S1 Nov 2002 S1 Solutions

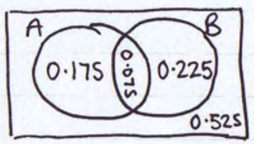
1a) Cheaper / Easier to collect results from a sample. These results can be statistically analysed and providing the sample was unbiased be extended to the whole population.

b) i) Normal distribution   ii) Discrete Uniform Distribution.

2 $P(AAA) = \frac{60}{125} \times \frac{59}{124} \times \frac{58}{123} = 0.108$ (10.8%)

b) $P(AAS) + P(ASA) + P(SAA) = 3 \times P(AAS) = 3 \times \frac{85}{125} \times \frac{84}{124} \times \frac{40}{123} = 0.449$ (45%)

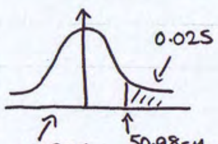
3 If Independent $P(A \cap B) = P(A) \times P(B) = 0.25 \times 0.3 = 0.075$

b) 

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.475$

$P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{0.175}{0.7} = \frac{1}{4} = 0.25$

4 $L \sim N(\mu, 0.5^2)$ $Z = \frac{L-\mu}{\sigma}$



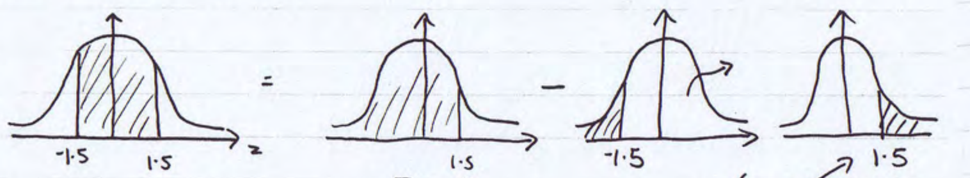
$P(L > 50.98) \Rightarrow P(Z > \frac{50.98-\mu}{0.5}) = 0.025$

$\Rightarrow P(Z < \frac{50.98-\mu}{0.5}) = 0.975 = \Phi(1.96)$

$\Rightarrow \frac{50.98-\mu}{0.5} = 1.96 \Rightarrow \mu = 50.98 - \frac{1.96}{2} = \underline{50}$ ✓

b) $P(49.25 < L < 50.75) \Rightarrow P(\frac{49.25-50}{0.5} < Z < \frac{50.75-50}{0.5})$

$\Rightarrow P(-1.5 < Z < 1.5)$



$= 2\Phi(1.5) - 1 = 0.8664$

So $P(L > 50.75)$ or $P(L < 49.25) = 1 - 0.8664 = 0.1336$

$P(2 \text{ Unusable}) = 0.1336 \times 0.1336 = 0.0178$ (1.8%)

5) $n = 8$, $\bar{s} = \frac{\sum s}{n} = 6.06$ $\bar{t} = \frac{\sum t}{n} = 8.125$

$S_{ss} = \sum s^2 - \frac{(\sum s)^2}{n} = 108.08$ and $S_{tt} = 173.68$, $S_{st} = 129.17$

$y = a + bx$, $b = \frac{S_{xy}}{S_{xx}}$ $a = \bar{y} - b\bar{x}$ so $t = p + qs \Rightarrow q = \frac{S_{st}}{S_{ss}} = 1.195$

$p = \bar{t} - q\bar{s} \Rightarrow p = 0.88$

$t = 0.88 + 1.195s$

b) $s = x - 6$, $y - 20 = t$ $\Rightarrow$ $y - 20 = 0.88 + 1.195(x-6) \Rightarrow y = 20.88 + 1.195x - 7.17$

$y = 13.71 + 1.195x$

c) PMCC for x and y is the same as s and t since to go from $(x,y) \to (s,t)$ each point is moved 6 left and 20 down, i.e gradient of the least squares regression line will be identical.

$r = \frac{S_{st}}{\sqrt{S_{ss}S_{tt}}} = 0.943$

6

x	-2	-1	0	1	2
$P(X=x)$	α	0.2	0.1	0.2	β

$\sum P(X=x) = 1 \Rightarrow \alpha + 0.2 + 0.1 + 0.2 + \beta = 1$ $\Rightarrow \alpha + \beta = 0.5$

a) $E(X) = -2\alpha - 0.2 + 0.2 + 2\beta = -0.2$

$\Rightarrow \alpha - \beta = 0.1$

$\Rightarrow \underline{\alpha = 0.3}$, $\underline{\beta = 0.2}$

b) $F(0.8) = P(-2) + P(-1) + P(0) = \alpha + 0.2 + 0.1 = 0.6$

c) $Var(X) = E(X^2) - [E(X)]^2 \Rightarrow 2.4 - (-0.2)^2 = \underline{2.36}$

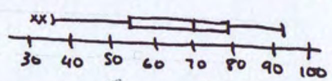
$E(X^2) = 4 \times 0.3 + 1 \times 0.2 + 1 \times 0.2 + 2 \times 0.2 = 2.4$

d) $E(3X-2) = 3E(X) - 2 = \underline{-2.6}$ $Var(2X+6) = 2^2 Var(X) = \underline{9.44}$

7) Mode = $\underline{78}$, $n = 50 \Rightarrow Q_1 = x_{13} = \underline{56}$ $Q_2 = \frac{1}{2}(x_{25} + x_{26}) = \underline{70}$ $Q_3 = x_{38} = \underline{7}$

IQR $= Q_3 - Q_1 = 22$ $Q_1 - 1.0(Q_3 - Q_1) = 34$

$Q_3 + 1.0(Q_3 - Q_1) = 100$



$\sigma^2 = \frac{\sum x^2}{n} - \mu^2 = 242.19 \Rightarrow \sigma = 15.56$ ~~$\bar{x} = \mu$~~ $\mu = \frac{\sum x}{n} = 67.26$

Skew $= \frac{3(\mu - Q_2)}{\sigma} = -0.53$

Weak negative skew

negative skew if $Q_2 - Q_1 > Q_3 - Q_2$

$Q_2 - Q_1 = 14$

$Q_3 - Q_2 = 8$

$14 > 8$ so negative skew