S1 Specimen (MA)

Q1a) A statistical process created in order to make predictions about the behaviour of a real world scenario / problem.

b) Number shown on a fair die after rolling.

ii) The height of adult males.

Q2) $T \sim N(\mu, \sigma^2)$

$P(T > 55) = 0.05$

$P\left(Z > \frac{55-\mu}{\sigma}\right) = 0.05$

$\Rightarrow \frac{55-\mu}{\sigma} = 1.6449$ // (from tables).

$P(T < 10) = 0.09$ $\quad (0.1\% \Rightarrow 0.001)$

$P\left(Z < \frac{10-\mu}{\sigma}\right) = 0.001$

$\Rightarrow -\frac{(10-\mu)}{\sigma} = 3.0902$ //

$\mu - 10 = 3.0902\sigma$ ~ ①

$55 - \mu = 1.6449\sigma$ ~ ②

① + ② : $45 = (3.0902 + 1.6449)\sigma$

$$\therefore \sigma = \frac{45}{3.0902 + 1.6449} = \boxed{9.50}$$

and $\mu = \boxed{39.4}$ (sub back σ into ①/②).

(Q3a) $\sum P(X = x) = 1$; $k(1+2+3+4+5) = 1$

$$\therefore k = \frac{1}{15}$$

□

b) $E(2X+3) = 2E(X) + 3 = 2\left(\frac{11}{3}\right) + 3 = \boxed{\frac{31}{3}}$

$\left[E(X) = \frac{1}{15}(1 + 2(2) + 3(3) + 4(4) + 5(5)) = \frac{11}{3}//\right]$

c) $E(X^2) = \frac{1}{15}(1 + 4(2) + 9(3) + 16(4) + 25(5))$

$= 15//$

$$Var(X) = E(X^2) - (E(X))^2 = 15 - \left(\frac{11}{3}\right)^2 = \frac{14}{9}$$

$$\therefore Var(2X-4) = 4Var(X) = 4 \times \frac{14}{9} = \boxed{\frac{56}{9}}$$

Q4a) $S_{xy} = (484) - \frac{(143)(391)}{15} = -\frac{48653}{15}$

$S_{xx} = 2413 - \frac{(143)^2}{15} = \frac{15746}{15}$

$b = \frac{S_{xy}}{S_{xx}} = \frac{-48653}{15746} = \boxed{-3.09}$

$a = \bar{y} - b\bar{x} = \frac{391}{15} - (-3.09)\left(\frac{143}{15}\right) = \boxed{55.524}$

$\therefore y = 55.52 - 3.09x$ //

but $y = h - 100$, $x = s - 20$

$\Rightarrow h - 100 = 55.52 - 3.09(s - 20)$

$\Rightarrow \boxed{h = 217.32 - 3.09s}$

b) For every extra RPM, the lifetime of the drill reduces by $\simeq$ 3 hours.

c) $s = 30$: $h = 217.32 - 3.09(30) = \boxed{124.6}$ hours.

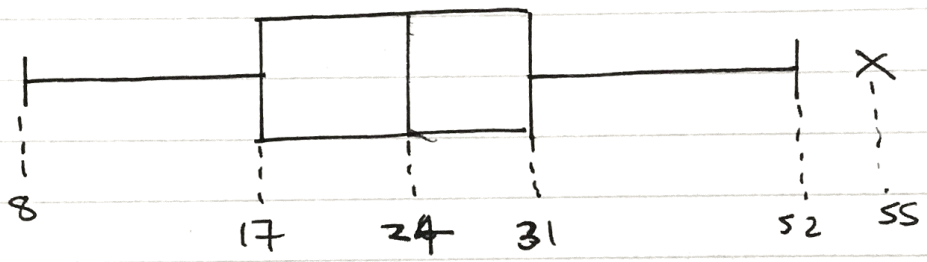
(Q5a) ADV : - not affected by extreme values.
- provide another measure of spread rather than variance (ie IQR).

DISADV : - Not all data is directly used.
- Not always easy to work out, (eg with grouped freq. distributions interpolation is required.)

b) - shows maximum / minimum values.
- shows skewness.
- shows values of all Quartiles.

c) $Q_1 - 1.5(Q_3 - Q_1) = -4$ //

$Q_2 + 1.5(Q_3 - Q_1) = 52$ //



d) - median for B > median for A
24 > 22
(Teaches from B on avg. travel further).

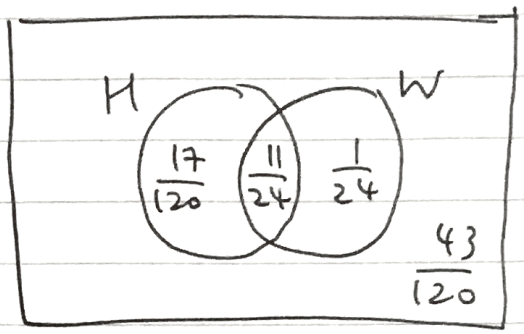
- Range of B > Range of A

- Both distributions are symmetrical.
$Q_3 - Q_2 = Q_2 - Q_1$ for both.

Q6a) $P(H \cap W) = P(H|W) \times P(W)$

$\therefore P(H \cap W) = \frac{11}{12} \times \frac{1}{2} = \boxed{\frac{11}{24}}$

b)



$1 - \left(\frac{17}{120} + \frac{11}{24} + \frac{1}{24}\right) = \frac{43}{120}$

$\frac{3}{5} - \frac{11}{24} = \frac{17}{120}$

$\frac{1}{2} - \frac{11}{24} = \frac{1}{24}$

c) $\frac{17}{120} + \frac{1}{24} = \boxed{\frac{11}{60}}$

d) $\frac{43}{120}$.

e) Possibilities : Couple 1 Couple 2

equal probability
- (H)(W) (H')(W')
- (H')(W') (H)(W)
- (H)(W') (H')(W) — equal probability
- (H')(W) (H)(W') — equal probability

$P(\text{required}) = \left[\left(\frac{11}{24}\right)\left(\frac{43}{120}\right)\right] \times 2 + \left[\left(\frac{17}{120}\right)\left(\frac{1}{24}\right)\right] \times 2$

$= \boxed{\frac{49}{144}}$