds correct for M1 M1 can be also be gained from freq per 1000 – 14, 44, 52, 18, 3.5 (at least 4 correct) and A1 for all correct or freq per 500 - 7, 22, 26, 9, 1.75 Accept any suitable unit for fd, eg freq per 1000, BUT NOT FD per 1000 Allow fds correct to at least three dp If fd not explicitly given, M1 A1 can be gained from all heights correct (within one square) on histogram (and M1A0 if at least 4 correct) Allow restart with correct heights if given fd wrong
		A1 G1(L1)	For fd's all correct linear scales on both axes and label on vertical axis Label required on vert axis **IN RELATION** to first M1 mark ie fd or frequency density or if relevant freq/1000, etc (NOT fd/1000, but allow fd×1000, etc) Accept f/w or f/cw (freq/width or freq/class width) Ignore horizontal label and allow horizontal scale to start at 500 Can also be gained from an accurate key
	<u>INCORRECT DIAGRAMS:</u> Frequency diagrams can get M0, A0, G0, G1, G0 MAXIMUM Thus frequency density = frequency × width, frequency/midpoint etc gets MAX M0A0G0G1G0 **Frequency polygons MAX M1A1G0G0G0**	G1(W1)	Width of bars Must be drawn at 500, 1000etc NOT 499.5 or 500.5 etc NO GAPS ALLOWED Must have linear scale. No inequality labels on their own such as 500≤S<1000, etc but allow if a clear horizontal linear scale is also given.

Question			Answer	Marks	Guidance
				G1(H1) [5]	Height of bars FT of heights *dep* on at least 3 heights correct and all must agree with their fds If fds not given and one height is wrong then max M1A0G1G1G0 – visual check only (within one square) –no need to measure precisely
6	(ii)		Do not know exact highest and lowest values so cannot tell what the midrange is. **OR** No and a counterexample to show it may not be 2750 **OR** (500 + 5000) / 2 = 2750. But very unlikely to be absolutely correct but probably close to the true value. Some element of doubt needed. Allow 'Likely to be correct'	E1 [1]	Allow comment such as 'Highest value could be 5000 and lowest could be 500 therefore midrange could be 2750' NO mark if incorrect calculation Sight of 1750 AND 3000 (min and max of midrange) scores E1
6	(iii)		Mean = $\frac{(750\times 7)+(1250\times 22)+(1750\times 26)+(2500\times 18)+(4000\times 7)}{80}$ $=\frac{151250}{80}=1891$ $\Sigma x^2 f=(750^2\times 7)+(1250^2\times 22)+(1750^2\times 26)+(2500^2\times 18)+(4000^2\times 7)$ = 3937500 + 34375000 + 79625000 + 112500000 + 112000000 =342437500 $Sxx=342437500-\frac{151250^2}{80}=56480469$ $s=\sqrt{\frac{56480469}{79}}=\sqrt{714943}=846$ Only an estimate since the data are grouped.	M1 A1 M1 A1 E1 indep [5]	For midpoints (at least 3 correct) No marks for mean or sd unless using midpoints Answer must **NOT** be left as improper fraction CAO Accept correct answers for mean (1890 or 1891) and sd (850 or 846 or 845.5) from calculator even if eg wrong S_{xx} given For sum of at least 3 correct multiples fx^2 Allow M1 for anything which rounds to 342400000 Only penalise once in part (iii) for over specification, even if mean and standard deviation both over specified. Allow SC1 for RMSD 840.2 or 840 from calculator Or for any mention of midpoints or 'don't have actual data' or 'data are not exact' oe

Question			Answer	Marks	Guidance
6	(iv)		$\bar{x} - 2s = 1891 - (2 \times 846) = 199$ Allow 200	M1	For either. FT any positive mean and their positive sd/rmsd for M1 Only follow through numerical values, not variables such as *s*, so if a candidate does not find *s* but then writes here 'limit is 40.76+ 2 × standard deviation', do NOT award M1 No marks in (iv) unless using $\bar{x} + 2s$ or $\bar{x} - 2s$
			$\bar{x} + 2s = 1891 + (2 \times 846) = 3583$ Allow 3580 or 3600	A1	For both (FT) Do **NOT** penalise over specification here as it is not the final answer
			So there are probably some outliers	E1 **[3]**	Must include an element of doubt Dep on upper limit in range 3000 – 5000 Allow comments such as 'any value over 3583 is an outlier' Ignore comments about possible outliers at lower end.
6	(v)		Number of cars over 2000 cm^3 = 25/80 × 2.5 million = 781250 So duty raised = 781250 × £1000 = £781 million	M1 M1 indep A1 **[3]**	For 25/80× 2.5 million or (18+7) /80× 2.5 million For something × £1000 even if this is the first step CAO NB £781250000 is over specified so only 2/3
6	(vi)		Because the numbers of cars sold with engine size greater than 2000 cm^3 might be reduced due to the additional duty.	E1 **[1]**	Allow any other reasonable suggestion Condone 'sample may not be representative' Allow 'sample is not of **NEW** cars'

Question			Answer	Marks	Guidance
7	(i)		$P(X = 0) = 0.4 \times 0.5^4 = 0.025$ **NB ANSWER GIVEN**	M1 A1 **[2]**	For 0.5^4
7	(ii)		$P(X = 1) = (0.6 \times 0.5^4) + (4 \times 0.4 \times 0.5 \times 0.5^3)$ $= 0.0375 + 0.1 = 0.1375$ **NB ANSWER GIVEN**	M1* M1* M1* dep A1 **[4]**	For 0.6×0.5^4 seen as a single term (not multiplied or divided by anything) For $4 \times 0.4 \times 0.5^4$ Allow 4×0.025 Watch out for incorrect methods such as (0.4/4) 0.1 **MUST** be justified For sum of both , dep on both M1's
7	(iii)			G1 G1 **[2]**	For labelled linear scales on both axes Dep on attempt at vertical line chart. Accept P on vertical axis For heights – visual check only but last bar taller than first and fifth taller than second and fourth taller than third. Lines must be thin (gap width > line width). All correct. Zero if vertical scale not linear Everything correct but joined up tops G0G1 MAX Everything correct but f poly G0G1 MAX Everything correct but bar chart G0G1 MAX Curve only (no vertical lines) gets G0G0 Best fit line G0G0 Allow transposed diagram

Question			Answer	Marks	Guidance
7	(iv)		'Negative' or 'very slight negative'	E1 **[1]**	E0 for symmetrical but E1 for (very slight) negative skewness even if also mention symmetrical Ignore any reference to unimodal
7	(v)		$E(X) = (0\times0.025) + (1\times0.1375) + (2\times0.3) + (3\times0.325) + (4\times0.175) + (5\times0.0375)$ $= 2.6$ $E(X^2) = (0\times0.025) + (1\times0.1375) + (4\times0.3) + (9\times0.325) + 16\times0.175) + (25\times0.0375) = 0 + 0.1375 + 1.2 + 2.925 + 2.8 + 0.9375 = 8$ $\text{Var}(X) = 8 - 2.6^2$ $= 1.24$	M1 A1 M1* M1* dep A1 **[5]**	For Σrp (at least 3 terms correct) CAO For $\Sigma r^2 p$ (at least 3 terms correct) for – their $E(X)^2$ FT their E(X) provided $\text{Var}(X) > 0$ USE of $E(X-\mu)^2$ gets M1 for attempt at $(x-\mu)^2$ should see $(-2.6)^2$, $(-1.6)^2$, $(-0.6)^2$, 0.4^2, 1.4^2, 2.4^2 (if $E(X)$ correct but FT their $E(X)$) (all 5 correct for M1), then M1 for $\Sigma p(x-\mu)^2$ (at least 3 terms correct) Division by 5 or other spurious value at end gives max M1A1M1M1A0, or M1A0M1M1A0 if $E(X)$ also divided by 5. Unsupported correct answers get 5 marks.
7	(vi)		$P(\text{Total of 3}) = (3\times0.325\times0.025^2) + (6\times0.3\times0.1375\times0.025) + 0.1375^3 = 3\times0.000203 + 6\times0.001031 + 0.002600 =$ $0.000609 + 0.006188 + 0.002600 = 0.00940$ $(= 3\times13/64000 + 6\times33/32000 + 1331/512000)$	M1 M1 M1 A1 **[4]**	For decimal part of first term 0.325×0.025^2 For decimal part of second term $0.3\times0.1375\times0.025$ For third term – ignore extra coefficient All M marks above depend on triple probability products CAO: AWRT 0.0094. Allow 0.009 with working.

NOTE RE OVER-SPECIFICATION OF ANSWERS

If answers are grossly over-specified, deduct the final answer mark in every case. Probabilities should also be rounded to a sensible degree of accuracy. In general final non probability answers should not be given to more than 4 significant figures. Allow probabilities given to 5 sig fig. In general accept answers which are correct to 3 significant figures when given to 4 or 5 significant figures.
If answer given as a fraction and as an over-specified decimal – ignore decimal and mark fraction.

ADDITIONAL NOTES RE Q5

Comparison with 95% method
If 95% seen anywhere then
B1 for P($X \leq 3$)
B1 for 0.9891
M1* for comparison with 95% dep on B1
A1* for significant oe
E1*

Smallest critical region method:

Either:
Smallest critical region that 4 could fall into is {4,5,6,7,8,9,10,11,12,13,14,15, 16, 17, 18} gets B1 and has size 0.0109 gets B1, This is < 5% gets M1*, A1*, E1* as per scheme
NB These marks only awarded if 4 used, not other values.

Use of *k* method with no probabilities quoted:

P($X \geq 3$) = 1 – P($X \leq 2$) > 5%
P($X \geq 4$) = 1 – P($X \leq 3$) < 5%
These may be seen in terms of *k* or *n*.
Either $k = 4$ or $k – 1 = 3$ so $k = 4$ gets SC1
so CR is {4,5,6,7,8,9,10,11,12,13,14,15, 16, 17, 18} gets another SC1and conclusion gets another SC1

Use of *k* method with one probability quoted:

1 - 0.9891 < 5% or 0.0109 < 5% gets B0B1M1
P($X \leq k$ - 1) = P($X \leq 3$)
so $k – 1 = 3$ so $k = 4$ (or just $k = 8$)
so CR is {4,5,6,7,8,9,10,11,12,13,14,15, 16, 17, 18} and conclusion gets A1E1

Two tailed test done but with correct H_1: $p>0.05$
Hyp gets max B1B1B1
if compare with 5% ignore work on lower tail and mark upper tail as per scheme but withhold A1E1
if compare with 2.5% no marks B0B0M0A0E0

Line diagram method
B1 for squiggly line between 3 and 4 or on 4 exclusively (ie just one line), B1*dep* for arrow pointing to right, M1 0.0109 seen on diagram from squiggly line or from 4, A1E1 for correct conclusion

Bar chart method
B1 for line clearly on boundary between 3 and 4 or within 4 block exclusively (ie just one line), B1*dep* for arrow pointing to right, M1 0.0109 seen on diagram from boundary line or from 8, A1E1 for correct conclusion.

Using P(Not faulty) method
H_0: $p=0.95$, H_1: $p<0.95$ where p represents the prob that a frame is faulty gets B1B1B1.
$P(X\leq 14)=0.0109 < 5\%$ So significant, etc gets B1B1M1A1E1

NB
If H_0: $p=0.5$, H_1: $p>0.5$, etc seen, but then revert to 0.05 in working allow marks for correct subsequent working. However if 0.5 used consistently throughout, then max B1 for definition of p and possibly B1 for notation $P(X \geq 4)$.