

# 4766 Statistics 1

## Section A

<b>Q1</b> (i)	<p>(With <math>\sum fx = 7500</math> and <math>\sum f = 10000</math> then arriving at the mean)</p> <p>(i) £0.75 scores (B1, B1)</p> <p>(ii) 75p scores (B1, B1)</p> <p>(iii) 0.75p scores (B1, B0) (incorrect units)</p> <p>(iv) £75 scores (B1, B0) (incorrect units)</p> <p><b>After B0, B0</b> then sight of <math>\frac{7500}{10000}</math> scores SC1. SC1 or an answer in the range £0.74 - £0.76 or 74p – 76p (both inclusive) scores SC1 (units essential to gain this mark)</p> <p><u>Standard Deviation: (CARE NEEDED here with close proximity of answers)</u></p> <ul style="list-style-type: none"> <li>• 50.2(0) using divisor 9999 scores B2 (50.20148921)</li> <li>• 50.198 (= 50.2) using divisor 10000 scores B1 (<i>rmsd</i>)</li> <li>• If divisor is <u>not</u> shown (or calc used) and only an answer of 50.2 (i.e. <u>not</u> coming from 50.198) is seen then award B2 on b.o.d. (default)</li> </ul> <p><b>After B0 scored</b> then an attempt at <math>S_{xx}</math> as evident by either</p> $S_{xx} = (5000 + 200000 + 25000000) - \frac{7500^2}{10000} (= 25199375)$ <p style="text-align: center;">or</p> $S_{xx} = (5000 + 200000 + 25000000) - 10000(0.75)^2$ <p style="text-align: center;"><b>scores (M1) or M1ft ‘their 7500<sup>2</sup>’ or ‘their 0.75<sup>2</sup>’</b></p> <p>NB The <u>structure</u> must be correct in both above cases with a max of <u>1 slip only after applying the f.t.</u></p>	<p>B1 for numerical mean (0.75 or 75 seen) B1dep for correct units attached</p> <p>B2 correct s.d. (B1) correct rmsd</p> <p>(B2) default</p> <p><math>\sum fx^2 = 25,205,000</math></p> <p><b>Beware</b> <math>\sum x^2 = 25,010,100</math></p> <p><b>After B0 scored</b> then (M1) or M1f.t. for attempt at <math>S_{xx}</math></p> <p><i>NB full marks for correct results from recommended method which is use of calculator functions</i></p>	<b>4</b>
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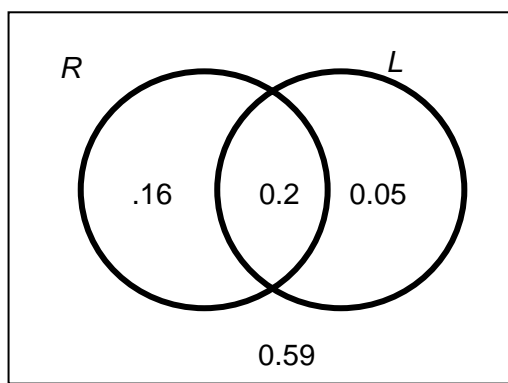
<b>(ii)</b>	<p>P(Two £10 or two £100)</p> $= \frac{50}{10000} \times \frac{49}{9999} + \frac{20}{10000} \times \frac{19}{9999}$ $= 0.0000245 + 0.0000038 = (0.00002450245 + 0.00000380038)$ $= 0.000028(3) \text{ o.e.} = (0.00002830283)$ <p><u>After M0, M0</u> then <math>\frac{50}{10000} \times \frac{50}{10000} + \frac{20}{10000} \times \frac{20}{10000}</math> o.e.</p> <p>Scores SC1 (ignore final answer but SC1 may be implied by sight of <math>2.9 \times 10^{-5}</math> o.e.)</p> <p>Similarly, <math>\frac{50}{10000} \times \frac{49}{10000} + \frac{20}{10000} \times \frac{19}{10000}</math> scores SC1</p>	<p>M1 for either correct product seen (ignore any multipliers)</p> <p>M1 sum of both correct (ignore any multipliers)</p> <p>A1 CAO (as opposite with no rounding)</p> <p>(SC1 case #1)</p> <p>(SC1 case #2) <u>CARE</u> answer is also <math>2.83 \times 10^{-5}</math></p>	<b>3</b>
<b>TOTAL</b>		<b>7</b>	
<b>Q2 (i)</b>	<p>Either <math>P(\text{all correct}) = \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{1} = \frac{1}{720}</math></p> <p>or <math>P(\text{all correct}) = \frac{1}{6!} = \frac{1}{720} = 0.00139</math></p>	<p>M1 for 6! Or 720 (sioc) or product of fractions</p> <p>A1 CAO (accept 0.0014)</p>	<b>2</b>
<b>(ii)</b>	<p>Either <math>P(\text{picks T, O, M}) = \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = \frac{1}{20}</math></p> <p>or <math>P(\text{picks T, O, M}) = \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! = \frac{1}{20}</math></p> <p>or <math>P(\text{picks T, O, M}) = \frac{1}{\binom{6}{3}} = \frac{1}{20}</math></p>	<p>M1 for denominators</p> <p>M1 for numerators or 3!</p> <p>A1 CAO</p> <p>Or M1 for <math>\binom{6}{3}</math> or 20 <u>sioc</u></p> <p>M1 for <math>1/\binom{6}{3}</math></p> <p>A1 CAO</p>	<b>3</b>
<b>TOTAL</b>		<b>5</b>	
<b>Q3 (i)</b>	<p><math>p = 0.55</math></p>	<p>B1 cao</p>	<b>1</b>
<b>(ii)</b>	<p><math>E(X) = 0 \times 0.55 + 1 \times 0.1 + 2 \times 0.05 + 3 \times 0.05 + 4 \times 0.25 = 1.35</math></p> <p><math>E(X^2) = 0 \times 0.55 + 1 \times 0.1 + 4 \times 0.05 + 9 \times 0.05 + 16 \times 0.25 = 0 + 0.1 + 0.2 + 0.45 + 4 = (4.75)</math></p> <p><math>\text{Var}(X) = \text{'their'} 4.75 - 1.35^2 = 2.9275 \text{ awfw } (2.9275 - 2.93)</math></p>	<p>M1 for <math>\sum rp</math> (at least 3 non zero terms correct)</p> <p>A1 CAO (no 'n' or 'n-1' divisors)</p> <p>M1 for <math>\sum r^2 p</math> (at least 3 non zero terms correct)</p> <p>M1 dep for – their <math>E(X)^2</math> provided <math>\text{Var}(X) &gt; 0</math></p> <p>A1 cao (no 'n' or 'n-1' divisors)</p>	<b>5</b>
<b>(iii)</b>	<p><math>P(\text{At least 2 both times}) = (0.05 + 0.05 + 0.25)^2 = 0.1225 \text{ o.e.}</math></p>	<p>M1 for <math>(0.05 + 0.05 + 0.25)^2</math> or <math>0.35^2</math> seen</p> <p>A1 cao: awfw <math>(0.1225 - 0.123)</math> or <math>49/400</math></p>	<b>2</b>

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		TOTAL	<b>8</b>
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<p><b>Q4</b> (i)</p>	<p><math>X \sim B(50, 0.03)</math></p> <p>(A) <math>P(X = 1) = \binom{50}{1} \times 0.03 \times 0.97^{49} = 0.3372</math></p> <p>(B) <math>P(X = 0) = 0.97^{50} = 0.2181</math>  <math>P(X &gt; 1) = 1 - 0.2181 - 0.3372 = 0.4447</math></p>	<p>M1 <math>0.03 \times 0.97^{49}</math> or <math>0.0067(4)\dots</math>                      M1 <math>\binom{50}{1} \times pq^{49}</math> (<math>p+q=1</math>)                      A1 CAO                      (awfw 0.337 to 0.3372)                      or                      0.34(2s.f.) or 0.34(2d.p.)                      but not just 0.34</p> <p>B1 for <math>0.97^{50}</math> or 0.2181                      (awfw 0.218 to 0.2181)                      M1 for  <math>1 - (\text{'their' } p(X=0) + \text{'their' } p(X=1))</math>                      must have both probabilities                      A1 CAO                      (awfw 0.4447 to 0.445)</p>	<p><b>3</b></p> <p><b>3</b></p>
<p>(ii)</p>	<p>Expected number = <math>np = 240 \times 0.3372 = 80.88 - 80.93 = (81)</math>  <i>Condone <math>240 \times 0.34 = 81.6 = (82)</math> but for M1 A1ft.</i></p>	<p>M1 for <math>240 \times \text{prob (A)}</math>                      A1FT</p>	<p><b>2</b></p>
<p>TOTAL</p>			<p><b>8</b></p>
<p><b>Q5</b> (i)</p>	<p><math>P(R) \times P(L) = 0.36 \times 0.25 = 0.09 \neq P(R \cap L)</math>                      Not equal so not independent. (Allow <math>0.36 \times 0.25 \neq 0.2</math> or <math>0.09 \neq 0.2</math> or <math>\neq p(R \cap L)</math> so not independent)</p>	<p>M1 for <math>0.36 \times 0.25</math> or <math>0.09</math> seen                      A1 (numerical justification needed)</p>	<p><b>2</b></p>
<p>(ii)</p>		<p>G1 for two overlapping circles labelled</p> <p>G1 for 0.2 and either 0.16 or 0.05 in the correct places</p> <p>G1 for all 4 correct probs in the correct places (including the 0.59)</p> <p>The last two G marks are independent of the labels</p>	<p><b>3</b></p>
<p>(iii)</p>	<p><math display="block">P(L R) = \frac{P(L \cap R)}{P(R)} = \frac{0.2}{0.36} = \frac{5}{9} = 0.556 \text{ (awrt 0.56)}</math></p> <p>This is the probability that Anna is late given that it is raining. (must be in context)                      Condone 'if' or 'when' or 'on a rainy day' for 'given that' but <b>not</b> the words 'and' or 'because' or 'due to'</p>	<p>M1 for <math>0.2/0.36</math> o.e.                      A1 cao</p> <p>E1 (indep of M1A1)  <b>Order/structure must be correct i.e. no reverse statement</b></p>	<p><b>3</b></p>
<p>TOTAL</p>			<p><b>8</b></p>

## Section B

<b>Q6</b> <b>(i)</b>	Median = 4.06 – 4.075 (inclusive)  $Q_1 = 3.8$ $Q_3 = 4.3$  Inter-quartile range = $4.3 - 3.8 = 0.5$	B1cao  B1 for $Q_1$ (cao) B1 for $Q_3$ (cao)  B1 ft for IQR must be using t-values not locations to earn this mark	<b>4</b>
<b>(ii)</b>	Lower limit ‘their 3.8’ – $1.5 \times$ ‘their 0.5’ = (3.05) Upper limit ‘their 4.3’ + $1.5 \times$ ‘their 0.5’ = (5.05) Very few if any temperatures <u>below 3.05 (but not zero)</u> None <u>above 5.05</u> ‘So few, if any outliers’ scores SC1	B1ft: must have -1.5 B1ft: must have +1.5 E1ft dep on -1.5 and $Q_1$ E1ft dep on +1.5 and $Q_3$  Again, must be using t-values NOT locations to earn these 4 marks	<b>4</b>
<b>(iii)</b>	Valid argument such as ‘Probably not, because there is nothing to suggest that they are not genuine data items; (they do not appear to form a separate pool of data.)’ Accept: exclude outlier – ‘measuring equipment was wrong’ or ‘there was a power cut’ or ref to hot / cold day [Allow suitable valid alternative arguments]	E1	<b>1</b>
<b>(iv)</b>	Missing frequencies 25, 125, 50	B1, B1, B1 (all cao)	<b>3</b>
<b>(v)</b>	Mean = $(3.2 \times 25 + 3.6 \times 125 + 4.0 \times 243 + 4.4 \times 157 + 4.8 \times 50) / 600$  $= 2432.8 / 600 = 4.05(47)$	M1 for at least 4 midpoints correct and being used in attempt to find $\sum ft$  A1cao: awfw (4.05 – 4.055) ISW or rounding	<b>2</b>
<b>(vi)</b>	New mean = $1.8 \times$ ‘their 4.05(47)’ + 32 = 39.29(84) to 39.3 New s = $1.8 \times 0.379$ $= 0.682$	B1 FT M1 for $1.8 \times 0.379$ A1 CAO awfw (0.68 – 0.6822)	<b>3</b>
		TOTAL	<b>17</b>

<p><b>Q7</b> <b>(i)</b></p>	<p><math>X \sim B(10, 0.8)</math></p> <p><b>(A) Either</b> <math>P(X = 8) = \binom{10}{8} \times 0.8^8 \times 0.2^2 = 0.3020</math> (awrt)</p> <p><i>or</i> <math>P(X = 8) = P(X \leq 8) - P(X \leq 7)</math> <math>= 0.6242 - 0.3222 = 0.3020</math></p> <p><b>(B) Either</b> <math>P(X \geq 8) = 1 - P(X \leq 7)</math> <math>= 1 - 0.3222 = 0.6778</math></p> <p><i>or</i> <math>P(X \geq 8) = P(X = 8) + P(X = 9) + P(X = 10)</math> <math>= 0.3020 + 0.2684 + 0.1074 = 0.6778</math></p>	<p>M1 <math>0.8^8 \times 0.2^2</math> or 0.00671...</p> <p>M1 <math>\binom{10}{8} \times p^8 q^2</math>; (p+q=1) Or <math>45 \times p^8 q^2</math>; (p+q=1) A1 CAO (<b>0.302</b>) not 0.3</p> <p>OR: M2 for 0.6242 – 0.3222 A1 CAO</p> <p>M1 for 1 – 0.3222 (s.o.i.) A1 CAO awfw 0.677 – 0.678 or M1 for sum of ‘their’ p(X=8) plus correct expressions for p(x=9) and p(X=10)</p> <p>A1 CAO awfw 0.677 – 0.678</p>	<p><b>3</b></p> <p><b>2</b></p>
<p><b>(ii)</b></p>	<p>Let <math>X \sim B(18, p)</math> Let <math>p</math> = probability of delivery (within 24 hours) (for population)</p> <p><math>H_0: p = 0.8</math> <math>H_1: p &lt; 0.8</math></p> <p><math>P(X \leq 12) = 0.1329 &gt; 5\%</math> ref: [pp =0.0816]</p> <p>So not enough evidence to reject <math>H_0</math></p> <p>Conclude that there is not enough evidence to indicate that less than 80% of orders will be delivered within 24 hours</p> <p>Note: use of critical region method scores M1 for region {0,1,2,...,9, 10} M1dep for 12 does not lie in critical region then A1dep E1dep as per scheme</p>	<p>B1 for definition of <math>p</math></p> <p>B1 for <math>H_0</math> B1 for <math>H_1</math></p> <p>M1 for probability 0.1329</p> <p>M1dep strictly for comparison of 0.1329 with 5% (seen or clearly implied)</p> <p>A1dep on both M’s</p> <p>E1dep on M1,M1,A1 for conclusion in context</p>	<p><b>7</b></p>

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(iii)	<p>Let <math>X \sim B(18, 0.8)</math>  <math>H_1: p \neq 0.8</math>  <b>LOWER TAIL</b>  <math>P(X \leq 10) = 0.0163 &lt; 2.5\%</math>  <math>P(X \leq 11) = 0.0513 &gt; 2.5\%</math></p> <p><b>UPPER TAIL</b>  <math>P(X \geq 17) = 1 - P(X \leq 16) = 1 - 0.9009 = 0.0991 &gt; 2.5\%</math>  <math>P(X \geq 18) = 1 - P(X \leq 17) = 1 - 0.9820 = 0.0180 &lt; 2.5\%</math></p> <p>So critical region is <math>\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 18\}</math> o.e.  Condone <math>X \leq 10</math> and <math>X \geq 18</math> or <math>X = 18</math> but <b>not</b> <math>p(X \leq 10)</math> and <math>p(X \geq 18)</math>  Correct CR without supportive working scores SC2 max after the 1<sup>st</sup> B1 (SC1 for each fully correct tail of CR)</p>	<p>B1 for <math>H_1</math></p> <p>B1 for 0.0163 or 0.0513 seen</p> <p>M1dep for either correct comparison with <b>2.5%</b> (<b>not 5%</b>) (seen or clearly implied)</p> <p>A1dep for correct lower tail CR (must have zero)</p> <p>B1 for 0.0991 or 0.0180 seen</p> <p>M1dep for either correct comparison with <b>2.5%</b> (<b>not 5%</b>) (seen or clearly implied)</p> <p>A1dep for correct upper tail CR</p>	<p><b>7</b></p>
		TOTAL	<b>19</b>