MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

STATISTICS 1, S1

Practice Paper S1-A

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You **may** use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that you may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 The box-and-whisker plot in Fig. 1 illustrates the scores, out of 80, of 120 people in a diving competition.

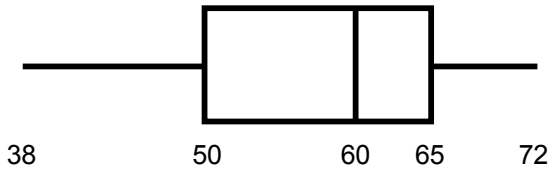


Fig. 1

Draw a cumulative frequency graph to illustrate these data. [4]

2 100 people attend a music festival. They are asked which, if any, of the instruments piano, cello, violin they play.

Their answers are illustrated in Fig. 2.

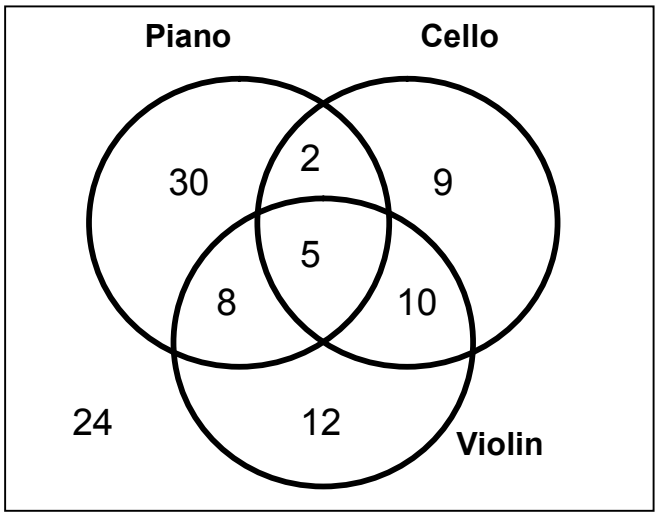


Fig. 2

A person is chosen at random from those attending the festival and asked which of the three instruments he or she plays.

Find the probability that this person plays

(**i**) the piano,

(**ii**) exactly one of the other instruments given that he or she plays the piano. [4]

3 In a year group of three classes the distribution of sexes is given in the table below.

	Class 1	Class 2	Class 3
Males	10	11	9
Females	15	9	9

Three students are selected, one from each class, at random.

Find the probability that

(i) all 3 are male, [2]

(ii) only one is male. [3]

4 A train company runs a non-stop service from Oxbridge to Camford. The numbers of passengers on the 07:30 service on 20 weekdays were as follows.

184	193	195	189	173
175	171	178	174	163
184	162	171	154	199
217	187	169	183	186

(i) Calculate the median and the inter-quartile range. [3]

(ii) Using the inter-quartile range, show that there is just one outlier. Find the effect of its removal on the median and the inter-quartile range. [4]

5 A random sample of cyclists were asked how many days they had used their bicycles in the last week. The results are given in the following table.

Number of days (x)	0	1	2	3	4	5	6	7
Frequency (f)	15	10	9	5	7	24	8	2

(i) Illustrate the distribution using a suitable diagram and describe its shape. [3]

(ii) Calculate the mean and the standard deviation, s, of the data. Give your answers to 4 decimal places. [3]

(iii) As a reward for taking part in the survey, the cyclists' names are entered for a draw. There are 3 identical prizes. In how many ways can the 3 winners be chosen ? [2]

6 In one turn of the game of *Polopoly* a player throws three ordinary dice, the score being the largest of the numbers appearing face up. The score, X, is given by the probability distribution given in the following table.

r	1	2	3	4	5	6
$P(X = r)$	$\frac{1}{216}$	$\frac{7}{216}$	$\frac{19}{216}$	$\frac{37}{216}$	$\frac{61}{216}$	$\frac{91}{216}$

(i) Find $E(X)$ and $Var(X)$. [4]

(ii) Find the probability that the player will score a total of exactly 10 in two turns. [4]

Section B (36 marks)

7 A survey is conducted to find which type of property people live in and whether the property is owned or rented by its occupier. The results for a particular region of the country are as follows.

Type of Property	Proportion of each type	Proportion of properties	
		Owned	Rented
Detached / semi-detached	45%	75%	25%
Terraced house	35%	50%	50%
Flat / bedsit	20%	35%	65%

A property is chosen at random.

(i) Construct a tree diagram to represent the information in the table. [3]

(ii) Find the probability that the property is owned. [3]

(iii) Find the probability that the property is a terraced house or rented. [4]

(iv) Given that the property is owned, calculate the probability that it is a terraced house. [3]

Two properties are now chosen at random.

(v) Find the probability that they are

(*A*) of the same type,

(*B*) of different types. [5]

8 Phil likes rifle shooting at an amusement arcade. He reckons that he can hit the target on 3 out of 4 shots on average. Each "go" at the amusement arcade consists of 10 independent shots at a moving target. A prize is awarded if at least 9 shots hit the target.

(i) Show that the probability that Phil wins a prize in one "go" is 0.244, correct to 3 significant figures. [2]

(ii) Phil has 3 "goes". Find the probability that he wins

(*A*) exactly one prize,

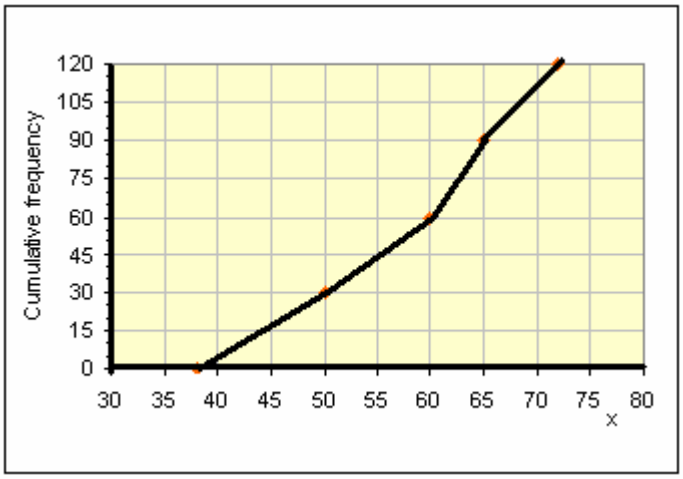
(*B*) at least one prize. [6]

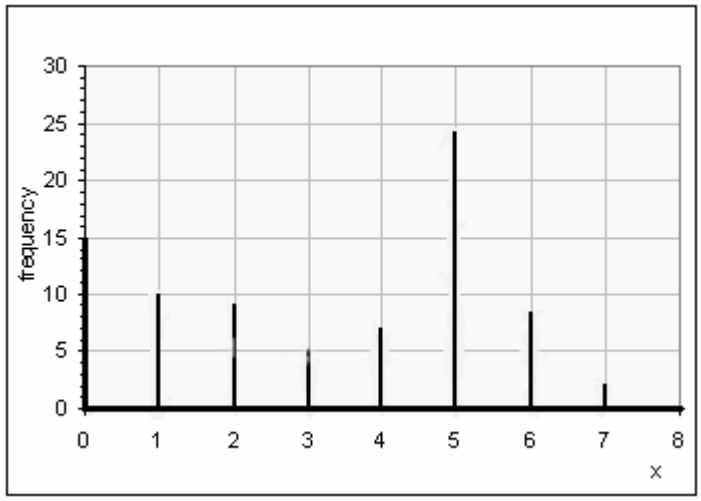
(iii) How many "goes" does Phil need to have so that the probability of winning at least one prize is more than 90% ? [4]

Val is less experienced at rifle shooting. She thinks that she has an even chance of hitting the target with one shot. Phil thinks that she has a better chance of hitting the target. He conducts a hypothesis test at the 10% significance level by getting Val to have 10 shots at the target.

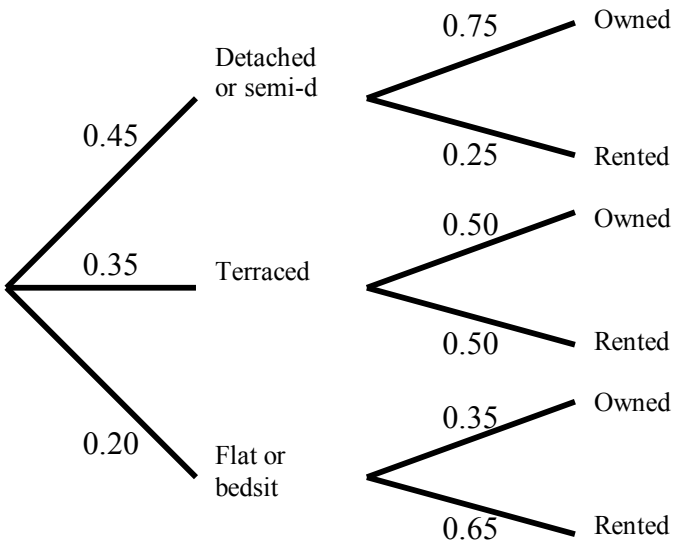
(iv) Write down suitable hypotheses for this test in terms of p, the probability that Val hits the target, giving a reason for your alternative hypothesis. [3]

(v) Find the least number of times Val should hit the target to suggest that Phil is correct. [3]

Qu		Answer	Mark	Comment
Section A				
1			G1 G1 G1 G1 **4**	Correctly scaled axes, with attempted ogive. Maximum & minimum points plotted Median plotted Quartiles plotted *Curve or line segments accepted*
2	**(i)**	P(plays piano) $= \frac{45}{100}$ *or* 0.45	M1 A1 **2**	For (30 + 8 + 5 + 2)
	(ii)	P(plays one other instrument | plays piano) $= \frac{10}{45} = \frac{2}{9}$	M1 A1 **2**	For $\frac{n}{45}$
3	**(i)**	P(all 3 male) $= \frac{10}{25}\times\frac{11}{20}\times\frac{9}{18} = \frac{11}{100}$ *or* 0.11	M1 A1 **2**	Product of 3 terms
	(ii)	P(1 male) $= \frac{10}{25}\times\frac{9}{20}\times\frac{9}{18}+\frac{15}{25}\times\frac{11}{20}\times\frac{9}{18}+\frac{15}{25}\times\frac{9}{20}\times\frac{9}{18}$ $= \frac{39}{100}$ *or* 0.39	M1 M1 A1 **3**	Product of 3 terms Digits correct on top of at least one
4	**(i)**	Median = 180.5 Inter-quartile range = 188 – 171 = 17 [*or* = 188.5 – 171 = 17.5]	B1 M1 A1 **3**	For median For sensible attempt at finding IQR
	(ii)	$Q_1 - 1.5\times IQR = 171 - 1.5\times 17 = 145.5$ $Q_3 + 1.5\times IQR = 188 + 1.5\times 17 = 213.5$ Hence only data item outside the interval [145.5, 213.5] is 217. If 217 is removed, median drops to 178 IQR becomes 187 – 171 = 16 *or* 187.5 – 171 = 16.5 *or* 186.25 – 170.5 = 15.75	E1 E1 B1 B1 **4**	For showing 217 is > 1.5 ×IQR above Q_3 For showing there are no values < 1.5× IQR below Q_1 For effect on median For effect on IQR

5	**(i)**	 Distribution is bimodal	G1 G1 B1 **3**	For linear scales on both axes For heights of lines of vertical line chart For comment
	(ii)	Mean $= \frac{253}{80} = 3.1625$ days (to 4 d.p.) Standard deviation $= \sqrt{\frac{1189 - 80 \times 3.1625^2}{79}}$ $= \sqrt{4.922626582} = 2.2187$ (to 4 d.p.)	B1 M1 A1 **3**	For mean For variance
	(iii)	Number of ways of choosing the 3 winners $= {}^{80}C_3 = 82160$	M1 A1 **2**	For ${}^{n}C_3$
6	**(i)**	$E(X) = \Sigma r P(X = r) = \frac{1}{216}(1 \times 1 + 2 \times 7 + ... + 6 \times 91)$ $= \frac{1071}{216} = 4.96$ (to 3 s.f.) $\Sigma r^2 P(X = r) = \frac{1}{216}(12 \times 1 + 22 \times 7 + ... + 62 \times 91)$ $= \frac{5593}{216}$ $\Rightarrow$ Var(X) $= \frac{5593}{216} - \left(\frac{1071}{216}\right)^2 = 1.31$ (to 3 s.f.)	M1 A1 M1 A1 **4**	For $\Sigma r P(X = r)$ For $\Sigma r^2 P(X = r)$
	(ii)	P(score exactly 10 in 2 turns) = P(4, 6) + P(5, 5) + P(6, 4) $= \frac{37}{216} \times \frac{91}{216} + \frac{61}{216} \times \frac{61}{216} + \frac{91}{216} \times \frac{37}{216}$ = 0.224 (to 3 s.f.)	M1 M1 M1 A1 4	For $\geq$ 2 pairs **soi** For a product of 2 correct probabilities For sum of 3 correct products

Total = 36

Qu		Answer	Mark	Comment
Section B				
7	**(i)**		B1 B1 B1 **3**	For overall structure For 1st set branches For 2nd set branches
	(ii)	P(property is owned) $= 0.45 \times 0.75 + 0.35 \times 0.50 + 0.20 \times 0.35$ $= 0.5825$	M1 M1 A1 **3**	For one product For sum of 3 prods
	(iii)	P(property terraced or rented) = P(terraced) + P(rented) – P(terraced and rented) $= 0.35 + (1 - 0.5825) - 0.35 \times 0.50$ $= 0.5925$ *or* $0.45 \times 0.25 + 0.35 + 0.20 \times 0.65 = 0.5925$	M1 M1 A1 A1 *or* M1 A1 M1 A1 **4**	For "addition law" for terms *or* For 2 products For sum
	(iv)	P(property terraced | owned) $= \dfrac{\text{P(property terraced and owned)}}{\text{P(property owned)}}$ $= \dfrac{0.35 \times 0.5}{0.5825} = 0.30$ (2 s.f.)	M1 M1 A1 **3**	For numerator For quotient
	(v)	P(each is the same type of property) $= 0.45^2 + 0.35^2 + 0.20^2$ $= 0.365$ P(each is a different type of property) = 1 – 0.365 = 0.635 [*or* $2 \times 0.45 \times 0.35 + 2 \times 0.45 \times 0.20 + 2 \times 0.35 \times 0.20$ $= 0.635$]	M1 M1 A1 M1 A1 **5**	For "p^2" For sum of 3 squares For "1 – their 0.365"

8	**(i)**	[Let $X \sim \mathrm{B}(10, 0.75)$] P(Phil wins a prize) = $\mathrm{P}(X \geq 9) = 1 - \mathrm{P}(X \leq 8) = 1 - 0.7560$ [*or* $= 10 \times 0.75^9 \times 0.25 + 0.75^{10}$ $= 0.1877\ldots + 0.0563\ldots$] $\mathrm{P}(X \geq 9) = 0.244$ (to 3 s.f.)	M1 A1 **2**	For use of tables
	(ii)(A)	[Let $Y \sim \mathrm{B}(3, 0.244)$] $\mathrm{P}(Y = 1) = 3 \times 0.244 \times 0.756^2$ $= 0.418$ (3 s.f.)	M1 M1 A1 **3**	For "0.244×0.756^2" For "$3 \times p \times q^2$"
	(ii)(B)	$\mathrm{P}(Y \geq 1) = 1 - \mathrm{P}(Y = 0) = 1 - 0.756^3$ $= 0.568$ (3 s.f.)	M1 M1 A1 **3**	For "0.756^3" For "$1 - p^3$"
	(iii)	[Let n represent the number of goes, then] Require $1 - 0.756^n > 0.9 \Rightarrow 0.756^n < 0.10$ *By trial:* $1 - 0.756^8 = 0.893 < 0.90$ $1 - 0.756^9 = 0.919 > 0.90$ *or by logs:* $n \log(0.756) < \log(0.10)$ $\Rightarrow \quad n > \dfrac{\log(0.10)}{\log(0.756)} = 8.23$ hence Phil needs to have 9 goes.	M1 M1 M1 A1 **4**	For "$1 - 0.756^3$" For inequality For attempt at solving inequality
	(iv)	H_0: $p = 0.5$ H_1: $p > 0.5$ since we want to see if Val is more likely to hit the target than not.	B1 B1 E1 **3**	For null hypothesis For alternative hypothesis For reason
	(v)	Using binomial tables for $n = 10$: $\mathrm{P}(X \geq 7) = 1 - \mathrm{P}(X \leq 6) = 1 - 0.8281$ $= 0.1719 > 0.10$ $\mathrm{P}(X \geq 8) = 1 - \mathrm{P}(X \leq 7) = 1 - 0.9453$ $= 0.0547 < 0.10$ So Val should hit the target at least 8 times.	M1 M1 A1 **3**	For one comparison For 2nd comparison

Total = 36